Exploration

On Maxwell-Dirac Isomorphism

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Abstract

In this paper, we discuss Maxwell-Dirac isomorphism and quantum entanglement.

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1. Introduction

In its simplest form, the features of quantum theory can be reduced to: (a) a wave function description of microscopic entities; and (b) entanglement. Entanglement is a key property that makes quantum information theory different from its classical counterpart [14].

But what is entanglement? Wootter gives one of clearest description [13]:

> In both classical mechanics and quantum mechanics, one can define a pure state to be a state that is as completely specified as the theory allows. In classical mechanics a pure state might be represented by a point in phase space. In quantum mechanics it is a vector in a complex vector space. Perhaps the most remarkable feature of quantum mechanics, a feature that clearly distinguishes it from classical physics, is this: for any composite system, there exist pure states of the system in which the parts of the system do not have pure states of their own. Such states are called entangled.

According to Scolarici and Solombrino [5]:

> The essential difference in the concept of state in classical and quantum mechanics is clearly pointed out by the phenomenon of entanglement, which may occur whenever the product states of a compound quantum system are superposed. Entangled states play a key role in all controversial features of QM; moreover, the recent developments in quantum information theory have shown that entanglement can be considered a concrete physical resource that it is important to identify, quantify and classify.

Nonetheless, they concluded that "our research has pointed out a puzzling situation, in which the same state of a physical system is entangled in CQM, while it seems to be separable in QQM."

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While entanglement is usually considered as purely quantum effect, it by no means excludes the possibility to describe it in a classical way.

In this regard and from the history of QM, we learn that there were many efforts to describe QM features in a more or less classical picture. For example, in 1927 Einstein presented his version of the hidden variable theory of QM, starting from Schrödinger's picture, which seems to influence his later insistence that "*God does not play dice*" [6][7].

Efforts have also been made to extend QM to QQM (quaternionic quantum mechanics), for instance by Stephen Adler from IAS [8].

In recent decades, however, another route began to appear, which may be called the Maxwell-Dirac isomorphism route, where it can be shown that there is close link between Maxwell's equations of classical electromagnetism and the Dirac equation of the electron. Intuitively, this may suggest that there is a one-to-one correspondence between the electromagnetic wave and quantum wave function. But can it offer a classical description of entanglement? This problem will be explored in the next sections.

2. A Few Alternatives of a Realistic Maxwell-Dirac Isomorphism

There are some papers dealing with the formal connection between classical electrodynamics and wave mechanics, especially there are some existing proofs on the Maxwell-Dirac isomorphism. We will review here two derivations of the Maxwell-Dirac isomorphism, i.e., by Hans Sallhofer and Volodimir Simulik. In the last section, we will also discuss a third option, i.e., by exploring the Maxwell-Dirac isomorphism through quaternionic language.

a. Sallhofer's method

Summing up one of Sallhofer's papers [1], he says that under the sufficiently general assumption of periodic time dependence, the following connection exists between source-free electrodynamics and wave mechanics:

$$\sigma \cdot \begin{bmatrix} rotE + \frac{\mu}{c}\frac{\partial}{\partial t}H = 0 \\ rotH - \frac{\varepsilon}{c}\frac{\partial}{\partial t}H = 0 \\ div\varepsilon E = 0 \\ div\mu H = 0 \end{bmatrix}_{divE=0} \equiv \left[(\gamma \cdot \nabla + \gamma^{(4)}\partial_4)\Psi = 0\right] \qquad (1)$$

That is, the multiplication of source-free electrodynamics by the Pauli-vector yields wave mechanics [1].

In simple terms, this result can be written as follows:

$$P \cdot M = D, \tag{2}$$

where P = Pauli vector, M = Maxwell's equations and D = the Dirac equation.

