**Article**

# Interpretation of Dark Energy in Evolving Quantum Cosmology

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## Abstract

Quantum gravity (QG) is a wide-range physical model intended for understanding built-in cosmological quantum phenomena on small scale as well as large scale distances. So far, progress in this direction is nominal and the general theory of relativity (GTR) needs serious review with reference to 'quantum cosmology'. In this context, assuming that the Planck scale Hubble parameter and Mach's principle play a crucial role in cosmic evolution, we propose a toy model of evolving flat space quantum cosmology. We would like to suggest that:1) cosmic temperature is directly proportional to the cosmic ordinary matter density and current ordinary matter density and is about 0.0434 times the current critical density; 2) the current cosmic radius is about 11.13Gpc and seems to constitute around 18 Hubble spheres; 3) the current dark energy density can be identified with energy density spent in passing from current expected critical temperature to current actual temperature; 4) the ratio of current critical temperature to current actual temperature can be called the current thermal stretching factor and seems to be connected to the ratio of the Planck scale and current Hubble parameters; and 5) the Planck scale expansion velocity is 1.414*c* and the current expansion velocity is 2.60*c*.

**Keywords**:General relativity, quantum cosmology, thermal stretching factor, dark energy, dark matter, cosmic expansion velocity, Hubble's law and galactic rotation curves.

## 1. Introduction

By considering the 'Planck scale' as a characteristic limit of the evolving universe and 'Mach's principle' as a deep cosmic probe, an 'evolving flat space quantum cosmology' can be developed in a quantum gravity approach [1-4].In this toy model, by fitting the current ordinary matter density with current cosmic temperature and current Hubble parameter, we try to estimate the current cosmic radius and cosmic matter content. Proceeding further, we proposed a simple method for understanding dark energy with a new cosmic thermal stretching factor. We sincerely put forward that our toy model is coherent in fitting most of the observable current cosmic physical parameters and makes simple extrapolation to past and future.

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## 1.1 About Inflation

Weighing the big crunch, estimating the energy content of big crunch, understanding the materialistic nature of the big crunch, the span/duration of formation of the big crunch, estimating the intensity/power of big bang, the span/duration of the big bang and correlations between the big crunch, the big bang and the Planck scale seem to be very important in understanding 'conservation of energy' and 'inflation' on cosmic scales. To understand these points, further study is required at a very fundamental level and is beyond the scope of current science [5,6]. Even though the majority of modern scientists believe in 'inflation' [7,8], based on Planck2013 data, a serious debate is going on among the founders of inflation [9,10]. In their published paper [Inflationary schism. Physics Letters B 736 (2014) 142-146], Anna Ijjas, Paul J. Steinhardt and Abraham Loeb raise the following question- *If classic inflation is outdated and a failure, are we willing to accept postmodern inflation, a construct that lies outside of normal science? Or is it time to seek an alternative cosmological paradigm?* It is quite surprising. Future science, engineering and technology may resolve the issue. Nevertheless, to understand the ground reality, we are working on understanding the concepts of 'inflation' in a quantum gravitational approach.

## 1.2 About Quantum Cosmology

According to M. Bojowald [1]: 1) "Quantum cosmology is based on the idea that quantum physics should apply to anything in nature, including the whole universe. Quantum descriptions of all kinds of matter fields and their interactions are well known and can easily be combined into one theory - leaving aside the more complicated question of unification, which asks for a unique combination of all fields based on some fundamental principles or symmetries. Nevertheless, quantizing the whole universe is far from being straightforward because, according to general relativity, not just matter but also space and time are physical objects. They are subject to dynamical laws and have excitations (gravitational waves) that interact with each other and with matter. *Quantum cosmology is therefore closely related to quantum gravity, the quantum theory of the gravitational force and space-time. Since quantum gravity remains unfinished, the theoretical basis of quantum cosmology is unclear.* And to make things worse, there are several difficult conceptual problems to be overcome" and 2) "*We remain far from a proper understanding of quantum cosmology, especially when physics at the Planck scale is involved*. At the same time, research on quantum cosmology has led to progress in our understanding of generally covariant quantum systems and often showed unexpected effects of quantum space-time."

According to T. Padmanabhan [3]: "One natural - and in fact, inevitable - contribution to cosmological constant arises from the energy density of quantum vacuum fluctuations. The trouble is, we do not know how to compute the gravitational effects of quantum fluctuations of the vacuum from first principles. Naive estimates suggest that this will give $\Lambda\left(\frac{G\hbar}{c^3}\right)\approx 1$ which misses the correct result by 120 orders of magnitude! It is possible to get around this difficulty and get the correct value but only if we are prepared to make some extra assumptions. The appearance of $G$ and $\hbar$ together strongly suggests that the problem of dark energy needs to be addressed by quantum gravity. None of the currently popular models of quantum gravity has anything meaningful to say on this issue (let alone predict its correct value). *In fact, explaining the observed value of the dark energy is the acid test for any quantum gravity model and all the models currently available flunk this test.* There is no doubt that, when we eventually figure this out, it will lead to as drastic a revolution in our conceptual understanding as relativity and quantum theory did."

According to C.Sivaram [4]: "Although there has been a considerable spurt of recent interest in research in several formal aspects of quantum gravity including considerable mathematical progress, the subject still remains enigmatic and remote from other areas of physics. Despite several suggestions and complex models, no clear cut consistent consensus on uniting quantum theory and gravity has emerged. It would appear as if quantum gravity has no implications or impact on the rest of everyday mundane physics which depends on measurement or observation of well-defined physical quantities or properties that characterize a system or a substance. We shall see that this is not strictly true. *It is possible to carry out calculations of the effects of quantum gravity on certain systems and come out with numbers*! This has been known for some time especially in the case of a weak field in a linearized theory."

## 2. Concepts and Relations Pertaining to Quantum Cosmology

### 2.1 Nomenclature

1) $\left(\Omega_{OM}\right) =$ *Ratio of ordinary matter density to critical density.*
2) $\left(\Omega_{DM}\right) =$ *Ratio of dark matter density to critical density.*
3) $\left(\Omega_{DE}\right) =$ *Ratio of dark energy density to critical energy density.*
4) $H =$ *Hubble parameter.*
5) $V_{\exp} =$ *Cosmic expansion velocity.*
6) $M_{OM} =$ *Cosmic ordinary mass.*
7) $M_{DM} =$ *Cosmic dark matter content.*
8) $R =$ *Cosmic radius associated with* $M_{OM}$ *and* $M_{DM}$
9) $T_c \cong \left(\frac{3H^2c^2}{8\pi Ga}\right)^{\frac{1}{4}} =$ *Cosmic critical temperature.*
10) $\left(\lambda_{max}\right) =$ *Cosmic thermal wavelength and* $T =$ *Cosmic temperature* $= \frac{2.898\times10^{-3}\,^{\circ}\text{K.m}}{\left(\lambda_{max}\right)}$.
11) $\sqrt{\gamma} =$ *Cosmic thermal stretch factor=Ratio of critical temperature to actual temperature.*

**Note1**: For the above symbols, subscript $t$ denotes time dependent value, 0 denotes current value and $pl$ denotes Planck scale value.

