Article

A Decelerating Anisotropic Bianchi Type-VI$_0$ Cosmological Model in General Relativity

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Abstract

We have investigated a decelerating anisotropic Bianchi type-VI$_0$ cosmological model in general relativity. We have obtained a determinant solution by assuming $A = B^n$ and $\rho + \bar{p} = 0$. Some physical and geometrical properties of the model are also discussed.

Keywords: Bianchi type-VI$_0$, bulk viscous, spacetime, cosmology, relativity.

1. Introduction

Cosmology, the branch of astronomy that deals with evolution and the study of large structures of the universe. In the past few consecutive years, string cosmology models have been considered. Cosmic strings play an important role in study of the early stages of universe and galaxy formation. Phase transitions in the early universe can give rise to microscopic topological defects: vacuum domain walls, strings, walls bounded by strings and monopoles connected by strings [1, 2]. The model formed by a massive string, which was initiated by Letelier, was used as a Bianchi type-I and "Kantowski-Sachs" type of cosmological model [3, 4]. The basic virtues of inflation in the deflationary picture has been discussed by Gasperini *et al.* [5].

Bianchi type I-IX cosmological models are important in the sense of strings, isotropic and homogeneous aspects etc. In past five decades, relativists have been interested in constructing a string cosmological model. Borrow [6] has investigated a Bianchi type-VI$_0$ model of universe and explained a cosmological problem. Ruban [7] and Collins [8] discussed some exact solutions for Bianchi type-VI$_0$ models for perfect fluid distributions, satisfying a specific equation of state. Ellis and McCollum [9] found a solution of Einstein's field equation for Bianchi type-VI$_0$ spacetime in a stiff fluid. Dunn and Tupper [10] obtained the solution of a class of Bianchi type-VI$_0$ perfect fluid cosmological models associated with an electromagnetic field. Reddy and Rao [11] presented on some Bianchi type cosmological models in the biometric theory of gravitation. Shri Ram [12] presented an algorithm for generating an exact perfect fluid solution for Einstein's field equation, not satisfying the equation of state, for a spatially homogeneous cosmological model of Bianchi type-VI$_0$. Singh and Singh [13] have obtained the solution for string cosmological models with a magnetic field in general relativity. Some exact solutions of a string

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cosmological model have investigated by several researchers [14-17]. Xing-Xiang [18-20] has obtained a solution of a Bianchi string cosmological model with bulk viscosity and a magnetic field. A Bianchi type-III for cloud string cosmological model was described by Tikekar & Patel [21]. Chakraborty *et al.* [22, 23] investigated a string cosmological model in general relativity. In a Bianchi Type-VI$_0$ string cosmological model, Tikekar and Patel [24] obtained some exact solutions. Bianchi Type-I and Bianchi Type-III models were investigated by Bali *et al.* [25-28].

Two parameters of Einstein's field equation, cosmological constant ∧ and gravitational constant G, play the role of a coupling constant between geometry and matter in Einstein's field equation. Shrimali and Joshi [29-32] obtained the solution of a Bianchi type-III cosmological model in general relativity. Pradhan and Bali [33] obtained the solution for a magnetized Bianchi type-VI$_0$ barotropic massive string universe with a decaying vacuum energy density. Verma and Ram [34] investigated the solutions for Bianchi type-VI$_0$ bulk viscous fluid models with variable gravitational and cosmological constants. Pradhan *et al.* [35,36] obtained a dark energy model in a Bianchi Type-VI$_0$ context.

Recently, Bali and Poonia [37] investigated a Bianchi Type-VI$_0$ inflationary cosmological model in general relativity. Tyagi *et al.* [38-40] obtained a Bianchi type-VI$_0$ homogeneous cosmological model for an anti-stiff perfect fluid for time dependent ∧ in general relativity, an inhomogeneous cosmological model for stiff perfect fluid distribution in general relativity and a barotropic perfect fluid in creation field theory with a time-dependent cosmological model. Bali *et al.* [41] and Bhoyar *et al.* [42] have investigated Bianchi type-VI$_0$ models in general relativity.

2. Field Equation

We consider a Bianchi type-VI$_0$ spacetime metric in the form of

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2mx} dy^2 + C^2 e^{2mx} dz^2 \tag{1}$$

Where A, B and C are functions of time t and m is constant. The energy momentum tensor for a bulk viscous fluid distribution is given by

$$T_i^j = (\rho + \overline{p}) v_i v^j + \overline{p} g_i^j \tag{2}$$

$$\overline{p} = p - \xi v^i_{;i}$$

Here, $\rho, p, \overline{p}, \lambda$ and H are energy densities, isotropic pressure, bulk viscous pressure and string tension density and Hubble's parameter respectively. The velocity vector of fluid satisfies

$$v_i v^j = -1 = -u_i u^j \tag{3}$$

$$u^i v_i = 0 \tag{4}$$

The vector $u_i u^j$ describes the direction of string or direction or anisotropy.
The Einstein field equation

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G T_{ij} + \wedge g_{ij} \tag{5}$$

