Article

The Project of the Quantum Relativity

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Abstract

The intrinsic unification of the quantum theory and relativity has been discussed here in the light of the last developments. Such development is possible only on the way of the serious deviation from traditional assumptions about a priori spacetime structure and the Yang-Mills generalization of the well known $U(1)$ Abelian gauge symmetry of the classical electrodynamics. In fact, more general gauge theory should be constructed. Formally we deal with the quantum version of the gauge theory of the deformable bodies - the gauge theory of the deformable quantum state. More physically this means that the distance between quantum states is strictly defined value whereas the distance between bodies (particle) is an approximate value, at best. Thereby, all well known solid frames and clocks even with corrections of special relativity should be replaced by the flexible and anholonomic quantum setup. Then Yang-Mills arguments about the spacetime coordinate dependence of the gauge unitary rotations should be reversed on the dependence of the spacetime structure on the gauge transformations of the flexible quantum setup. One needs to build “inverse representation” of the unitary transformations by the intrinsic dynamical spacetime transformations. In order to achieve such generalization one needs the general footing for gauge fields and for ”matter fields”. Only fundamental pure quantum degrees of freedom like spin, charge, hyper-charges, etc., obey this requirement. One may assume that they correspond some fundamental quantum motions in the manifold of the unlocated quantum states (UQS’s). Then “elementary particles” will be represented as a dynamical process keeping non-linear coherent superposition of these fundamental quantum motions.

Keywords: Quantum theory, relativity, quantum relativity, Yang-Mills generalization.

1 Introduction

Quantum mechanics (QM) is not logically closed and cannot be such a theory [1, 2, 3]. Developments of quantum field theory, theory of elementary particles (in the framework of the Standard Model), and recent astronomical observation clearly tell that initial assumption about Minkowski spacetime structure in the vicinity of “elementary” quantum particles was too simple. Probably, Einstein was correct and in this matter: bodies don’t move in spacetime. If we apply this assumption to extended quantum particles like electrons then it will be agreed with the experimental impossibility to find their finite “effective” radius: one may say that this simply is zero since quantum particles move in a different space. Better to say that the radius of elementary particle does not have an invariant sense (relative a choice of setup) since it is state-dependent. If one assumes that the “real placement” of quantum particles is some Hilbert space of the quantum states then the most general physically motivated invariant is the action and, therefore, there is the problem of the separation of momentum from distance and energy from time interval and, generally, physical dynamical variables from geometric parameters.

The state-dependent dynamics was already demonstrated due to essential achievements of QM itself in the framework of so-called Complex Mechanics (CM) [4, 5]. In fact, relativistic QM needs a modification as well as the Newton’s mechanics was generalized to relativistic kinematics and dynamics under the influence of the Maxwell electrodynamics. *Quantum geometry should be related to the state-dependent invariants of elementary particles since the fundamental quantum degrees of freedom are invariant relative changes of quantum setups.* The infinitesimal version of such invariance for two slightly different setups S_1 and S_2 will be realized for two slightly different values of the boson electromagnetic-like field. This field taking the place of the functional argument of the total (Schrödinger) quantum state “cum location” whereas the unlocated quantum state (UQS) correspond to QDF’s [8, 9, 7, 6, 2, 3].

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The gauge field is commonly treated as the mean of the momentum "improvements" in respect with the gradient transformations due to introduction of the (non-affine) connection in the fiber bundle over physical spacetime. Such construction looks very realistic as the direct generalization of the definitely correct the Abelian gauge symmetry $U(1)$ of the classical electrodynamics. Nevertheless, such generalization leads to heavy artificial problems in QFT. Besides this, the separation between gauge fields and the "fields of matter" thereby obtains the forever legitimation which cannot be accepted from the principle point of view. The unification of the relativity and quantum principles is possible on the level of the quantum degrees of freedom (QDF) that are common for all kinds of physical fields. Namely, more general version of the gauge invariance relative the local projective coordinates transformations will be used. The spacetime and its transformations will be built "from inside" due to separation from the isotropy subgroup $H = U(1) \times U(N-1)$ of $G = SU(N)$ acting on the quantum state space of rays $CP(N-1)$ by the diffeomorphic coset transformations $G/H = SU(N)/S[U(1)\times U(N-1)] = CP(N-1)$. This approach means that the Yang-Mills arguments about the spacetime coordinate dependence of the gauge unitary rotations should be reversed on the dependence of the spacetime structure on the gauge transformations of the flexible quantum setup.