**Note2**: $\beta \cong$ A new number related with quantum constants $\cong 4.96511423\left(\frac{45}{128\pi^7}\right)^{\frac{1}{4}} \cong 0.51572.$

## 2.2 Proposed New Concepts

Based on Mach's principle and quantum gravity, we imagine our universe as a quantum gravity object and consider the following *concepts*. With further study, they can be grouped into *two or three assumptions*. At any stage of cosmic evolution:

1) $\sqrt{\left(\frac{3H_t^2c^2}{8\pi G\left(aT_t^4\right)}\right)} \cong \left[1+\ln\left(\frac{H_{pl}}{H_t}\right)\right] \cong \gamma_t$ plays a crucial role in evolution of the entire cosmos.
2) $\left(T_c\right)_t \cong \left(\frac{3H_t^2c^2}{8\pi Ga}\right)^{\frac{1}{4}}$ can be called cosmic critical temperature.
3) Actual temperature is $\sqrt{\gamma_t} \cong \sqrt{1+\ln\left(\frac{H_{pl}}{H_t}\right)}$ times smaller than critical temperature and $\sqrt{\gamma_t} \cong \sqrt{1+\ln\left(\frac{H_{pl}}{H_t}\right)}$ can be called the cosmic thermal stretching factor.
4) Cosmic thermal wavelength, $\left(\lambda_{max}\right)_t$, is inversely proportional to, $\left(\Omega_{OM}\right)_t$.
5) Space-time curvature follows $G\left(M_{OM}+M_{DM}\right)_t \cong R_tc^2$.

**Note3:** We are working on understanding the correlations between $(z+1)$ and $\sqrt{\gamma_t}$. Redshift seems to be a cosmological expansion 'result' whereas $\sqrt{\gamma_t} \cong \sqrt{1+\ln\left(\frac{H_{pl}}{H_t}\right)}$ seems to be an inherent cosmological quantum gravitational 'constraint' that decides the cosmic instantaneous thermal sate. See sections 3 and 6.

## 2.3 Choosing the Magnitude of $H_0$

1) As per2015 Planck data [11]: $H_0 \cong \left(67.31 \pm 0.96\right)$ km/sec/Mpc and the present temperature of CMB radiation is $T_0 \cong \left(2.722 \pm 0.027\right)$ K.
2) According to advanced observational data analysis by A.G. Riess et al. [12], the current best value of $H_0 \cong \left(73.24 \pm 1.74\right)$ km/sec/Mpc.
3) With reference to $T_0 \cong 2.722$ K and our proposed set of concepts, we choose $H_0 \cong$ 70km/sec/Mpc$\cong 2.26853\times10^{-18} sec^{-1}$. This value seems to lie in between (67.31 and 73.24) km/sec/Mpc.

## 2.4 Role of the Planck Scale in Entire Cosmic Evolution

*So far, no mainstream cosmological model implemented the Planck scale in current cosmic evolution*. In this complicated situation, we attempt to implement the 'Planck scale' in the evolution of the entire cosmos with a positive approach. With further study, our approach can be developed for better understanding. Based on quantum gravity, we define the Planck scale Hubble parameter $H_{pl} \cong \sqrt{\frac{c^5}{G\hbar}} \cong 1.855\times10^{43}\ \sec^{-1}$. To proceed further, we define that,

$$\sqrt{\left(\frac{3H_t^2c^2}{8\pi G\left(aT_t^4\right)}\right)} \cong \gamma_t \cong \left[1+\ln\left(\frac{H_{pl}}{H_t}\right)\right] \tag{1}$$

where $H_t$ is the time-dependent Hubble parameter. If $H_{pl} \cong 1.854921\times10^{43}\ sec^{-1}$, one can choose different values of $\gamma$ in between $\gamma_{pl} \cong 1$ and $\gamma_0 \cong 141.2564$. For each value of $\gamma$, one can get a corresponding $H$ and all other physical parameters can be estimated.

## 2.5 Semi Empirical Relations Connected with Quantum Gravity

With reference to the set of concepts at any stage of cosmic evolution, we choose the following set of 'semi-empirical model relations'. One can modify them for a better understanding. At any arbitrary point of time,

1) Temperature of the CMB radiation,

$$\left.\begin{aligned}
&\left(\lambda_{max}\right)_t \cong \frac{1}{\left(\Omega_{OM}\right)_t}\left(\frac{c}{\sqrt{H_{pl}H_t}}\right) \text{ and} \\
&T_t \cong \frac{2.898\times10^{-3}\ {}^\circ\text{Km}}{\left(\lambda_{max}\right)_t} \cong \left(\Omega_{OM}\right)_t \times \frac{h\sqrt{H_{pl}H_t}}{4.965114k_B} \\
&\cong \left[1+\ln\left(\frac{H_{pl}}{H_t}\right)\right]^{-\frac{1}{2}}\left(\frac{3H_t^2c^2}{8\pi Ga}\right)^{\frac{1}{4}} \cong \left[\gamma_t\right]^{-\frac{1}{2}}\left(\frac{3H_t^2c^2}{8\pi Ga}\right)^{\frac{1}{4}}
\end{aligned}\right\} \tag{2}$$

where $\left(\frac{c}{\sqrt{H_{pl}H_t}}\right)$ can be called the Planck-Hubble mean length.

2) Ordinary matter density ratio,

$$\left(\Omega_{OM}\right)_t \cong \left(\frac{\left(M_{OM}\right)_t}{\frac{4\pi}{3}R_t^3}\right) \div \left(\frac{3H_t^2}{8\pi G}\right) \cong \frac{0.51572}{\sqrt{\gamma_t}} \cong \frac{\beta}{\sqrt{\gamma_t}} \tag{3}$$

3) Cosmic radius,

$$R_t \cong \sqrt{\frac{2}{\left[(\Omega_{OM})_t + (\Omega_{DM})_t\right]}}\left(\frac{c}{H_t}\right) \tag{4}$$

## 3. Interpreting Dark Energy and Estimating Dark Matter

In a heuristic approach, if one is willing to consider the relations proposed in section2.5, the magnitude of the current and Planck scale cosmological physical parameters can be fitted/predicted. It needs further study.