We can also say that wave mechanics is a solution-transform of electrodynamics. Here, one has to bear in mind that the well-known circulatory structure of the wave functions, manifest in Dirac's hydrogen solution, is not introduced just by the Pauli-vector [1].

b. Simulik's method

Simulik described another derivation of the Maxwell-Dirac isomorphism. In one of his papers [2], he wrote a theorem suggesting that Maxwell's equations of source-free electrodynamics which can be written as follows:

$$\begin{gathered} rotE + \frac{\mu}{c}\frac{\partial}{\partial t}H = 0 \\ rotH - \frac{\varepsilon}{c}\frac{\partial}{\partial t}H = 0 \\ divE = 0 \\ divH = 0 \end{gathered} \tag{3}$$

are equivalent to the Dirac-like equation [2]:

$$\left[\gamma.\nabla - \begin{pmatrix} \varepsilon 1 & 0 \\ 0 & \mu 1 \end{pmatrix} \frac{1}{c}\frac{\partial}{\partial t}\right]\Psi^{c1} = 1, \tag{4}$$

where in the usual representation

$$\gamma = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \tag{5}$$

and σ are the well-known Pauli matrices.

c. Maxwell-Dirac isomorphism through Quaternionic language

In text books, quantum theory is based on complex numbers of the form $a_0+a_1 i$, with i being the imaginary unit $i^2 = -1$. It has long been known that an alternative quantum mechanics can be based on the quaternion or hyper-complex numbers of the form $a_0 + a_1 i + a_2 j + a_3 k$, with i, j, k being three non-commuting imaginary units [8].

On the other hand, recognizing that Maxwell's equations were originally formulated in terms of quaternionic language, some authors investigated whether there could be a formal correspondence between Maxwell's and Dirac's equations. Kravchenko and Arbab are a few researchers who worked on this problem. And also the present authors arrived at a similar conclusion despite using different procedures based on the Gersten decomposition of the Dirac equation [4].

This MD isomorphism can also be extended further to the classical description of boson mass, which was usually called the Higgs boson [3], so it may be a simpler option compared to scale symmetry theory.

3. Quaternionic QM & Entanglement

Singh & Prabakaran are motivated to examine the geometry of a two-qubit quantum state using the formalism of the Hopf map. The "quaternions" again come in handy in studying the two-qubit state. [10]

In his exposition of Quaternionic Quantum Mechanics, Singh concluded that [9]:

> Having established the compatibility of the Hopf fibration representation with the conventional theory for unentangled states, let us, now, address the issue of measurability of entanglement in this formalism. In the context, "Wootters' Concurrence" and the related "Entanglement of Formation" constitute well accepted measures of entanglement, particularly so, for pure states. …It follows that any real linear combination of the "magic basis" would result in a fully entangled state with unit concurrence. Conversely, any completely entangled state can be written as a linear combination in the "magic basis" with real components, up to an overall phase factor. In fact, these properties are not unique to a state description in the "magic basis" and hold in any other basis that is obtained from the "magic basis" by an orthogonal transformation…

In a rather different way, Najarbashi et al. explored quaternionic Möbius transformations, which can be useful in theoretical physics in areas such as quaternionic quantum mechanics, quantum conformal field theory and quaternionic computations [11]. They found that "*as in the case of two-qubits, both octonionic stereographic projection and Möbius transformation are entanglement sensitive*."

5. Discussions & Conclusion

Despite its enormous practical success, many physicists and philosophers alike agree that quantum theory is full of contradictions and paradoxes that are difficult to solve consistently. Even after 90 years, experts still do not agree about what to make of it.

In the meantime, the problem of the formal connection between electrodynamics and wave mechanics has attracted the attention of a number of authors, especially there are some existing proofs on Maxwell-Dirac isomorphism. Here the author reviews two derivations of the Maxwell-Dirac isomorphism by Hans Sallhofer and Volodimir Simulik as well as quaternion language.

While this paper does not conclusively answer the question of whether the Maxwell-Dirac isomorphism and especially its quaternionic formulation can offer a classical description of entanglement, we have mentioned some recent discussions on this topic such the Hopf map and quaternionic Möbius transformations.

This paper was inspired by an old question: *Is there a consistent and realistic description of the wave function, both classically and quantum mechanically?* It can be expected that the above discussions will shed some light on such an old problem especially in the context of the physical meaning of the quantum wave function. This is reserved for further investigations.[1]

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[1] More lengthy discussions on old problems related to QM will appear in our forthcoming book, with title: Old Problems and New Horizons in World Physics, to be released by this year.