2 Quantum relativity

The principle of Quantum Relativity (QR) (I called this principle initially as "super-relativity" [19, 20]) assumes the invariance of physical properties of "quantum particles", i.e. their quantum numbers like mass, spin, charge, etc., lurked behind two amplitudes $|\Psi_1>, |\Psi_2>$ in two setups S_1 and S_2. The invariant content of these properties will be discussed here under the infinitesimal variation of the "flexible quantum setup" described by the amplitudes $|\Psi(\pi, p, q)>$ due to a small variation of the boson electromagnetic-like field $P^\alpha(p, q)$ serving as the coefficient functions of LDV's $\boldsymbol{D}_\alpha = \Phi^i_\alpha \frac{\partial}{\partial \pi^i} + c.c.$ on the complex projective Hilbert space $CP(N-1)$ of QDF's [16]. I put here short explanations for the clarity.

The mathematical formulation of the QR principle is based on the *similarity* of any physical systems ("setup", if somebody wants) which are built on the "elementary" particles. This similarity is obvious only on the level of pure quantum degrees of freedom of quantum particles. Therefore, all "external" details of the "setup" should be discarded as non-essential and only the ratios of the components of the "unitary spin" like ($\pi^1 = \frac{\psi^2}{\psi^1}, ..., \pi^{N-1} = \frac{\psi^N}{\psi^1}$) should be taken into account. These ratios are the local projective coordinates in the complex projective Hilbert space $CP(N-1)$. One may think about these coordinates as parameters of the "shape of quantum particle" in the spirit of the [12]. This "shape" is unlocated quantum state (UQS) of the "unitary spin". These coordinates are analog of an *angle* in the trigonometry that is the invariant characteristic of all similar triangles. Thereby, the coefficients functions Φ^i_α of the generators of $SU(N)$ defined as the Lie derivative of the ratios π^i under the infinitesimal unitary variation

$$\Phi^i_\sigma = \lim_{\epsilon \to 0} \epsilon^{-1} \left\{ \frac{[\exp(i\epsilon\lambda_\sigma)]^i_m \psi^m}{[\exp(i\epsilon\lambda_\sigma)]^j_m \psi^m} - \frac{\psi^i}{\psi^j} \right\} = \lim_{\epsilon \to 0} \epsilon^{-1} \{\pi^i(\epsilon\lambda_\sigma) - \pi^i\}, \tag{2.1}$$

[16] may be treated as the *special functions of this "unitary spin"* as the analog of the *sin* and *cos* functions. The parameter ϵ has different physical sense depends on the choice of the $SU(N)$ generator.

The operators $\boldsymbol{D}_\alpha = \sum_i \Phi^i_\alpha \frac{\partial}{\partial \pi^i}$ comprise the unholonomic basis - the "flexible quantum setup" (FQS) whose "orientation" will be given by the gauge electromagnetic-like fields [16] that will be found. One should take into account that dynamical spacetime (DST) must be introduce intrinsically only by the means of the geometry of the $CP(N-1)$ and FQS (quantum geometry).

Such approach dictates the new formulation of the inertia principle [9, 7, 6] and a new expression for the unified energy-momentum-potential of the massive particle like electron together with four-potential. New equation cannot contain the mass as a free parameter but as consequence of the natural geometric restriction. Physically this means that ordinary separation the mass from the acceleration is not allowed. Acceleration is perfectly defined for a material point, angle velocity and kinetic momentum applicable for a classical solid body but in the case of "elementary particle" these notions are not so clear because they are state-dependent and depend on an environment. How we should take into account this dependence? The good allusion gives the gauge theory of the classical deformable body [12]. One should distinguish the "total quantum state" (cum location) as an analog of the spatial coordinates of the system of material points with their "orientation coordinates", and the "unlocated quantum state" of the quantum vacuum (QV) as an analog of the "unlocated shape coordinates".

3 Quantum vacuum

The quantum vacuum (QV) being understood as the motion of the quantum degrees of freedom (QDF's) under the unitary transformations comprises the manifold of the unlocated quantum states (UQS's). These "elementary" motions (say, spin/charge currents in $CP(3)$ discussed below) replace "elementary particles" of the Standard Model. Its localizable in DST excitations then realized as known "elementary particles". The intrinsic "unitary field" acting without super-selection rule continuously splits the multiplete of the spin, charge, hypercharge, etc., into zones. QDF's acts as unified "chiral" field whose dynamics will be discussed properly.

The fundamental quantum degrees of freedom like spin, charge, hyper-charges, etc., are common for gauge and matter fields. These fundamental quantum motions take the place in the manifold of the UQS's which described by the rays of states $|\psi>\in C^N$ of the "unitary spin" $S : 2S+1 = N$. Physics requires to use in this background the local coordinates of UQS's and the state-dependent generators of the unitary group $G = SU(N)$ [19, 20, 16]. This nonlinear representation of the $SU(N)$ group on the coset manifold $G/H = SU(N)/S[U(1)\times U(N-1)] = CP(N-1)$ is primary and this is independent on the spacetime manifold. The last one should be introduced in a special section of the fiber bundle over $CP(N-1)$ [19, 20, 10, 8, 9, 7, 6]. The breakdown of the global $SU(N)$ symmetry down to the isotropy subgroup $H_{|\psi>} = U(1)\times U(N-1)$ of the some quantum state $|\psi>$ has natural geometric counterpart in $CP(N-1)$.