If $T_0 \cong 2.722\,\text{K}$, $(\lambda_{max})_0 \cong 1.06466$ mm and $H_0 \cong 2.26853\times10^{-18}\,sec^{-1} \cong$ 70km/sec/Mpc

Based on this data, it is possible to understand the current energy density spent in thermal expansion with a cosmic stretching factor due to cosmic evolution.

By considering $\sqrt{\gamma_t}$ as a representation of the thermal stretching factor, if $T_t$ is the observable temperature due to expansion with stretching, then

1) $aT_t^4$ can be considered to be the actual thermal energy density due to expansion with stretching and $a(T_c)_t^4$ can be considered to be the critical thermal energy density due to expansion without stretching.
2) To bring temperature from critical $(T_c)_t$ to actual $T_t$, loss or spent thermal energy density can be expressed as

$$(\in_{loss})_t \cong a\left[(T_c)_t - T_t\right]^4 \cong a\left[\sqrt{\gamma_t}T_t - T_t\right]^4 \cong a\left[T_t\left(\sqrt{\gamma_t}-1\right)\right]^4 \tag{5}$$

3) The ratio of loss in thermal energy density to critical energy density can be identified with 'dark energy density ratio' and can be expressed as

$$\left\{(\in_{loss})_t \Big/ \left(\frac{3H_t^2c^2}{8\pi G}\right)\right\} \cong \frac{\left(\sqrt{\gamma_t}-1\right)^4}{\gamma_t^2} \cong (\Omega_{DE})_t \tag{6}$$

$$(\in_{loss})_t \cong (\Omega_{DE})_t\left(\frac{3H_t^2c^2}{8\pi G}\right) \cong \left\{\frac{\left(\sqrt{\gamma_t}-1\right)^4}{\gamma_t^2}\right\}\left(\frac{3H_t^2c^2}{8\pi G}\right) \tag{7}$$

4) By knowing or predicting ordinary matter density and by estimating dark energy density, dark matter density can be estimated with Friedmann's density ratio sum rule. It can be expressed as

$$(\Omega_{DM})_t \cong 1-\left(\frac{\beta}{\sqrt{\gamma_t}}\right)-\left(\frac{\left(\sqrt{\gamma_t}-1\right)^4}{\gamma_t^2}\right) \tag{8}$$

5) Without considering $\sqrt{\gamma_0}$ as a stretching factor, by this time, cosmic temperature will be around 32.35 K. By this time, cosmic thermal wavelength seems to be stretched by a factor of $\sqrt{\gamma_0} \cong 11.885$ and seems to have reached a temperature of 2.72 K. Energy density spent/lost in bringing the cosmic temperature from 32.35 K to the actual 2.72 K will be $a[32.35-2.722]^4 \cong a(29.63)^4 \cong 5.832\times10^{-10}$ J.m$^{-3}$. It is 0.7 times the critical density. In this way, starting from the Planck scale, dark energy can be estimated directly. With reference to current values, it is possible to show that

$$(\Omega_{DE})_t \cong \left\{(\in_{loss})_0 \Big/ \left(\frac{3H_0^2c^2}{8\pi G}\right)\right\} \cong \frac{\left(\sqrt{\gamma_0}-1\right)^4}{\gamma_0^2} \cong 0.7036 \tag{9}$$

where $\sqrt{\gamma_0} \cong 11.8851$ and $\sqrt{\gamma_0}-1 \cong 10.8851$

$$(\Omega_{DM})_0 \cong 1-\left(\frac{\beta}{\sqrt{\gamma_0}}\right)-\frac{\left(\sqrt{\gamma_0}-1\right)^4}{\gamma_0^2} \cong 0.253 \tag{10}$$

6) For the Planck scale

$$(\Omega_{DE})_{pl} \cong \left\{(\in_{loss})_{pl} \Big/ \left(\frac{3H_{pl}^2c^2}{8\pi G}\right)\right\} \cong \frac{\left(\sqrt{\gamma_{pl}}-1\right)^4}{\gamma_{pl}^2} \cong 0 \tag{11}$$

where $\sqrt{\gamma_{pl}} \cong 1$ and $\sqrt{\gamma_{pl}}-1 \cong 0$

$$(\Omega_{DM})_{pl} \cong 1-\left(\frac{\beta}{\sqrt{\gamma_{pl}}}\right)-\left(\frac{\left(\sqrt{\gamma_{pl}}-1\right)^4}{\gamma_{pl}^2}\right) \cong 0.48428 \tag{12}$$

7) This procedure is very simple and can be applied to the past and future without any difficulty. An important point to be noted is that the procedure follows Friedmann's density ratio sum rule as well as conservation of energy based on the 'time-dependent Hubble parameter'-based (instantaneous) critical energy density. It needs further study. See the following picture 1.The blue curve represents ordinary matter density ratio, the red curve indicates dark matter density ratio and the green curve indicates the dark energy density ratio.

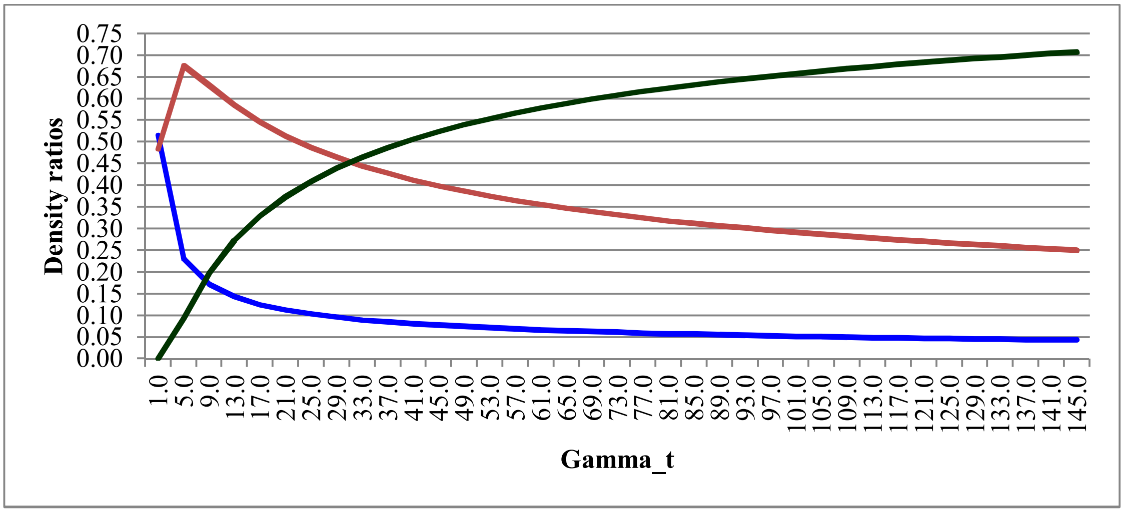


**Figure 1:** Cosmic Density Ratio Break Up

## 4. Current and Planck Scale Cosmic Physical Parameters

Based on the above relations (1) to (12), for the current case,

$$(M_{OM})_0 \cong (\Omega_{OM})_0 \left(\frac{3H_0^2}{8\pi G}\right) \cong 6.77\times10^{52}\ \text{kg} \tag{13}$$

$$(M_{DM})_0 \cong (\Omega_{DM})_0 \left(\frac{3H_0^2}{8\pi G}\right) \cong 3.95\times10^{53}\ \text{kg} \tag{14}$$

$$\left[(M_{OM})_0 + (M_{DM})_0\right] \cong \left[(\Omega_{OM})_0 + (\Omega_{DM})_0\right]\left(\frac{3H_0^2}{8\pi G}\right) \cong 4.623\times10^{53}\ \text{kg} \tag{15}$$

See Table-1 for various cosmic physical parameters associated with current and Planck scales.

