Article

Bianchi Type-II Inflationary Universe with Massive String Source in General Relativity

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Abstract

This study is about an inflationary cosmological model in the presence of scalar field with a flat potential and a massive string source in general relativity in the framework of locally rotational symmetric(LRS) Bianchi type-II space-time. An exact solution of the field equations is determined using the fact that scalar expansion is proportional to shear scalar and a hybrid expansion for the average scale factor of the model. We have determined the scalar field, energy density and tension density of the string in the model. We find all the dynamical parameters of the model and discuss their physical importance.

Keywords: BianchiType-II, inflationary universe, flat potential, general relativity.

1.Introduction

The study of inflationary cosmology is gaining importance , in recent years, because of the recent scenario of accelerated expansion of the universe[1-3]. It is said that when the universe undergoes an exponential expansion, in this scenario, then it is called ‘inflation’ . Many important problems in modern cosmology like homogeneity, the isotropy, flatness of observed universe and primordial monopole problems can be addressed to using inflation. Classical scalar fields, in particular, self interacting scalar fields have an important role in the discussion of inflationary cosmology. cosmic microwave background(CMB) observations have also confirmed the inflationary scenario in general relativity. The inflationary cosmological model was first discussed by Starobinsky[4]. But this scenario had become famous through an interesting paper by Guth[6] which suggested that the inflation is due to false vacuum energy and after the inflation the universe will be filled with bubbles. The concept of Higgs field with potential $v(\varphi)$ plays a vital role in this discussion. Several authors have discussed different aspects of the inflationary universe in general relativity and various aspects of scalar field φ in the evolution of the universe.[7-17].

The exact physical situation of our early universe remains a mystery even today. It is believed that immediately after the big bang, the universe might have undergone symmetry breaking and

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phase transitions resulting in topologically stable defects which are known as strings, domain walls and monopoles(Kibble[18]).Out of these topological defects strings play a vital role in the structure formation of the universe. It is interesting to note that strings can give rise to density perturbations leading to galaxy formation[19-20].The study of gravitational effects of cosmic strings is significant because they possess stress energy and are coupled to gravitational field. Cosmological models with string source, in general relativity, have been investigated by many authors[21-27].

Cosmological models with anisotropic back ground are attracting more and more attention of research workers because of the fact that the recent experimental data as well as theoretical arguments support the existence of anisotropic expansion phase of the universe which evolves into isotropic one at late times. Therefore, spatially homogeneous and anisotropic Bianchi models (I-IX) are important to study the universe in its early stages of evolution. Hence many authors have studied Bianchi models in the presence of different physical sources. In particular, Bali and Jain [28] has discussed inflationary scenario in LRS Bianchi Type I space-time in the presence of mass less scalar field with flat potential. Reddy et al. [29] have investigated inflationary scenario in Kantowski-Sachs space-time while Reddy[30] has discussed Bianchi type-V inflationary universe in general relativity. sKatore et al.[31] have studied Einstein-Rosen inflationary universe in general relativity while Reddy et al.[32] investigated axially symmetric inflationary universe. Reddy and Naidu[33] discussed Kaluza-Klein inflationary universe in general relativity. A higher dimensional Bianchi type-I inflationary universe has been obtained by Katore et al.[34]. Recently, Bali and Kumari[35] have discussed inflationary scenario in Bianchi type-VI_0 space time with flat potential and bulk viscosity in general relativity.

The above discussion inspired us to take up the investigation of inflationary cosmological model in the presence of mass less scalar field with flat potential and a massive string source in LRS Bianchi type-II space time. The work in this article is planned as follows: Section-2 deals with the metric and field equations. In Sec-3 we present the solution of the field equations and the model. Sec-4 is concerned with the physical discussion of the model. The last section contains summary and concluding remarks.

2.Metric and field equations

LRS Bianchi type-II metric can be written in the form

$$ds^2 = -dt^2 + X^2(dx - z\ dy)^2 + Y^2(dy^2 + dz^2) \tag{1}$$

where X and Y are functions cosmic time t only.