The coset manifold $G/H_{|\psi>} = SU(N)/S[U(1)\times U(N-1)] = CP(N-1)$ contains locally unitary transformations *deforming* "initial" quantum state $|\psi>$. This means that $CP(N-1)$ contains physically distinguishable, "deformed" quantum states. Thereby the unitary transformations from $G = SU(N)$ of the basis in the Hilbert space may be identified with the unitary state-dependent gauge field $U(|\psi>)$ that may be represented by the N^2-1 unitary generators as functions of the local projective coordinates $(\pi^1, ..., \pi^{N-1})$ [9]. This manifold resembles the "shape space" of the deformable body [12, 9, 7, 6]. But now it is the manifold of the deformed physically distinguishable UQS's, i.e. the geometric, invariant counterpart of the quantum interaction or self-interaction. Then the classical acceleration is merely an "external" consequence of this complicated quantum dynamics in the some section of the frame fiber bundle over $CP(N-1)$.

Now I will introduce the necessary construction of the internal dynamics of QDF's in terms of the local coordinates π^k of UQS's. Thereby they will live in the geometry of $CP(N-1)$ with the Fubini-Study metric tensor

$$G_{ik^*} = (1/\kappa)[(1+\sum|\pi^s|^2)\delta_{ik} - \pi^{i^*}\pi^k](1+\sum|\pi^s|^2)^{-2}, \tag{3.1}$$

where κ is holomorphic sectional curvature of the $CP(N-1)$ [15]. The contra-variant metric tensor field

$$G^{ik^*} = \kappa(\delta^{ik} + \pi^i\pi^{k*})(1+\sum|\pi^s|^2), \tag{3.2}$$

is inverse to the G_{ik^*} thereby

$$G_{ik^*}G^{i^*q} = \delta^q_k. \tag{3.3}$$

The affine connection agrees with the Fubini-Study metric is as follows

$$\Gamma^i_{mn} = \frac{1}{2}G^{ip^*}\left(\frac{\partial G_{mp^*}}{\partial\pi^n} + \frac{\partial G_{p^*n}}{\partial\pi^m}\right) = -\frac{\delta^i_m\pi^{n^*} + \delta^i_n\pi^{m^*}}{1+\sum|\pi^s|^2}. \tag{3.4}$$

The curvature tensor of Riemann in holonomic basis is proportional to the constant section curvature since

$$R^i_{klm^*} = \kappa^2(\delta^i_l G_{km^*} + \delta^i_k G_{lm^*}) \tag{3.5}$$

[15].

4 The flexible quantum reference frames

The flexible quantum setup inherently connected with local projective coordinates will be built from so-called local dynamical variables (LDV's) [16]. These LDV's realize a non-linear representation of the unitary global $SU(N)$ group in the Hilbert state space C^N. Namely, N^2-1 generators of $G = SU(N)$ may be divided in accordance with the Cartan decomposition: $[B,B] \in H, [B,H] \in B, [B,B] \in H$. The $(N-1)^2$ generators

$$\Phi^i_h \frac{\partial}{\partial \pi^i} + c.c. \in H, \quad 1 \le h \le (N-1)^2 \tag{4.1}$$

of the isotropy group $H = U(1) \times U(N-1)$ of the ray (Cartan sub-algebra) and $2(N-1)$ generators

$$\Phi^i_b \frac{\partial}{\partial \pi^i} + c.c. \in B, \quad 1 \le b \le 2(N-1) \tag{4.2}$$

are the coset $G/H = SU(N)/S[U(1) \times U(N-1)]$ generators realizing the breakdown of the $G = SU(N)$ symmetry. Notice, the partial derivatives are defined here as usual: $\frac{\partial}{\partial \pi^i} = \frac{1}{2}(\frac{\partial}{\partial \Re \pi^i} - i\frac{\partial}{\partial \Im \pi^i})$ and $\frac{\partial}{\partial \pi^{*i}} = \frac{1}{2}(\frac{\partial}{\partial \Re \pi^i} + i\frac{\partial}{\partial \Im \pi^i})$.