**Table1:** Current and Planck scale cosmic physical parameters

| Current scale | Planck scale |
|---|---|
| $H_0 \cong 70\text{km/sec/Mpc} \cong 2.26853\times10^{-18}\,sec^{-1}$ | $H_{pl} \cong \sqrt{\frac{c^5}{G\hbar}} \cong 1.855\times10^{43}\ \sec^{-1}$ |
| $\gamma_0 \cong \left[1+\ln\left(\frac{H_{pl}}{H_0}\right)\right] \cong 141.2564$ | $\gamma_{pl} \cong \left[1+\ln\left(\frac{H_{pl}}{H_{pl}}\right)\right] \cong 1$ |
| $(\Omega_{OM})_0 \cong \left(\beta/\sqrt{\gamma_0}\right) \cong 0.04341$ | $(\Omega_{OM})_{pl} \cong \left(\beta/\sqrt{\gamma_{pl}}\right) \cong 0.5157$ |
| $(\Omega_{DM})_0 \cong 0.253$ | $(\Omega_{DM})_{pl} \cong 0.48428$ |
| $(\Omega_{DE})_0 \cong 0.704$ | $(\Omega_{DE})_{pl} \cong 0$ |

| | |
|---|---|
| $T_0 \cong (\Omega_{OM})_0 \times \frac{h\sqrt{H_{pl}H_0}}{4.965114k_B} \cong 2.722\text{ K}$ | $T_{pl} \cong (\Omega_{OM})_{pl} \times \frac{hH_{pl}}{4.965114k_B}$ $\cong 9.247\times10^{31}\text{ K}$ |
| $R_0 \cong \sqrt{\frac{2}{\left[(\Omega_{OM})_t + (\Omega_{DM})_t\right]}}\frac{c}{H_0} \cong 11.13\text{ Gpc}$ | $R_{pl} \cong \sqrt{\frac{2}{\left[(\Omega_{OM})_{pl} + (\Omega_{DM})_{pl}\right]}}\left(\frac{c}{H_{pl}}\right)$ $\cong 2.9845\times10^{-35}\text{ m}$ |
| $(M_{OM})_0 \cong (\Omega_{OM})_0\left(\frac{3H_0^2}{8\pi G}\right)\left(\frac{4\pi}{3}R_0^3\right)$ $\cong 6.77\times10^{52}\text{ kg}$ | $(M_{OM})_{pl} \cong (\Omega_{OM})_{pl}\left(\frac{3H_{pl}^2}{8\pi G}\right)\left(\frac{4\pi}{3}R_{pl}^3\right)$ $\cong 1.59\times10^{-8}\text{ kg}$ |
| $(M_{DM})_0 \cong (\Omega_{DM})_0\left(\frac{3H_0^2}{8\pi G}\right)\left(\frac{4\pi}{3}R_0^3\right)$ $\cong 3.95\times10^{53}\text{ kg}$ | $(M_{DM})_{pl} \cong (\Omega_{DM})_{pl}\left(\frac{3H_{pl}^2}{8\pi G}\right)\left(\frac{4\pi}{3}R_{pl}^3\right)$ $\cong 1.49\times10^{-8}\text{ kg}$ |
| $\left[(M_{OM})_0 + (M_{DM})_0\right] \cong 4.623\times10^{53}\text{ kg}$ | $\left[(M_{OM})_{pl} + (M_{DM})_{pl}\right] \cong 3.08\times10^{-8}\text{ kg}$ |

## 5. Estimating Cosmic Age

With reference to the Planck scale cosmic age of $\approx \frac{1}{H_{pl}} \cong \sqrt{\frac{G\hbar}{c^5}}$, current cosmic age of $\approx \frac{1}{H_0}$ and the standard cosmology-based cosmic age of 380,000 years pertaining to 3000 K, with trial and error, we developed the following semi-empirical relation. We are working on understanding its physical background and it needs further study.

$$(t \times H_t) \approx \left[1 + \ln\left(\frac{H_t}{H_0}\right)\right] \cong (\gamma_0 - \gamma_t) + 1 \tag{16}$$

Based on this relation, the cosmic age corresponding to a temperature of $\approx 3000$ K, a Hubble parameter of $\approx 2.5\times10^{-12} sec^{-1}$ and $\gamma_t \approx 127.344$ could be around 189,022 years. This is roughly half of the current estimation of 380,000 years.

## 6. Cosmic Redshift Associated with Temperature

Based on the proposed $\gamma_t$, redshift associated with the cosmic scale factor and cosmic Temperature ratio can be expressed in the following way:

1) Inverse of the cosmic scale factor can be expressed with

$$\left.\begin{array}{l}\frac{1}{a_t} \cong (z+1) \cong \frac{T_t}{T_0} \cong \frac{(\lambda_{max})_0}{(\lambda_{max})_t} \cong \left(\frac{(\Omega_{OM})_t}{(\Omega_{OM})_0}\right)\sqrt{\frac{H_t}{H_0}} \cong \sqrt{\frac{\gamma_0}{\gamma_t}}\exp\left(\frac{\gamma_0-\gamma_t}{2}\right) \\ \left.\begin{array}{l} \rightarrow z \cong \left(\frac{T_t}{T_0}\right)-1 \cong \left(\frac{(\lambda_{max})_0}{(\lambda_{max})_t}\right)-1 \\ \cong \left\{\left(\frac{(\Omega_{OM})_t}{(\Omega_{OM})_0}\right)\sqrt{\frac{H_t}{H_0}}\right\}-1 \cong \left\{\sqrt{\frac{\gamma_0}{\gamma_t}}\exp\left(\frac{\gamma_0-\gamma_t}{2}\right)\right\}-1 \end{array}\right\} \end{array}\right\} \quad (17)$$

2) The time-dependent Hubble parameter can be expressed with

$$H_t \cong \left(\frac{(\Omega_{OM})_0}{(\Omega_{OM})_t}\right)^2 (z+1)^2 H_0 \cong \left(\frac{\gamma_t}{\gamma_0}\right)(z+1)^2 H_0 \cong e^{(\gamma_0-\gamma_t)} H_0 \quad (18)$$

## 7. Galactic Rotational Curves at the Core Radius

With reference to the currently believed role of dark matter in galaxies [13-16] in a quantitative approach, we noticed that

$$v_{gr} \approx \left[\frac{(\Omega_{OM})_0}{(\Omega_{DM})_0}\right]^{\frac{1}{2}} \sqrt{\frac{GM_g}{R_g}} \approx 0.4142\sqrt{\frac{GM_g}{R_g}} \quad (19)$$

where, $M_g$ = mass of galaxy, $R_g$ =core radius of galaxy and $v_{gr}$ =galactic rotation speed.