The Einstein Field equations (in gravitational units G = c = 1) in case of mass less scalar field φ with potential $v(\varphi)$ are given by

$$R_{ij} - \frac{1}{2} g_{ij} R = -(T_{ij}^{(\varphi)} + T_{ij}^{(s)}) \tag{2}$$

where $T_{ij}^{(\varphi)}$ and $T_{ij}^{(s)}$ are the energy momentum tensors corresponding to mass less scalar field with potential $v(\varphi)$ and for massive string source, respectively, which are given by

$$T_{ij}^{(\varphi)} = \varphi_{,i}\varphi_{,j} - \left(\frac{1}{2}\varphi_{,k}\varphi^{,k} + v(\varphi)\right) \tag{3}$$

$$T_{ij}^{(s)} = \rho u_i u_j - \lambda x_i x_j \tag{4}$$

Here ρ is the energy density of the string cloud, u^i is the four velocity of the string cloud, x^i is the string direction and λ is the tension density of the string. In this case we consider string along x-direction.

Also, we have

$$u^i u_i = -x^i x_j = -1 \tag{5}$$

$$\rho = \rho_p + \lambda \tag{6}$$

where ρ_p is the rest energy density of particles attached to the string and λ , here, may be positive or negative(Letelier[21]).The conservation of energy tensors in Eqs.(3) and (4) lead to

$$\varphi^{,i}_{;i} = -\frac{dv}{d\varphi} \tag{7}$$

$$\dot{\rho} + \rho\left(\frac{\dot{X}}{X} + 2\frac{\dot{Y}}{Y}\right) - \lambda\frac{\dot{X}}{X} = 0 \tag{8}$$

where an over head dot denotes differentiation with respect to cosmic time t.

Now using co moving coordinates explicit Einstein field equations (2) for the metric (1) with the help of Eqs.(3)-(7) can be written as

$$2\frac{\dot{X}}{X}\frac{\dot{Y}}{Y} + \left(\frac{\dot{Y}}{Y}\right)^2 - \frac{1}{4}\frac{X^2}{Y^4} = -\frac{\dot{\varphi}^2}{2} + v(\varphi) + \rho \tag{9}$$

$$2\frac{\ddot{Y}}{Y}+\left(\frac{\dot{Y}}{Y}\right)^2-\frac{3}{4}\frac{X^2}{Y^4}=\frac{\dot{\varphi}^2}{2}+v(\varphi)-\lambda \tag{10}$$

$$\frac{\ddot{X}}{X}+\frac{\ddot{Y}}{Y}+\frac{\dot{X}\dot{Y}}{XY}+\frac{1}{4}\frac{X^2}{Y^4}=\frac{\dot{\varphi}^2}{2}+v(\varphi) \tag{11}$$

$$\ddot{\varphi}+\dot{\varphi}\left(\frac{\dot{X}}{X}+2\frac{\dot{Y}}{Y}\right)=-\frac{dv}{d\varphi} \tag{12}$$

The cosmological parameters of the space-time(1) which are significant in the solution of field equations and in the discussion of cosmology are defined as follows:

The average scale factor $a(t)$ and volume V of the space time are given by

$$V=a^3=XY^2 \tag{13}$$

The average Hubble parameter is

$$H=\frac{1}{3}\left(\frac{\dot{X}}{X}+2\frac{\dot{Y}}{Y}\right) \tag{14}$$

The scalar expansion isS

$$\theta=3H=\left(\frac{\dot{X}}{X}+2\frac{\dot{Y}}{Y}\right) \tag{15}$$

The shear scalar is

$$\sigma^2=\frac{1}{3}\left(\frac{\dot{X}}{X}-\frac{\dot{Y}}{Y}\right)^2 \tag{16}$$

The anisotropy parameter is

$$A_h=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \tag{17}$$

The deceleration parameter is

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1 \tag{18}$$

3. Inflationary model

Here we are interested in inflationary solutions of the field equations (9)-(12), we consider the flat region wherein the potential is constant[6]. i.e.