Here $\Phi^i_\sigma, \quad 1 \le \sigma \le N^2-1$ are the coefficient functions of the generators of the non-linear $SU(N)$ realization. They give the infinitesimal shift of the i-component of the generalized coherent state driven by the σ-component of the unitary field $\exp(i\epsilon\lambda_\sigma)$ rotating by the generators of $AlgSU(N)$ and they are defined as follows:

$$\Phi^i_\sigma = \lim_{\epsilon \to 0} \epsilon^{-1} \left\{ \frac{[\exp(i\epsilon\lambda_\sigma)]^i_m \psi^m}{[\exp(i\epsilon\lambda_\sigma)]^j_m \psi^m} - \frac{\psi^i}{\psi^j} \right\} = \lim_{\epsilon \to 0} \epsilon^{-1} \{\pi^i(\epsilon\lambda_\sigma) - \pi^i\}, \tag{4.3}$$

[16].

Quantum reference frames (QRF) will be used as an analog of the classical deformable solid body. One needs the QRF as an internal quantum analog of the "setup" since the spacetime distance should be replaced by the distance between UQS's (Fubini-Study metric) and the QRF "orientation" will given by the functional coefficients (affine gauge fields of the energy-momentum and electromagnetic-like potentials).

The main idea of the affine quantum gauge theory is as follows: *the curvature tensor of the group sub-manifold $CP(N-1)$ is the non-singular tensorial source of the electromagnetic, etc. interactions.* Thereby, the curvature of the $SU(N)$ is the true reason of such anholonomy as the geometric phase. The physics is free from singularities. Degeneracy and singularity are merely the mathematical properties of the mapping and they are false reasons of the fictional "electric" and "magnetic" fields. It should be noted that so-called "covariant derivative" in spacetime including Abelian or non-Abelian gauge potential will be replaced by the true covariant derivative in the affine connection agrees with Fubini-Study metric in $CP(N-1)$ [7, 6].

More technically one may note that the Riemann tensor of the curvature in $CP(N-1)$ guarantees the most general gauge invariance due to its pure locality of the action: quantum physics is the same anywhere. Locality of the vector field of LDV's instead of the bi-locality of the probabilistic approach rids us from the measurement dependence from pre-history and post-history [10] and from the misty assumption of Multiverse. The transversal and longitudinal gauge fields of Jacobi clearly related to the curvature tensor in $CP(N-1)$ [7, 6].

The operator of the curvature tensor in the two-dimension direction (α, β) in the adjoint representation of $SU(N)$ acting on the vector field $X^n(\pi, \pi^*)$ is as follows

$$\begin{aligned} R(D_\alpha, D_\beta)X^k = \{[\nabla_{D_\alpha}, \nabla_{D_\beta}] - \nabla_{[D_\alpha, D_\beta]}\}X^k \\ = \{(D_\alpha \Phi^i_\beta - D_\beta \Phi^i_\alpha)\Gamma^k_{in} + (\Phi^i_\alpha \Phi^{s^*}_\beta - \Phi^i_\beta \Phi^{s^*}_\alpha)R^k_{ins^*} \\ + \Phi^m_\alpha \Gamma^k_{mp} \Phi^i_\beta \Gamma^p_{in} - \Phi^i_\beta \Gamma^k_{ip} \Phi^m_\alpha \Gamma^p_{mn} - C^\gamma_{\alpha\beta} \Phi^i_\gamma \Gamma^k_{in}\}X^n. \end{aligned} \tag{4.4}$$

This operator was initially calculated in [10] without clear physical interpretation. I show that this operator is the **tensorial charge** of the multipole of the unitary field. Indeed, it is poli-linear operator in fields and this gives the Coriolis tensor describing vortexes of UQS's in $CP(N-1)$. It is important that at the origin ($\pi^1 = ... = \pi^{N-1} = 0$) all terms will be equal zero, besides

$$\begin{aligned} R(D_\alpha, D_\beta)(0)X^k = (\Phi^i_\alpha \Phi^{s^*}_\beta - \Phi^i_\beta \Phi^{s^*}_\alpha)R^k_{ins^*}(0)X^n \\ = \kappa(\Phi^i_\alpha \Phi^{s^*}_\beta - \Phi^i_\beta \Phi^{s^*}_\alpha)(\delta^k_n \delta_{is^*} + \delta^k_i \delta_{ns^*})X^n \\ = \kappa(\Phi^k_\alpha \Phi^{n^*}_\beta - \Phi^k_\beta \Phi^{n^*}_\alpha)X^n \end{aligned} \tag{4.5}$$

where $\alpha, \beta = b$ in horizontal direction, since all vertical components $\Phi^i_h(0) = 0$.

Now it may be assumed that the unification of the fundamental interactions is possible in new manner: the different components of the single universal tensorial charge will be correspond to different kinds of interactions.