Relation (19) needs further study with respect to frame-dragging effects, galactic self-rotation speed, the distribution of stars in the galaxy and the distribution of dark matter in the galaxy. See the following picture2. We arranged galactic rotation speeds in ascending order. The red curve represents our approximation and the blue curve represents the MSTG fit. An interesting observation is that starting from a galactic mass of $0.13\times10^{10} M_{\odot}$ to a galactic mass of $33\times10^{10} M_{\odot}$, our approximation seems to be in line with the MSTG fit. See Table-2 for data.

In Table-2,

1) Column-1 represents the galaxy name.
2) Column-2 represents the galactic mass estimation from MSTG data.
3) Column-3 represents the estimated tolerance of galactic mass estimation from MSTG data.
4) Column-4 represents the galactic core radius estimation from MSTG data.
5) Column-5 represents the estimated tolerance of galactic core radius from MSTG data.
6) Column-6 represents the estimated MSTG model of revolving speeds of orbiting stars.
7) Column-7 represents the tolerance in the estimated MSTG model of revolving speeds of orbiting stars.
8) Column-8 represents our approximation for the galactic rotation speed at the core radius.

**Figure 2:**Galactic Rotation Speeds Arranged in Ascending Order

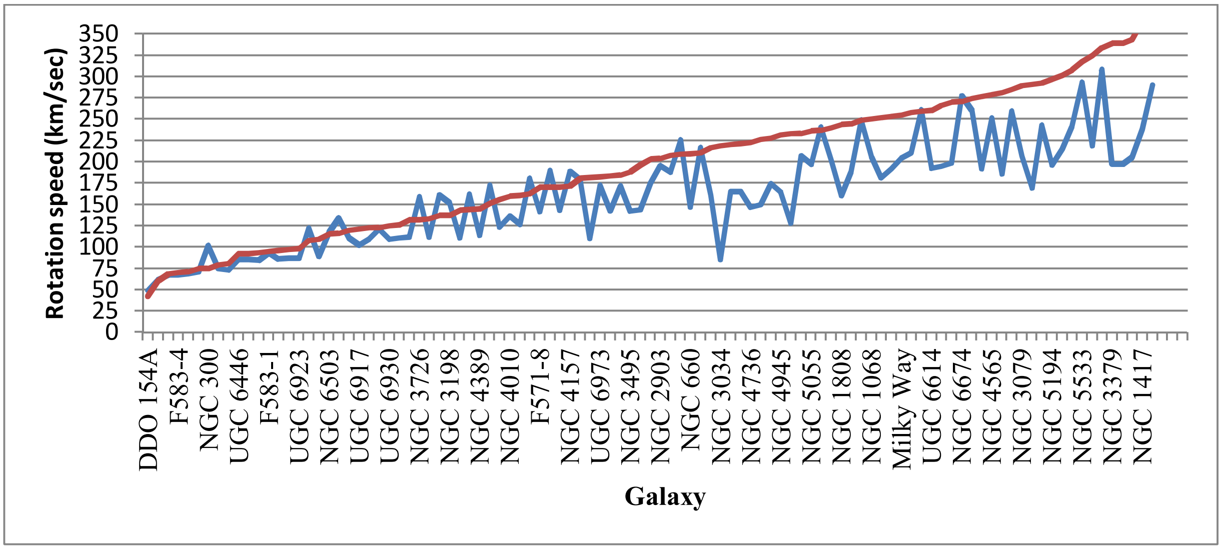


**Table2:** Fitting Galactic Rotation Speeds at Core Galaxy Radius

| Galaxy Name | Galaxy Mass ($10^{10} M_{\odot}$) | Tolerance in Galaxy Mass ($10^{10} M_{\odot}$) | Galaxy core radius (kpc) | Tolerance in galaxy core radius (kpc) | Rotation speed from MSTG estimations (km/sec) | Tolerance in rotation speed from MSTG estimations (km/sec) | Our approximation for rotation speed at core radius (km/sec) |
|---|---|---|---|---|---|---|---|
| Dwarf (LSB & HSB) Galaxies | | | | | | | |
| DDO 154[A] | 0.13 | 0.02 | 0.53 | 0.07 | 48.9 | 2.4 | 42.6 |
| DDO 168 | 0.42 | 0.09 | 0.66 | 0.08 | 67.1 | 4.7 | 68.5 |
| DDO 170 | 0.4 | 0.04 | 0.82 | 0.07 | 61.9 | 2.3 | 60.0 |
| F583-4 | 0.38 | 0.04 | 0.57 | 0.05 | 67.2 | 2.4 | 70.2 |
| NGC 55 | 1.17 | 0.07 | 0.99 | 0.05 | 84.4 | 2 | 93.4 |
| NGC 1560 | 0.79 | 0.05 | 0.93 | 0.04 | 74.9 | 1.7 | 79.2 |
| NGC 2708 | 9.43 | 1.1 | 0.66 | 0.05 | 218.7 | 10.8 | 324.8 |
| NGC 3109 | 0.78 | 0.04 | 1.15 | 0.04 | 68.6 | 1.3 | 70.8 |
| NGC 3877 | 8.65 | 0.53 | 1.31 | 0.06 | 164.8 | 4.3 | 220.8 |
| NGC 3949 | 6.51 | 0.3 | 0.99 | 0.03 | 164.5 | 3.2 | 220.4 |
| NGC 3972 | 4.09 | 0.23 | 1.18 | 0.05 | 126.8 | 2.9 | 160.0 |
| NGC 4062 | 2.98 | 0.17 | 0.43 | 0.02 | 149.4 | 3.4 | 226.2 |
| NGC 4085 | 5.11 | 0.54 | 1.12 | 0.07 | 142 | 6.1 | 183.5 |
| NGC 4096 | 1.07 | 0.07 | 0.24 | 0.01 | 110.1 | 2.8 | 181.4 |
| NGC 4389 | 4.4 | 1.02 | 1.56 | 0.18 | 113.9 | 10.6 | 144.3 |
| NGC 4569 | 6.23 | 0.51 | 0.39 | 0.03 | 205 | 7 | 343.5 |
| NGC 5585 | 1.17 | 0.07 | 0.94 | 0.04 | 85.7 | 1.8 | 95.9 |
| UGC 2259 | 0.77 | 0.02 | 0.48 | 0.01 | 88.8 | 1 | 108.8 |
| UGC 3691 | 2.83 | 0.14 | 0.86 | 0.03 | 123.5 | 2.3 | 155.9 |
| UGC 6399 | 1.34 | 0.08 | 1.05 | 0.04 | 86.7 | 2 | 97.1 |
| UGC 6446 | 0.83 | 0.04 | 0.73 | 0.04 | 85.1 | 1.4 | 91.6 |
| UGC 6818 | 1.31 | 0.53 | 1.5 | 0.32 | 73.1 | 10.8 | 80.3 |
| UGC 6917 | 2.06 | 0.11 | 1.04 | 0.05 | 102.1 | 2.2 | 120.9 |
| UGC 6923 | 0.96 | 0.17 | 0.74 | 0.1 | 86.5 | 5.6 | 97.9 |
| UGC 7089 | 0.86 | 0.08 | 1.15 | 0.07 | 71.1 | 2.3 | 74.3 |
| LSB GALAXIES | | | | | | | |
| F563-1 | 2.26 | 0.16 | 1.06 | 0.07 | 110.4 | 2.7 | 125.5 |
| F568-3 | 3.08 | 0.41 | 1.58 | 0.13 | 110.9 | 5.2 | 120.0 |
| F571-8 | 5.46 | 0.84 | 1.4 | 0.14 | 141.2 | 8 | 169.7 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| F583-1 | 1.56 | 0.12 | 1.28 | 0.06 | 93.2 | 2.3 | 94.9 |
| NGC 247 | 2.27 | 0.17 | 1.11 | 0.06 | 109.4 | 2.8 | 122.9 |
| NGC 598 | 1.78 | 0.04 | 0.64 | 0.01 | 110.9 | 0.8 | 143.3 |
| NGC 1003 | 1.64 | 0.03 | 0.8 | 0 | 121.5 | 0.8 | 123.0 |
| NGC 1417 | 16.6 | 0.49 | 0.92 | 0.02 | 238.2 | 2.8 | 365.0 |
| NGC 3495 | 4.16 | 0.27 | 0.87 | 0.04 | 142.1 | 3.3 | 187.9 |
| NGC 3672 | 14.86 | 0.2 | 1.21 | 0.01 | 215.2 | 1.2 | 301.1 |
| NGC 3917 | 6.25 | 0.45 | 1.6 | 0.09 | 142.8 | 3.8 | 169.8 |
