Article

Dynamics of Kantowski-Sachs Universe with Magnetized Anisotropic Dark Energy

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Abstract

In this paper we study Kantowski-Sachs cosmological model in the presence of magnetized anisotropic fluid with dynamical equation of state. We obtain exact solutions to the field equations by utilizing the hybrid expansion law for the average factor of the model. The physical and kinematical behaviours of the cosmological model are discussed. We observe that the anisotropic universe approaches isotropy through the evolution of the universe. We also find that the universe is accelerating due to the dominance of the dark energy.

Keywords: Kantowski-Sachs, cosmological model, anisotropic dark energy, magnetic field, hybrid expansion law.

1 Introduction

The present-day observational evidences have confirmed the belief that universe in its present state is in the phase of accelerated expansion. The accelerated expansion of the universe is believed to be driven by mysterious energy with negative pressure dubbed as dark energy [1-5]. The nature and composition of dark energy (DE) is still a challenging problem to the cosmologists. The thermodynamical studies of DE reveals that the constituents of DE may be massless particles (bosons or fermions) whose collective behavior resembles with a kind of radiation fluid having negative pressure which is a kind of repulsive force acting as antigravity responsible for gearing up the universe. The cosmological constant is considered as the simplest DE candidate, but it needs to be fine-tuned to satisfy the current value of DE. Chaplygin gas as well as generalized chaplygin gas have been considered as other possible candidates for DE due to negative pressure [6-8]. Some cosmologists have suggested that interacting and non-interacting two fluids scenario are also possible DE candidate [9-11].

The anisotropy plays a significant role in the early stages of evolution of the universe and hence the study of spatially homogeneous and anisotropic cosmological models is of vital interest. The anisotropy of DE within the framework of Bianchi space-times has been found useful in generating arbitrary ellipsoidality to the universe and to fine tune the observed CMBR

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anisotropies [12]. This motivates several cosmologists to study cosmological models in the presence of anisotropic DE in general relativity. Several authors have studied Bianchi type-I model in the presence of an anisotropic DE. Rodrigues [13] obtained Bianchi type-I ΛCDM model whose DE component preserves non-dynamical character but yields anisotropic vacuum pressure. Koivisto and Mota [14] constructed a Bianchi type-I cosmological model containing interacting DE fluid with non-dynamical anisotropic equation of state (EoS) and perfect fluid. They have suggested that if the EoS is anisotropic, the expansion rate of the universe becomes direction dependent at late times and models with anisotropic EoS can explain some of the observed anomalies in CMB. Akarsu and Kilinc [15, 16] have suggested that the anisotropic fluid must not necessarily promote anisotropy in the expansion, such fluid may also support isotropic behavior of the universe.

The primordial magnetic fields have significant on CMB anisotropy depending on the direction of field lines [17, 18]. Several cosmologists have discussed the influence of magnetic field on the dynamics of the universe by analyzing anisotropic Bianchi models. Roy et al.[19], Bali et al.[20], Katore and Sancheti [21] have studied the effects of magnetic field on the dynamics of anisotropic Bianchi type-VI_0 cosmological model. King and Coles [18] discussed the dynamics of magnetized axisymmetric Bianchi type cosmological model with vacuum energy and examined the behaviors of scale factors perpendicular and parallel to the field lines. Sharif and Zubair [22] have discussed the dynamics of Bianchi type-I in the presence of an anisotropic DE and a uniform magnetic field. Sharif and Zubair [23] have also investigated Bianchi type-VI_0 cosmological models in the presence of electromagnetic field and anisotropic DE with constant deceleration parameter. Further, Sharif and Zubair [24] have discussed the dynamics of a Bianchi type-VI_0 cosmological model with an anisotropic fluid with anisotropic EoS and a uniform magnetic field. Recently, Sarkar [25] has discussed holographic DE model with linearly varying deceleration parameter, cosmic coincidence and the future singularity of the Kantowski-Sachs universe.

The purpose of the present work is investigate the nature of the magnetized anisotropic DE for the Kantowski-Sachs universe. The paper is organized as follows. In Sect.2, we present anisotropic Kantowski-Sachs model and Einstein's field equations in the presence of anisotropic fluid with anisotropic EoS and a uniform magnetic field. In Sect.3, we obtain exact solutions to the field equations using the hybrid expansion law for the average scale factor. The physical behaviors of the cosmological model discussed in Sect.4. Finally, we summarize the results in Sect. 5.