One may assume that that the curvature tensor

$$\begin{aligned} R(D_\mu, D_\nu)X^k = \{[\nabla_{D_\mu}, \nabla_{D_\nu}] - \nabla_{[D_\mu, D_\nu]}\}X^k \\ = \{(D_\mu \Phi^i_\nu - D_\nu \Phi^i_\mu)\Gamma^k_{in} + (\Phi^i_\mu \Phi^{s^*}_\nu - \Phi^i_\nu \Phi^{s^*}_\mu)R^k_{ins^*} \\ + \Phi^m_\mu \Gamma^k_{mp} \Phi^i_\nu \Gamma^p_{in} - \Phi^i_\nu \Gamma^k_{ip} \Phi^m_\mu \Gamma^p_{mn} - \tilde{C}^\lambda_{\mu\nu} \Phi^i_\lambda \Gamma^k_{in}\}X^n. \end{aligned} \tag{4.6}$$

being defined by the Dirac's vector fields in the two dimension direction (μ, ν) where $0 \leq \mu, \nu \leq 3$, $1 \leq \lambda \leq 15$ and $\tilde{C}^\lambda_{\mu\nu}$ is linear combination of the structure constants, will be related to the spacetime components of electromagnetic-like fields. Calculation of this tensor gives the result $R(D_1, D_2)(0)X^3 = (1-i)X^3$ of the complex rotation at the origin of the local map. This provides the mentioned above the "inverse representation" of the $CP(3)$ infinitesimal motions by the infinitesimal boost and rotation due to the anholonomy generated by the curvature of the $CP(3)$.

5 The total quantum state of the extended quantum electron (the quantum state cum location)

The imperturbable confidence in the collision method of "palpation" of the deep zone of "elementary" particles is close to the end because the physics-imposed limit of this method [13]. In fact this method of investigation is not applicable to the root problems of the self-interaction and stability of elementary particles since the typical energy of collisions is much higher than rest masses. Beside this, the Higgs-mechanism of the mass generation seems to me very questionable [14]. It is obvious that behind the success of the QFT and SM lies the shadow of the divergences and anomalies. Say, the oldest problem of the accelerated charged particle is one of the acute challenges for QFT, high energy physics, and for the theory of elementary particles. There is an interesting attempt to solve this problem in the spirit of my concept of the deformation of UQS [11]. Namely, the "backreaction of space" clearly close to the DST concept [8, 9, 7, 6]. Physically this concept is based on the absence of the solid scales of spatial distance and time interval at the subatomic area, therefore one needs some flexible (state-dependent) quantum setup and its appropriate mathematical description.

Attempt to build the QFT over UQS space $CP(N-1)$,i.e. the field theory where the spacetime separation between bodies (particles) was replaced by the distance between UQS's leads to the deep problem of the separation of the spacetime and energy-momentum variable since the invariant sense has only *action interval*. Thereby the notion of the "acceleration" as a reaction of the UQS on a deformation should be clarified.

Intrinsic introduction of the DST requires attachment of the local Lorentz reference frame to some UQS $|\psi>$ in $CP(N-1)$, not to a body. It is assumed that to infinitesimally close quantum states connected by the $H_{|\psi>} = U(1) \times U(N-1)$ correspond two infinitesimally close Lorentz reference frames. That is only infinitesimal state-dependent Lorentz transformations have a sense. These transformation should be separated from the gauge transformations given by the Jacobi vector field corresponding the geodesic rotations (deformations).

We deal with following problem. Let assume that there is a single quantum electron of Dirac. The problem of the self-interaction, i.e. "off-shell" zone of the dispersion law is in the focus of our attention. This may be presumably formulated as a "diffusion" of the mass-shell and short range deformation of the light cone (the long range deformation was studied initially by Einstein in the framework of general relativity) treated as the perturbation of the "square root of the cosmic potential" c^2 whose value defined in the Dirac theory by the eigenvalues of the unitary matrices $\frac{d\boldsymbol{x}}{dt} = [H, \boldsymbol{x}] = c\alpha_{\boldsymbol{x}}$. In order to do this one needs replace the fifteen $SU(4)$ Dirac matrices by the fifteen state-dependent vector fields which evidently show the deviation from the relativistic mass-shell relation.