| NGC 4010 | 5.56 | 0.88 | 1.62 | 0.17 | 136.2 | 7.9 | 159.2 |
| NGC 4183 | 2.04 | 0.11 | 0.85 | 0.05 | 111.3 | 2 | 133.1 |
| UGC 6446 | 0.83 | 0.04 | 0.73 | 0.04 | 85.1 | 1.4 | 91.6 |
| UGC 6614 | 11.36 | 1.79 | 1.24 | 0.22 | 192.3 | 11.9 | 260.1 |
| UGC 6930 | 2.17 | 0.13 | 1.03 | 0.06 | 109.5 | 2.2 | 124.7 |
| UGC 6983 | 2.12 | 0.16 | 0.9 | 0.07 | 111.5 | 2.8 | 131.9 |
| HSB GALAXIES | | | | | | | |
| IC 342 | 7.95 | 0.14 | 1.36 | 0.03 | 188.3 | 1.2 | 207.8 |
| Milky Way | 9.12 | 0.28 | 1.04 | 0.05 | 204.8 | 2.4 | 254.5 |
| NGC 224 | 20.19 | 0.3 | 1.84 | 0.04 | 259.6 | 1.6 | 284.7 |
| NGC 253 | 6.94 | 0.25 | 0.86 | 0.04 | 188 | 2.5 | 244.1 |
| NGC 300 | 2.03 | 0.17 | 2.7 | 0.19 | 101.7 | 2.9 | 74.5 |
| NGC 660 | 3.2 | 0.06 | 0.54 | 0.02 | 146.6 | 0.9 | 209.2 |
| NGC 801 | 20.07 | 2.09 | 2.65 | 0.24 | 240.3 | 10.2 | 236.5 |
| NGC 891 | 7.47 | 0.17 | 0.78 | 0.03 | 194.9 | 1.7 | 265.9 |
| NGC 1068 | 9.42 | 0.54 | 1.11 | 0.07 | 205.9 | 4.5 | 250.3 |
| NGC 1097 | 22.68 | 0.31 | 1.19 | 0.03 | 290.1 | 1.6 | 375.1 |
| NGC 1365 | 14.96 | 0.25 | 1.29 | 0.03 | 242.6 | 1.6 | 292.6 |
| NGC 1808 | 4.1 | 0.1 | 0.51 | 0.02 | 160.6 | 1.4 | 243.6 |
| NGC 2403 | 3.8 | 0.13 | 2.09 | 0.07 | 133.7 | 1.6 | 115.9 |
| NGC 2590 | 14.05 | 0.48 | 1.1 | 0.05 | 241 | 3.3 | 307.1 |
| NGC 2841 | 33.04 | 1.31 | 2.19 | 0.14 | 308.3 | 5.2 | 333.8 |
| NGC 2903 | 9.66 | 0.61 | 1.72 | 0.11 | 195.9 | 4.8 | 203.6 |
| NGC 2998 | 15.13 | 1.2 | 2.52 | 0.19 | 216.7 | 6.8 | 210.6 |
| NGC 3031 | 6.95 | 0.12 | 0.67 | 0.02 | 191.8 | 1.3 | 276.8 |
| NGC 3034 | 0.52 | 0.03 | 0.08 | 0.01 | 85 | 1.6 | 219.1 |
| NGC 3079 | 8.73 | 0.23 | 0.77 | 0.03 | 207.1 | 2.1 | 289.3 |
| NGC 3198 | 5.55 | 0.28 | 2.18 | 0.12 | 152.1 | 2.8 | 137.1 |
| NGC 3379 | 6.99 | 0.06 | 0.45 | 0.01 | 196.7 | 0.6 | 338.7 |
| NGC 3379 | 6.99 | 0.06 | 0.45 | 0.01 | 196.7 | 0.6 | 338.7 |
| NGC 3521 | 7.89 | 0.1 | 0.8 | 0.02 | 198.7 | 1 | 269.9 |
| NGC 3628 | 9.13 | 0.31 | 1.17 | 0.05 | 202.3 | 2.6 | 240.0 |
| NGC 3726 | 9.6 | 1.37 | 4.07 | 0.58 | 158.4 | 8.8 | 132.0 |
| NGC 3769 | 2.59 | 0.24 | 1.66 | 0.2 | 121.7 | 3.8 | 107.3 |
| NGC 3893 | 7.7 | 1 | 1.74 | 0.29 | 179.3 | 8.9 | 180.8 |
| NGC 3953 | 20.47 | 1.65 | 3.46 | 0.28 | 225.5 | 7.4 | 209.0 |
| NGC 3992 | 25.16 | 2.32 | 2.77 | 0.44 | 260.9 | 10 | 259.0 |
| NGC 4013 | 6.01 | 0.35 | 0.7 | 0.19 | 181.1 | 3.9 | 251.8 |
| NGC 4051 | 7.21 | 1.31 | 2.58 | 0.43 | 161.7 | 11.1 | 143.7 |
| NGC 4088 | 9.74 | 1.52 | 3.15 | 0.51 | 172.4 | 10.4 | 151.1 |
| NGC 4100 | 10.3 | 1.59 | 2.89 | 0.49 | 180.2 | 10.8 | 162.2 |
| NGC 4138 | 4.31 | 0.9 | 0.68 | 0.39 | 160.7 | 12.1 | 216.3 |
| NGC 4157 | 11.64 | 1.21 | 2.92 | 0.36 | 188.5 | 7.7 | 171.6 |
| NGC 4217 | 12.92 | 1.54 | 3.31 | 0.36 | 189.7 | 8.9 | 169.8 |
| NGC 4258 | 7.29 | 0.14 | 0.84 | 0.03 | 191.9 | 1.4 | 253.1 |
| NGC 4303 | 3.08 | 0.08 | 0.59 | 0.02 | 143.8 | 1.4 | 196.3 |
| NGC 4321 | 21.67 | 0.45 | 2.12 | 0.06 | 260.2 | 2.2 | 274.7 |
| NGC 4448 | 1.98 | 0.08 | 0.27 | 0.01 | 127.8 | 1.7 | 232.7 |
| NGC 4527 | 5.55 | 0.23 | 0.79 | 0.05 | 174.3 | 2.7 | 227.8 |
| NGC 4565 | 18.11 | 0.21 | 1.72 | 0.03 | 251.2 | 1.2 | 278.8 |
| NGC 4631 | 6.15 | 0.1 | 1.34 | 0.03 | 171.4 | 1 | 184.1 |
| NGC 4736 | 3.15 | 0.08 | 0.47 | 0.02 | 146.8 | 1.3 | 222.5 |
| NGC 4945 | 4.58 | 0.12 | 0.63 | 0.03 | 165.1 | 1.6 | 231.7 |
| NGC 5033 | 9.9 | 0.51 | 1.1 | 0.08 | 210.2 | 4.2 | 257.8 |
| NGC 5055 | 8.38 | 0.06 | 1.11 | 0.01 | 196.9 | 0.5 | 236.1 |
| NGC 5194 | 7.29 | 0.23 | 0.61 | 0.03 | 196.6 | 2.3 | 297.1 |
| NGC 5236 | 6.16 | 0.12 | 1.1 | 0.04 | 175.5 | 1.3 | 203.4 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| NGC 5457 | 10.2 | 0.27 | 1.39 | 0.04 | 206.5 | 2.1 | 232.8 |
| NGC 5533 | 28.81 | 1.92 | 2.11 | 0.23 | 293.2 | 8.2 | 317.5 |
| NGC 5907 | 4.59 | 0.26 | 0.4 | 0.05 | 169.3 | 3.5 | 291.1 |
| NGC 6503 | 1.98 | 0.06 | 1.1 | 0.05 | 117.4 | 1.3 | 115.3 |
| NGC 6674 | 32.48 | 2.38 | 3.27 | 0.33 | 277.7 | 8.6 | 270.8 |
| NGC 6946 | 8.95 | 0.65 | 3.54 | 0.27 | 161.2 | 4.5 | 136.6 |
| NGC 6951 | 6.22 | 0.22 | 0.58 | 0.03 | 185.8 | 2.5 | 281.4 |
| NGC 7331 | 21.47 | 0.76 | 2.56 | 0.1 | 248.9 | 3.6 | 248.9 |
| UGC 6973 | 6.41 | 0.45 | 1.43 | 0.12 | 172.5 | 4.5 | 181.9 |