$$v(\varphi)=\text{cons ta t}=v_0\,(\text{say}) \tag{19}$$

Now using Eq. (19), the field equations (9)-(12) reduce to

$$2\frac{\dot X}{X}\frac{\dot Y}{Y}+\left(\frac{\dot Y}{Y}\right)^2-\frac{1}{4}\frac{X^2}{Y^4}=-\frac{\dot\varphi^2}{2}+v_0+\rho \tag{20}$$

$$2\frac{\ddot Y}{Y}+\left(\frac{\dot Y}{Y}\right)^2-\frac{3}{4}\frac{X^2}{Y^4}=\frac{\dot\varphi^2}{2}+v_0-\lambda \tag{21}$$

$$\frac{\ddot X}{X}+\frac{\ddot Y}{Y}+\frac{\dot X\dot Y}{XY}+\frac{1}{4}\frac{X^2}{Y^4}=\frac{\dot\varphi^2}{2}+v_0 \tag{22}$$

$$\ddot\varphi+\dot\varphi\left(\frac{\dot X}{X}+2\frac{\dot Y}{Y}\right)=0 \tag{23}$$

From the Eqs.(20)-(23) we obtain the following equations:

$$\frac{\ddot X}{X}-\frac{\ddot Y}{Y}+\frac{\dot X\dot Y}{XY}-\left(\frac{\dot Y}{Y}\right)^2+\frac{X^2}{Y^4}=\lambda \tag{24}$$

$$\frac{\ddot X}{X}+\frac{\ddot Y}{Y}+3\frac{\dot X\dot Y}{XY}+\left(\frac{\dot Y}{Y}\right)^2=\rho+v_0 \tag{25}$$

$$\dot\varphi XY^2=\varphi_0 \tag{26}$$

Now Eqs.(24)-(26) are three independent equations [Eq.(8) being the consequence of conservation equation] in five unknowns

$X, Y, \lambda, \rho \text{and} \varphi$.

Hence we need two more conditions to get a determinate solution of the field equations. We use the following physically viable conditions:

(i)The scalar expansion of the space time is proportional to shear scalar which gives a relation between the metric potentials [36] given by

$$X = Y^{k}, \ k \neq 1 \tag{27}$$

is a positive constant which takes care of the anisotropy of the space time.

(ii) Usually ,in literature, many authors use either special law of variation for Hubble's parameter proposed by Berman[37] which yields constant deceleration parameter models or the hybrid expansion law given by Akarsu et al.[38] given by

$$a(t) = a_0 t^{\alpha_1} e^{\alpha_2 t} \tag{28}$$

where α_1 and α_2 are non-negative constants and a_0 is the present day value of $a(t)$. This yields variable deceleration parameter models. Here we use the hybrid law given by Eq.(28).
Now from Eqs.(13), (27) and (28) we obtain the metric potentials as

$$X = \left(a_0 t^{\alpha_1} e^{\alpha_2 t}\right)^{\frac{3k}{k+2}}$$
$$Y = \left(a_0 t^{\alpha_1} e^{\alpha_2 t}\right)^{\frac{3}{k+2}} \tag{29}$$

Using Eqs.(26) and (29) we obtain the scalar field with flat potential as

$$\varphi = \varphi_0 \int \left(t^{\alpha_1} e^{\alpha_2 t}\right)^{-3} dt + c_0 \tag{30}$$

Here φ_0 and c_0 are constants of integration.Now using Eqs.(29) and (30) in (1) we can represent the model(after a suitable choice of constants) as

$$ds^2 = -dt^2 + \left(t^{\alpha_1} e^{\alpha_2 t}\right)^{\frac{6k}{k+2}} (dx - z\, dy)^2 + \left(t^{\alpha_1} e^{\alpha_2 t}\right)^{\frac{6}{k+2}} (dy^2 + dz^2) \tag{31}$$

with the scalar field given by

$$\varphi = \varphi_0 \int \left(t^{\alpha_1} e^{\alpha_2 t}\right)^{-3} dt \tag{32}$$