Let me introduce new definition of the local DST as the special linear combinations of the Lie derivatives of the local projective coordinates (π^1, π^2, π^3) in directions given by the Dirac matrices in the Weyl representation. This construction is most transparent for the fundamental fermion like the electron. More general case of higher dimension should be discussed elsewhere. For this aim I will use the following set of the Dirac matrices

$$\gamma_t = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \gamma_1 = -i\sigma_1 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$
$$\gamma_2 = -i\sigma_2 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \gamma_3 = -i\sigma_3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}. \quad (5.1)$$

Then the corresponding coefficients of the $SU(4)$ generators will be calculated according to the equation

$$\Phi^i_\mu = \lim_{\epsilon \to 0} \epsilon^{-1} \left\{ \frac{[\exp(i\epsilon\gamma_\mu)]^i_m \psi^m}{[\exp(i\epsilon\gamma_\mu)]^j_m \psi^m} - \frac{\psi^i}{\psi^j} \right\} = \lim_{\epsilon \to 0} \epsilon^{-1} \{\pi^i(\epsilon\gamma_\mu) - \pi^i\}, \quad (5.2)$$

[9] that gives

$$\begin{aligned} \Phi^1_0(\gamma_t) &= i(\pi^3 - \pi^1\pi^2), \quad \Phi^2_0(\gamma_t) = i(1 - (\pi^2)^2), \quad \Phi^3_0(\gamma_t) = i(\pi^1 - \pi^2\pi^3); \\ \Phi^1_1(\gamma_1) &= -i(\pi^2 - \pi^1\pi^3), \quad \Phi^2_1(\gamma_1) = -i(-\pi^1 - \pi^2\pi^3), \quad \Phi^3_1(\gamma_1) = -i(-1 - (\pi^3)^2); \\ \Phi^1_2(\gamma_2) &= -i(i(\pi^2 + \pi^1\pi^3)), \quad \Phi^2_2(\gamma_2) = -i(i(\pi^1 + \pi^2\pi^3)), \quad \Phi^3_2(\gamma_2) = -i(i(-1 + (\pi^3)^2)); \\ \Phi^1_3(\gamma_3) &= -i(-\pi^3 - \pi^1\pi^2), \quad \Phi^2_3(\gamma_3) = -i(-1 - (\pi^2)^2), \Phi^3_3(\gamma_3) = -i(\pi^1 - \pi^2\pi^3). \end{aligned} \quad (5.3)$$

Such choice of the vector fields lead to the "imaginary" basic in local DST which conserves $4D$ Eucledian geometry along geodesic in $CP(3)$ for real four vectors (p^0, p^1, p^2, p^3) and correspondingly $4D$ pseudo-Eucledian geometry for four vectors (ip^0, p^1, p^2, p^3).

The complex DST "tangent vector" in μ direction defines the four complex shifts in DST that will be introduced as follows:

$$\frac{\partial}{\partial x^\mu} = \Phi^i_\mu \frac{\partial}{\partial \pi^i} \quad (5.4)$$

for $0 \le \mu \le 3$. In fact one may define the similar "tangent vector" in α direction

$$\frac{\partial}{\partial x^\alpha} = \Phi^i_\alpha \frac{\partial}{\partial \pi^i} \tag{5.5}$$

for $1 \le \alpha \le 15$ in the space R^{15} of the adjoint representation of the $SU(4)$. Thereby, the DST cannot be treated as the "space of events". It is rather 8-dimension subspace of the adjoint representation of the $SU(4)$. The quantum operator of the energy-momentum will be expressed in ordinary manner

$$\boldsymbol{P}_\mu = i\hbar \frac{\partial}{\partial x^\mu} = i\hbar \Phi^i_\mu \frac{\partial}{\partial \pi^i}. \tag{5.6}$$

The eight λ-matrices $(\lambda_4, \lambda_{11}), (\lambda_2, \lambda_{14}), (\lambda_1, \lambda_{13}), (\lambda_5, \lambda_{12})$ of the $AlgSU(4)$ were involved in the definition of the shift vector fields. There are additional seven λ-matrices $(\lambda_6, \lambda_7), (\lambda_9, \lambda_{10}), (\lambda_3), (\lambda_8), (\lambda_{15})$ involved in the definition of the boosts, rotations and gauge parametrization that all together with the eight λ-matrices comprise of the full set of the fifteenth matrices of the $AlgSU(4)$.

One may see that such definition of the "spacetime derivative" in x^μ direction provides the ordinary properties of the derivative in spacetime, namely: the linearity for the linear superposition

$$\frac{\partial[af(\pi) + bg(\pi)]}{\partial x^\mu} = \Phi^i_\mu \frac{\partial[af(\pi) + bg(\pi)]}{\partial \pi^i} = a\frac{\partial f(\pi)}{\partial x^\mu} + b\frac{\partial g(\pi)}{\partial x^\mu}, \tag{5.7}$$

symmetry for the multiplication of two functions

$$\frac{\partial[f(\pi)g(\pi)]}{\partial x^\mu} = \Phi^i_\mu \frac{\partial[f(\pi)g(\pi)]}{\partial \pi^i} = g(\pi)\frac{\partial f(\pi)}{\partial x^\mu} + f(\pi)\frac{\partial g(\pi)}{\partial x^\mu}, \tag{5.8}$$

and the chain rule for superposition of two functions

$$\frac{\partial f[g(\pi)]}{\partial x^\mu} = \Phi^i_\mu \frac{\partial f[g(\pi)]}{\partial \pi^i} = \Phi^i_\mu \frac{\partial f}{\partial g}\frac{\partial g(\pi)}{\partial \pi^i} = \frac{\partial f}{\partial g}\frac{\partial g(\pi)}{\partial x^\mu}. \tag{5.9}$$

Notice, that DST shift is "absolute" in the flexible reference frame since generated by physically essential (invariant) deformations of UQS by (π^1, π^2, π^3) variations.