# 8. Discussion and Conclusions

## 8.1 Cosmological Constant Problem

With reference to proposed concepts, the ratio of the Planck scale critical density to the current critical density is

$$\left(\frac{3H_{pl}^2c^2}{8\pi G}\right)\div\left(\frac{3H_0^2c^2}{8\pi G}\right)\cong\left(\frac{H_{pl}}{H_0}\right)^2\cong 6.685\times 10^{121} \quad (20)$$

We wish to put forward that this idea can be considered to be a characteristic tool for constructing a model of 'quantum gravity' with cosmic evolution.

## 8.2 The Horizon Problem

The 'horizon problem' is a problem with the standard cosmological model of the big bang. It points out that different regions of the universe have not 'contacted' each other because of the great distances between them, but nevertheless that they have the same temperature and other physical properties. If one is willing to consider the concept of 'matter causes space-time to curve', the 'horizon problem' can be understood. According to hot big bang model, during its evolution as the universe is expanding, thermal radiation temperature decreases and matter content increases. As matter content increases, based on Mach's principle, at any stage of evolution, it is possible to have an increasing radius of curvature, $R_t \cong \frac{G}{c^2}\left[\left(M_{OM}\right)_t+\left(M_{DM}\right)_t\right]$.

For the current case, $R_0 \cong \frac{G}{c^2}\left[\left(M_{OM}\right)_0+\left(M_{DM}\right)_0\right]\cong 11.13$ Gpc and there is no scope for 'causal disconnection' of distant visible matter.

## 8.3 Cosmic Inflation with Respect to Current Cosmic Size

Mainstream cosmologists believe that superluminal expansion period of the universe (called ''cosmic inflation'') ended by $10^{-32}$ seconds (a tiny fraction of a second) after the hot big bang. They believe that since that time, expansion initially decelerated (from gravity) and then, after about 6 billion years, began very slowly to accelerate (from dark energy). Many cosmologists proposed different starting mechanisms for initiating and fine-tuning the ostensible 'inflation'. In this context, we would like to stress the fact that, with $R_0 \cong \sqrt{\frac{2}{\left[(\Omega_{OM})_0 + (\Omega_{DM})_0\right]}}\left(\frac{c}{H_0}\right)$, the current estimated cosmic radius is 11.13Gpc.With respect to the proposed estimation/fit of the current cosmic radius/volume, the current belief in cosmic inflation can be reviewed and possibly can be relinquished.