2 Kantowski-Sachs Model and Field Equations

The line-element for the general anisotropic Kantowski-Sachs space-time is given by

$$ds^2 = dt^2 - A^2 dr^2 - B^2(d\theta^2 + \sin^2 d\phi^2) \tag{2.1}$$

where the scale factors A and B are functions of cosmic time t only. This space-time has one transverse direction r and two equivalent longitudinal directions θ and ϕ.

We assume that the universe is filled with an anisotropic fluid, and that there no electric field while the magnetic field is oriented along r-direction. King and Coles [18] and Jacob [26] have used the magnetized perfect fluid distribution to discuss the effect of magnetic field on the evolution of the universe. The energy-momentum tensor for an anisotropic fluid is taken as

$$T_i^j = \text{diag}\left[T_4^4, T_1^1, T_2^2, T_3^3\right] \tag{2.2}$$

$$= \text{diag}\left[\rho, -p_r, -p_\theta, -p_\phi\right] \tag{2.3}$$

where ρ is the energy density of the fluid, p_r, p_θ and p_ϕ are pressure in the direction of r, θ and ϕ respectively. The simplest generalization of EoS parameter of a fluid may be to determine the EoS parameter separately on each spatial axis preserving the diagonal form of the energy-momentum tensor in a consistent way with the considered metric. Allowing the anisotropy in the pressure of the fluid , and thus in its EoS parameter, will give rise to different possibilities for the evolution of the energy source. To see this we first parametrize the energy momentum tensor of the anisotropic fluid as follows:

$$\begin{aligned} T_i^j &= \text{diag}\left[1, -\omega_r, -\omega_\theta, -\omega_\phi\right] \\ &= \text{diag}\left[1, -\omega, -(\omega+\gamma), -(\omega+\delta)\right] \\ &= \text{diag}\left[1, -\omega, -(\omega+\delta), -(\omega+\delta)\right] \end{aligned} \tag{2.4}$$

since $T_2^2 = T_3^3$. Here $\omega_r = \omega$, $\omega_\theta = \omega + \delta$, $\omega_\phi = \omega + \delta$ are the directional EoS parameter on r, θ and ϕ axes respectively. The skewness parameter δ that the deviation from ω in the directions θ and ϕ. The parameter ω and δ are not necessarily constants.

The magnetic field is taken along only r-direction so that the electromagnetic field tensor F_{ij} has only non-vanishing components, viz. F_{23}=constant. Therefore, the energy-momentum tensor for the electromagnetic field E_{ij} has the following non-trivial components,

$$E_4^4 = E_1^1 = -E_3^3 = -E_2^2 = \rho_b \tag{2.5}$$

and

$$E_i^j = \text{diag}\left[\rho_b, \rho_b, -\rho_b, -\rho_b\right] \tag{2.6}$$

where ρ_b is the magnetic charge density.

The Einstein's field equations are

$$R_i^j - \frac{1}{2} g_i^j R = -(T_i^j + E_i^j) \tag{2.7}$$

in proper units $8\pi G = c = 1$. Choosing co-moving coordinates, the field equation (2.7) in term of the line element (2.1) are written as

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = -\omega\rho + \rho_b, \tag{2.8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = -(\omega+\delta)\rho - \rho_b, \tag{2.9}$$

$$\frac{2\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = \rho + \rho_b \tag{2.10}$$

where dot denote derivative with respect to time.
Here we assume that DE component and magnetic field interact minimally. Therefore the energy

conservation equation can be split up into components separately, namely energy momentum tensor for anisotropic dark energy and energy momentum tensor for the magnetic field which are respectively given as

$$\dot{\rho} + (1 + \omega + \delta)\rho \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right) = 0, \tag{2.11}$$

$$\rho_b = \frac{c}{B^4} \tag{2.12}$$

where c is an arbitrary constant.

we define some parameter for the Kantowski-Sachs model (2.1) which are important in cosmological observations. The average scale factor and the volume are given as

$$V = a^3 = AB^2. \tag{2.13}$$

The scalar expansion θ and shear scalar σ are given by

$$\theta = \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right), \tag{2.14}$$

$$\sigma^2 = \frac{1}{3} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right)^2. \tag{2.15}$$