The metric of the DST is state-dependent that may be demonstrated directly by the calculations of the square of the speed velocity $\frac{dS^2}{d\tau^2}$ of the geodesic distance in $CP(3)$. Let me assume that one has the 4D energy-momentum constant components $P^\mu = const$ taking the place of the "target parameters". Then along the geodesic given by the equation

$$\pi^1 = \frac{f^1}{g}\tan g\tau, \pi^2 = \frac{f^2}{g}\tan g\tau, \pi^3 = \frac{f^3}{g}\tan g\tau, \tag{5.10}$$

in the complex direction $(f^1 = c_1 + is_1, f^2 = c_2 + is_2, f^3 = c_3 + is_3)$ where $g = \sqrt{|f^1|^2 + |f^2|^2 + |f^3|^2}$ one has

$$\begin{aligned}\frac{dS^2}{d\tau^2} &= G_{ik^*}\frac{d\pi^i}{d\tau}\frac{d\pi^{k^*}}{d\tau} = G_{ik^*}P^\mu \Phi^i_\mu P^{\nu^*}\Phi^{k^*}_\nu + \Delta^2 \\ &= g_{\mu\nu^*}P^\mu P^{\nu^*} + \Delta^2 = -(P^0)^2 + (P^1)^2 + (P^2)^2 + (P^3)^2 + \Delta^2,\end{aligned} \tag{5.11}$$

where $\Delta^2 = G_{ik^*}P^a\Phi^i_a P^{b^*}\Phi^{k^*}_b = \eta_{ab^*}P^a P^{b^*}$ under the initial conditions $(f^1 = 1 + i, f^2 = 0, f^3 = 0)$. If one takes the different initial conditions for the rotated geodesic $(f^1 = 0, f^2 = 1 + i, f^3 = 0)$. Then one has

$$\frac{dS^2}{d\tau^2} = \tilde{G}_{ik^*}\frac{d\tilde{\pi}^i}{d\tau}\frac{d\tilde{\pi}^{k^*}}{d\tau} = \tilde{G}_{ik^*}P^\mu \tilde{\Phi}^i_\mu P^{\nu^*}\tilde{\Phi}^{k^*}_\nu + \tilde{\Delta}^2$$

$$= \tilde{g}_{\mu\nu^*} P^\mu P^{\nu^*} + \tilde{\Delta}^2 = -(P^0)^2 + (P^1)^2 + (P^2)^2 + (P^3)^2$$
$$+2[(P^0)^2 + (P^3)^2 - 2iP^0P^3](\cos g\tau)^2(\sin g\tau)^2 + \tilde{\Delta}^2. \quad (5.12)$$

"Diffusion" of the mass-shell is evident here but the scale of such diffusion is unknown since the value of the sectional curvature κ included in G_{ik^*} is a free parameter up to now. The key idea of the mass-shell diffusion closely connected with the non-separability of the inertial mass m from the acceleration $\frac{d^2x}{dt^2}$ in the Newton's expression for the force $\frac{dp}{dt} = m\frac{d^2x}{dt^2} = F$. One needs the quantum expression for the velocity of the energy-momentum variation. The simplest non-trivial expression of the quantum momentum in $CP(N-1)$ gives $P^i = \frac{d\pi^i}{d\tau}$ [9]. It was assumed that this momentum in $CP(N-1)$ may be expressed as the contraction of the $SU(N)$ generator $\Phi^i_\mu \frac{\partial}{\partial \pi^i} + c.c.$ in the projective representation and the energy-momentum $P_\mu(x)$ in the local DST that should be found due to the new formulation of the quantum inertia principle [8, 9, 7, 6]. *One will see below that the speed of the momentum variation will be treat now as field equation with localizable solution instead of the equation for trajectory of the point-wise particle.*

It is not the problem of propagation of the EM field an its value in the remote area in a remote reference frame. The problem is to find self-consistent self-field (I called this field as the "field-shell") in the "proper" reference frame intrinsically defined by the pure quantum means. There is no initially prescribed spacetime coordinates at all. One has initially pure UQS with three complex coordinates (π^1, π^2, π^3). There are fifteen vector fields of the adjoint representation of the $SU(4)$ generators concerning dipole, quadruple and octuple moments of in the "unitary field" of the coherent spin/charge degrees of freedom.