### 8.4 CMBR Fluctuations

Temperature fluctuations are directly proportional to actual galactic ordinary matter density fluctuations. Clearly speaking, observed hot spots and cold spots can be interpreted with higher and lower (ordinary) matter densities pertaining to galactic surroundings.

### 8.5 Various Cosmological Physical Parameters

See the following Figure3 for various cosmological physical parameters. In Figure-3, the x-axis represents $\gamma_t \cong (1 \text{ to } 141.256)$.On the y-axis:

a) Ln(Ht) represents the natural log of the decreasing cosmic Hubble parameter.
b) Ln(Tt) represents the natural log of decreasing cosmic temperature.
c) Ln(Mt) represents the natural log of increasing cosmic ordinary matter and dark matter.
d) Ln(Rt) represents the natural log of increasing cosmic radius.
e) Ln(Tt) represents the natural log of increasing cosmic time.

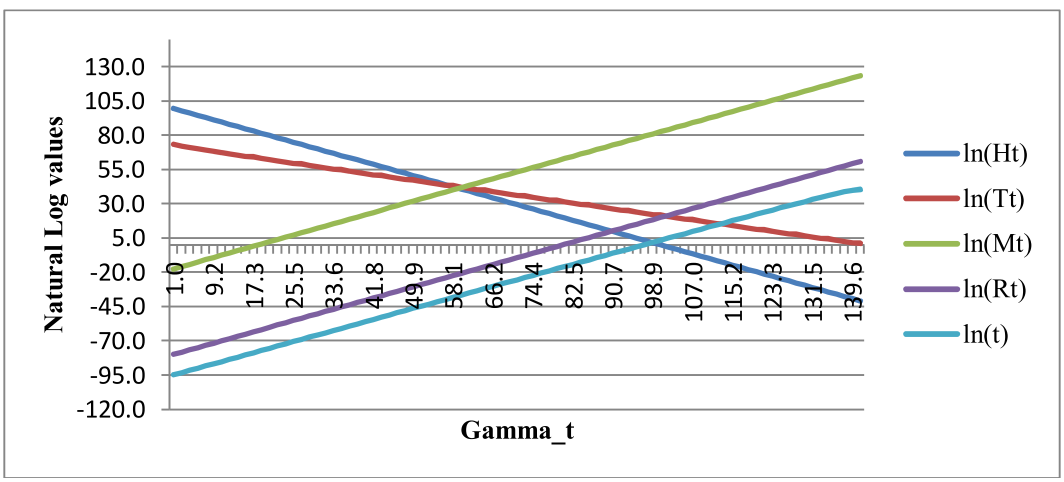


**Figure 3:** Natural log of various cosmic physical parameters

### 8.6 Hubble's Law and Cosmic Expansion Velocity

More recently, mainstream cosmologists have seriously begun working on 'eternal light speed expansion' [17-22]. In this context and from our earlier published papers based on ordinary matter density and Hubble's law, we came across different magnitudes of cosmic expansion velocities ranging from $2c$ to $12c$. We would like to propose that, by considering the decreasing density of ordinary matter and dark matter, starting from the Planck scale, it is possible to get an expression for the cosmic expansion velocity comparable to speed of light. It can be expressed as follows:

$$\left.\left(V_{\exp}\right)_t \cong R_t H_t \cong \sqrt{\frac{2}{\left[\left(\Omega_{OM}\right)_t + \left(\Omega_{DM}\right)_t\right]}}\,(c)\right\} \tag{21}$$

$$\frac{\left(V_{\exp}\right)_t}{c} \cong \sqrt{\frac{2}{\left[\left(\Omega_{OM}\right)_t + \left(\Omega_{DM}\right)_t\right]}} \tag{22}$$

Based on this expression, for the Planck scale, $\left(V_{\exp}\right)_{pl} \cong 1.414c$ and for the current scale, $\left(V_{\exp}\right)_0 \cong 2.60c.$ An interesting point to be highlighted is that after 14 billion years of cosmic expansion, the increment in expansion velocity seems to only be $\left[\left(V_{\exp}\right)_0 - \left(V_{\exp}\right)_{pl}\right] \cong 1.186c.$ We are working on accommodating this kind of approach in our future toy models.

### 8.7 Conclusion

In this context, we would like to suggest that:

a) Cosmic expansion, the lambda term, dark matter, cosmic temperature, inflation, cosmic acceleration and dark energy and vacuum energy are different concepts with which alternative models of GTR are emerging and are being extended in many ways.
b) Quantum gravity is a wide-range physical model intended for understanding built-in cosmological quantum phenomena on small scale as well as large scale distances. So far, progress in this direction is very nominal and 'GTR' needs serious review with reference to 'quantum cosmology' [23-29].
c) The current cosmic radius is about 11.13Gpc and the current cosmic sphere seems to constitute around 18 Hubble spheres and needs further study with respect to the Bayesian model average estimate of >251 Hubble spheres proposed by M. Vardanyan et al.[30].
d) With reference to particle physics, current technological limits on particle colliding energy, unidentified/unseen particles, unknown particle interactions and incomplete final unification scheme - to some extent, one can hopefully believe in the existence of dark matter. Even though its believed proportion is around 70% and numerous surveys are going on to detect dark energy, so far, no one has found a single clue for tracing its physical identity or

existence. In this identity crisis, it is reasonable to note that at least thermal energy density loss due to cosmic stretching seems to have some physical meaning and identity.

e) Even though subject of 'inflation' is very interesting, root causes of inflation are still very unclear. To understand ground reality, we are working on understanding the concepts of 'inflation' in a quantum gravitational approach to enable it to be incorporated into our toy model.

f) In standard cosmology, there exists no procedure in understanding dark energy, dark matter and ordinary matter in a unified approach. Our proposed concepts and relations can be recommended for further research.

g) Without knowing actual galactic distances and actual galactic receding speeds with a 100% confidence level, it may not be possible to decide the absolute nature of cosmic expansion rate. With reference to our proposed concepts, we are working on understanding the need of considering the observed galactic redshifts and their estimated distances in inferring the actual cosmic expansion rate.

h) Independent of galactic redshift data, we are working on finding alternative tools for understanding the cosmic expansion rate. In the future, with advanced science, engineering and technology, by considering $\frac{d\left(\lambda_{\max}\right)_0}{dt}$ or $\frac{d\left(T_0\right)}{dt}$ or $\frac{d\left(H_0\right)}{dt}$ or $\frac{d\left(\Omega_{OM}\right)_0}{dt}$ or $\frac{d\left(\Omega_{DM}\right)_0}{dt}$, we are confident that the absolute cosmic rate of expansion can be estimated.

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