R_{ij} is known as Ricci tensor and T_{ij} is the energy momentum tensor for matter.
The line element (1) and the field equation (5) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{m^2}{A^2} = -8\pi G \overline{p} + \wedge \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = -8\pi G \overline{p} + \wedge \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = -8\pi G \overline{p} + \wedge \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = 8\pi G \rho + \wedge \tag{9}$$

$$\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right) = 0 \tag{10}$$

The dot on B and C denotes ordinary differentiation with respect to t.
Integration of equation (10) gives

$$B = LC \tag{11}$$

Where, L is the constant of integration. Without loss of generality, we must take $L = 1$ so that

$$B = C \tag{12}$$

The expression for scalar expansion, shear scalar and spatial volume is

$$\theta = v^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{13}$$

$$\sigma^2 = \frac{1}{3} \left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC} \right) \tag{14}$$

$$S^3 = ABC \tag{15}$$

$$V = S^3 = ABC \tag{16}$$

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{17}$$

$$q = -1 + \frac{d}{dt}(H) \tag{18}$$

3. Solution of the Field Equation

We assumed that scalar expansion θ in this model is proportional to shear scalar σ

$$\theta \propto \sigma \tag{19}$$

So we have

$$A = B^n \tag{20}$$

Where n is any real number.
To get a determinate solution we need an extra condition. We assume

$$\rho + \overline{p} = 0 \tag{21}$$

I.e. the fluid is an anti-stiff fluid.

From equation (), we have

$$(n+1)\frac{\ddot{B}}{B} + (n^2 - 2\text{n} - 1)\frac{\dot{B}^2}{B^2} = 0 \tag{22}$$

$$\frac{\ddot{B}}{B} + \frac{(n^2 - 2\text{n} - 1)}{(n+1)}\frac{\dot{B}^2}{B^2} = 0 \tag{23}$$

Multiplying $\frac{B}{\dot{B}}$ in above equation, we have

$$\frac{\ddot{B}}{\dot{B}} + \frac{(n^2 - 2\text{n} - 1)}{(n+1)}\frac{\dot{B}}{B} = 0 \tag{24}$$

After integration with respect to B, we get

$$\log \dot{B} = -\left[\frac{(n^2 - 2\text{n}-1)}{(n+1)}\right]\log B + \log k_1 \tag{25}$$

$$\log \dot{B} = \log \frac{k_1}{B^{\left[\frac{(n^2-2\text{n}-1)}{(n+1)}\right]}}$$

$$B = [M\left(k_1 t + k_2\right)]^{\frac{1}{M}} \tag{26}$$

In above equation, $M = \dfrac{n(n-1)}{(n+1)}$ and $k_1 \neq 0$ and k_2 are integration constants. $n+1>0$

Hence, we obtain

$$A^2 = [M\left(k_1 t + k_2\right)]^{\frac{2n}{M}} \tag{27}$$

$$B^2 = [M\left(k_1 t + k_2\right)]^{\frac{2}{M}} \tag{28}$$

$$C^2 = [M\left(k_1 t + k_2\right)]^{\frac{2}{M}} \tag{29}$$

Therefore, equation (1) becomes

$$ds^2 = -dt^2 + [M\left(k_1 t + k_2\right)]^{\frac{2n}{M}} dx^2 + [M\left(k_1 t + k_2\right)]^{\frac{2}{M}} e^{-2mx} dy^2 + [M\left(k_1 t + k_2\right)]^{\frac{2}{M}} e^{2mx} dz^2 \tag{30}$$

$$ds^2 = -dt^2 + \left\{[MT]^{\frac{2n}{M}}\right\} dx^2 + \left\{[MT]^{\frac{2}{M}} e^{-2mx}\right\} dy^2 + \left\{[MT]^{\frac{2}{M}} e^{2mx}\right\} dz^2 \tag{31}$$

Where $T = k_1 t + k_2$

4. Geometrical and physical parameter

From this model, we can find other geometrical and physical parameters. The expressions for Hubble parameter H, expansion scalar θ, spatial volume V, shear scalar σ^2 and the deceleration parameter are respectively given by

$$H=\frac{n+2}{3}\left(\frac{\dot{B}}{B}\right) \tag{32}$$

$$H=\frac{n+2}{3}\left(\frac{k_1}{MT}\right) \tag{33}$$

$$\theta=\left(\frac{k_1(n+2)}{MT}\right) \tag{34}$$

$$\sigma=\frac{(n-1)k_1}{\sqrt{3}MT} \tag{35}$$

$$V=[\mathrm{MT}]^{\frac{n+2}{M}} \tag{36}$$

$$q=-1-\frac{(n+2)}{3MT^2} \tag{37}$$

$$\frac{\sigma}{\theta}=\frac{n-1}{\sqrt{3}(n+2)} \tag{38}$$

Case 1: For model n=2 and where constant k_1=1 and $M=\frac{n(n-1)}{(n+1)}=\frac{2}{3}$

$$H=\left(\frac{2}{T}\right) \tag{39}$$

$$\theta=\left(\frac{6}{T}\right) \tag{40}$$

$$\sigma=\frac{\sqrt{3}}{2T} \tag{41}$$

$$V=\left[\frac{2}{3}\mathrm{T}\right]^6 \tag{42}$$

$$q=-1-\frac{2}{T^2} \tag{43}$$

$$\frac{\sigma}{\theta} = \frac{1}{4\sqrt{3}} = \text{constant} \tag{44}$$

When T→0, the scalar expansion $\theta \to \infty$ (infinite). When T→∞, the scalar expansion $\theta \to 0$ and declaration parameter is always negative, therefore the model is decelerating. The spatial volume increases with time and becomes infinite for large values of T. The model is non-shearing and rotating as the universe is expanding. Since $\lim_{T\to\infty} \frac{\sigma}{\theta}$ = constant, anisotropy is maintained for all time and for large values of T, the model does not approach isotropy.

5. Conclusion

In this paper, we found that θ, σ and H decrease with growth of cosmic time and that V increases with time. Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$, anisotropy is maintained for all time. The model is therefore continuously non-shearing, expanding and rotating.

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