4.Some physical properties of the model

It may be observed that the model (31) along with Eq.(32) represents LRS Bianchi type-II inflationary model in the presence of mass less scalar field with flat potential and massive string source with the following dynamical parameters which play a vital role in the discussion of cosmology:

The spatial volume v is

$$v = \left(t^{\alpha_1} e^{\alpha_2 t}\right)^{3} \tag{33}$$

The average Hubble parameter is

$$H=\frac{\alpha_1}{t}+\alpha_2 \tag{34}$$

The scalar expansion is

$$\theta=3H=3\left(\frac{\alpha_1}{t}+\alpha_2\right) \tag{35}$$

The shear scalar is

$$\sigma^2=\frac{1}{3}\left(\frac{k-1}{k+2}\right)^2\left(\frac{\alpha_1+\alpha_2 t}{t}\right)^2 \tag{36}$$

The anisotropy parameter is

$$A_h=8\left(\frac{k-1}{k+2}\right)^2 \tag{37}$$

The deceleration parameter in the model is

$$q=\frac{\alpha_1}{(\alpha_1+\alpha_2 t)^2}-1 \tag{38}$$

From Eqs.(24) and (31) ,we obtain the tension in the string as

$$\lambda=\frac{3(k-1)}{(k+2)}\left(3(\frac{\alpha_1}{t}+\alpha_2)^2-\frac{\alpha_1}{t^2}\right)+\left(t^{\alpha_1}e^{\alpha_2 t}\right)^{\frac{3(2k-4)}{k+2}} \tag{39}$$

From Eqs.(25) and (31) we get the energy density of the string as

$$\rho=9(\frac{\alpha_1}{t}+\alpha_2)^2\left(\frac{(k^2+3k+1)}{(k+2)^2}\right)-\frac{3\alpha_1}{t^2}\left(\frac{k+1}{k+2}\right)-v_0 \tag{40}$$

From Eqs.(6),(39) and (40) we have the particle density of the string as

$$\rho_p=9(\frac{\alpha_1}{t}+\alpha_2)^2\left(\frac{(k^2+3k+1)}{(k+2)^2}\right)-\frac{3\alpha_1}{t^2}\left(\frac{k+1}{k+2}\right)-v_0-\frac{3(k-1)}{(k+2)}\left(3(\frac{\alpha_1}{t}+\alpha_2)^2-\frac{\alpha_1}{t^2}\right)+\left(t^{\alpha_1}e^{\alpha_2 t}\right)^{\frac{3(2k-4)}{k+2}} \tag{41}$$

From the above results we observe the following:

- The model, obtained here, has no initial singularity, i.e. t=0

- The spatial volume increases exponentially with time which shows that our model is an inflationary model.
- For late times the deceleration parameter is negative and hence we will have an universe with accelerated expansion.
- The average Hubble parameter, scalar expansion and shear scalar diverge at the initial epoch, t=0 and attain constant values as $t \to \infty$.
- The energy density and the particle density of the cosmic string diverge at t=0 and become constant for infinitely large t.
- The tension in the string diverges at t=0 and attains constant value at late times for k>1.
- It is interesting to note that for k=1, the model becomes isotropic and shear free.
- Our model will be useful to study the inflationary and hence accelerated expansion scenario of modern cosmology.

5. Summary

In this article, we have attempted to obtain an inflationary cosmological model in LRS Bianhi type-II space time in the presence of mass less scalar field with flat potential coupled with massive string source. A deterministic model is obtained by solving Einstein field equations using a relation between the metric potentials and a hybrid expansion law for the average scale factor of the universe. We have discussed the physical behavior of the model with the help of the dynamical parameters of our model. It is observed that our model represents an in inflationary universe leading to accelerated expansion scenario of modern cosmology.

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