The anisotropic parameter of the expansion is characterized by the mean and directional Hubble parameter and is defined as

$$A_m = \frac{1}{3} \sum_{i=1}^{3} \left(\frac{H_i - H}{H} \right)^2 \tag{2.16}$$

where

$$H = \frac{1}{3}\theta = \frac{1}{3} \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right) \tag{2.17}$$

is the mean Hubble parameter and $H_i (i = 1, 2, 3)$ are the directional Hubble parameter in the direction of r, θ and ϕ-axes respectively, and given by

$$H_1 = \frac{\dot{A}}{A}, H_2 = H_3 = \frac{\dot{B}}{B}. \tag{2.18}$$

3 Solution of Field Equations

We now obtain the exact solutions of field equations (2.8)-(2.12) by utilizing the hybrid expansion law (HEL) for the average scale factor a. Subtracting Eq.(2.8) from Eq.(2.9), it follows that

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{B}}{B} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) = \frac{1}{B^2} - \delta\rho - 2\rho_b. \tag{3.1}$$

Equation (3.1), after integration, leads to

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{X}{V} \exp\left[\int \left(\frac{1}{B^2} - \delta\rho - 2\rho_b\right) V dt\right] \tag{3.2}$$

where X is an integration constant. The integral term in Eq.(3.2) vanishes for

$$\frac{1}{B^2} - \delta\rho - 2\rho_b = 0. \tag{3.3}$$

Using Eq.(3.3) in Eq. (3.2), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{X}{V}. \tag{3.4}$$

From Eq. (2.13), we get

$$\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} = \frac{\dot{V}}{V}. \tag{3.5}$$

Solving Eqs.(3.4) and (3.5), we obtain

$$\frac{\dot{A}}{A} = \frac{2}{3}\frac{X}{V} + \frac{1}{3}\frac{\dot{V}}{V}, \tag{3.6}$$

$$\frac{\dot{B}}{B} = -\frac{1}{3}\frac{X}{V} + \frac{1}{3}\frac{\dot{V}}{V}. \tag{3.7}$$

From Eqs.(3.6) and (3.7), we can determine the scale factors A and B if V is known function of time t.

We consider the following ansatz for the average scale factor for the model

$$a(t) = kt^{\alpha}e^{\beta t} \tag{3.8}$$

where $k > 0$, $\alpha \geq 0$ and $\beta \geq 0$ are constants. This generalized form of the average scale factor is known as Hybrid expansion law (HEL). Clearly HEL leads to the power-law power-law cosmology for ($\beta = 0$) and to the exponential law cosmology for($\alpha = 0$). The case $\alpha > 0$ and $\beta > 0$ leads to a new cosmology arising for the HEL. [27] Kumar [28] studied the dynamics of the universe within the framework of a Bianchi type-V space-time in the presence of a perfect fluid composed of non-interacting matter and dynamical DE by applying HEL for the average scale factor.

Differencing Eq.(2.13) and substituting the values of V and $\dot{V}$ in Eqs. (3.6) and (3.7), we obtain

$$\frac{\dot{A}}{A} = (\frac{\alpha}{t} + \beta) + \frac{X}{3(t^{\alpha}e^{\beta t})^3}, \tag{3.9}$$

$$\frac{\dot{B}}{B} = (\frac{\alpha}{t} + \beta) - \frac{2X}{3(t^{\alpha}e^{\beta t})^3}. \tag{3.10}$$

Integration of Eqs. (3.9) and (3.10) yields the expressions for the scale factors as

$$A = (t^{\alpha}e^{\beta t}) \exp\left\{-\frac{X}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha, 3\beta t)\right\} \tag{3.11}$$

$$B = (t^{\alpha} e^{\beta t}) \exp\left\{\frac{2X}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha, 3\beta t)\right\} \tag{3.12}$$

where the constants of integration are taken unity and $\Gamma(s, x)$ is the lower incomplete gamma function.

Hence the metric (2.1) with scale factor $A(t)$ and $B(t)$ given by Eqs. (3.11) and (3.12) represents a Kantowski-Sachs cosmological model in the presence of magnetized anisotropic DE. The magnetic charge density ρ_b is given by

$$\rho_b = \frac{c}{t^{4\alpha} e^{4\beta t}} \exp\left\{-\frac{8X}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha, 3\beta t)\right\} \tag{3.13}$$

4 Physical and Kinematical Features of the Model

For the present Kantowski-Sachs universe, the energy density, EoS parameter and skewness parameter are obtained as

$$\begin{aligned} \rho &= 3\left(\frac{\alpha}{t}+\beta\right)^2 - 2\frac{(\frac{\alpha}{t}+\beta)X}{t^{3\alpha}e^{3\beta t}} + \frac{1}{t^{2\alpha}e^{2\beta t}} \exp\left[-\frac{4X}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha, 3\beta t)\right] \\ &- \frac{c}{t^{4\alpha}e^{4\beta t}} \exp\left[-\frac{8X}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha, 3\beta t)\right], \end{aligned} \tag{4.1}$$

$$\begin{aligned} \omega = -\frac{1}{\rho}\Bigg[\frac{(3\alpha-2)\alpha}{t^2} + 3\beta^2 + \frac{6\alpha\beta}{t} + \frac{4X^2}{3t^{6\alpha}e^{6\beta t}} + \frac{1}{t^{2\alpha}e^{2\beta t}} \exp\left\{-\frac{4X}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha, 3\beta t)\right\} \\ -\frac{c}{t^{4\alpha}e^{4\beta t}} \exp\left\{-\frac{8X}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha, 3\beta t)\right\}\Bigg], \end{aligned} \tag{4.2}$$

$$\delta = \frac{1}{\rho}\left[\frac{1}{t^{2\alpha}e^{2\beta t}} \exp\left\{-\frac{4X}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha, 3\beta t)\right\} - \frac{2c}{t^{4\alpha}e^{4\beta t}} \exp\left\{-\frac{8X}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha, 3\beta t)\right\}\right]. \tag{4.3}$$

The expansion scalar, shear scalar, mean Hubble parameter and anisotropic parameter have the values given as follows

$$\theta = 3H = 3\left(\frac{\alpha}{t}+\beta\right), \tag{4.4}$$

$$\sigma = \frac{X}{\sqrt{3}t^{3\alpha}e^{3\beta t}}, \tag{4.5}$$

$$A_m = \frac{2X^2}{9t^{6\alpha}e^{6\beta t}}\left(\frac{\alpha}{t}+\beta\right)^{-2}. \tag{4.6}$$

The deceleration parameter q has the value

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{\alpha}{(\alpha+\beta t)^2}. \tag{4.7}$$

From Eq. (3.13) we observe that the magnetic field reduces the energy density of anisotropic DE and increases the skewness parameter. The spatial volume of the model is zero at $t = 0$ and increases as $t \to \infty$. The energy density, scalar expansion, shear scalar and anisotropy parameter are all infinite. Thus the universe starts evolving with a big-bang singularity with infinite rate of expansion which slows down for the later times of the universe. The energy density of the anisotropic DE decreases with the increase of time and converges to $3\beta^2$ (constant) as $t \to \infty$. The expansion scalar is gradually decreasing and ultimately assumes the constant value 3β. As $t \to \infty$, the Eos parameter $\omega = -1$ and skewness parameter $\delta = 0$. The anisotropy parameter decreases as time increases and ultimately becomes zero as t tends to infinity. Hence the model attains isotropy at late times which is consistent with the recent observations that the universe is isotropic at large scale.

We also observe that the universe evolves with variable deceleration parameter and the transition from deceleration to acceleration takes place at the time

$$t = \frac{\sqrt{\alpha} - \alpha}{\beta} \tag{4.8}$$

which restricts α in the range $0 < \alpha < 1$. As $t \to \infty$, $q \sim -1$ which shows the inflationary behaviour of the universe for large time. This further indicates that the present-day is undergoing accelerated expansion. Thus, for sufficiently large times, we observe that $H \sim \beta$, $q \sim -1$, $\rho \sim 3\beta^2$, $\omega = -1$ which lead to the conclusion that the present universe asymptotically achieves the de Sitter phase and hence expands forever with the dominance of DE.

5 Conclusions

In this paper we have studied a Kantowski-Sachs cosmological model in the presence of magnetic field and anisotropic fluid with dynamical EoS. The exact solution of the field equation is obtained by utilizing the hybrid law for the average scale factor of the model. The physical and kinematical behaviours of the model are studied and analyzed. We have observed that the present cosmological exhibits transition from, early deceleration to late-time acceleration which is an essential feature of dynamic evolution of universe. The universe is anisotropic for all finite times and approaches to isotropy at late times. The role of magnetic field are also discussed. It is found that the magnetic field reduces the energy density whereas it increases the skewness parameter of the anisotropy. This anisotropic model may be useful while dealing with the issues of CMB anisotropy and structure formation in the early universe etc.

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