The numbers x_μ play the role of coordinates of the placeholder in the complex gradient of the action. In fact these coordinates should be initially separated from the full set of the variables $P^\alpha(q,p)$ of the total quantum state denoted by the $|\Psi(\pi, q, p)>$. The Hamiltonian vector field

$$\boldsymbol{H}(q,p,\pi) = \sum_{\alpha=1}^{N^2-1} \sum_{i=1}^{N} P^\alpha(q,p)\Phi^i_\alpha \frac{\partial}{\partial \pi^i} + c.c. \quad (5.13)$$

with

$$\frac{d\pi^i}{d\tau} = \frac{c}{\hbar} \sum_{\alpha=1}^{N^2-1} P^\alpha(q,p)\Phi^i_\alpha \quad (5.14)$$

lead to the "Schrödinger equation"

$$i\hbar \frac{|\Psi(\pi,q,p)>}{d\tau} = cP^\alpha \Phi^i_\alpha \frac{\partial|\Psi(\pi,q,p)>}{\partial \pi^i} + c.c. = C|\Psi(\pi,q,p)> . \quad (5.15)$$

Notice, the "setup" will be specified by the action that should be found due to solution of the Schrödinger-like field equations for the "total wave function cum location" $|\Psi(\pi^i,q,p)>$ of self-interacting quantum electron moving in DST like a material point with the rest dynamical mass $m(\pi,q,p)$ and continuous spin/charge variable (π^1,π^2,π^3). The coordinates (p,q) correspond to the shifts, rotations, boosts and gauge parameters of the local DST.

This means that the first equality in (5.15) is the tautology if

$$\frac{d\pi^i}{d\tau} = \frac{c}{\hbar} P^\alpha \Phi^i_\alpha; \frac{d\pi^{i*}}{d\tau} = \frac{c}{\hbar} P^{\alpha*} \Phi^{i*}_\alpha, \quad (5.16)$$

and the last one is the equation for the eigen-state problem. The most primitive motion of the QDF's is the motion along geodesic in $CP(N-1)$ thereby P^α taking the place of the generalized momentum-potentials (inertial term mc plus electromagnetic-like self-potentials), whereas their variations related to the Jacobi vector field describing electromagnetic-like fields.

The covariant derivative in the sense of the Fubini-Study metric of the right part should be zero

$$(P^\alpha \Phi^i_\alpha)_{;k} = \frac{\partial P^\alpha}{\partial \pi^k} \Phi^i_\alpha + P^\alpha (\frac{\partial \Phi^i_\alpha}{\partial \pi^k} + \Gamma^i_{kl} \Phi^l_\alpha) = 0. \quad (5.17)$$

Let me take initially only shifts in DST without rotations and boosts. Then in the equation (5.17) one will have the summation only of four terms

$$(P^\mu \Phi^i_\mu)_{;k} = \frac{\partial P^\mu}{\partial \pi^k}\Phi^i_\mu + P^\mu(\frac{\partial \Phi^i_\mu}{\partial \pi^k} + \Gamma^i_{kl}\Phi^l_\mu) = 0. \quad (5.18)$$

According our definition of the DST derivative for $k = i$ one may rewrite this as follows

$$\frac{\partial P^\mu}{\partial x^\mu} + P^\mu(\frac{\partial \Phi^i_\mu}{\partial \pi^i} + \Gamma^i_{il}\Phi^l_\mu) = 0. \quad (5.19)$$

Thus one has the field equation as the gauge restriction. For the parallel transported Φ^i_μ this coincides with the ordinary Lorentz gauge. This linear PDE has the traveling wave solutions (TWS), say, in the form $P^\mu = K^\mu + A^\mu F(\Phi^i_\mu)\tanh(C_0 + C_1 x + C_2 y + C_3 z + C_4 t) + B^\mu G(\Phi^i_\mu)\tanh(C_0 + C_1 x + C_2 y + C_3 z + C_4 t)^2 + H^\mu(\Phi^i_\mu)$. These solutions realize the state-dependent gauge conditions on the energy-momentum (potentials) and show that in such definition of the DST coordinates x^μ the complicated highly nonlinear field equations (5.18) transform into the linear PDE's (5.19) with localizable solutions. Thereby, the "wave front" of the action is given by the Schrödinger-like field equation (5.15) and the "rays" of localizable TWS taking the place of particles trajectories.

6 Discussion

Quantum Relativity is a new kind of the gauge theory: instead of the adaptation of the unitary transformations to spacetime location one needs to accommodate dynamical spacetime structure to the unitary field acting in the space of the pure quantum degrees of freedom. There are a lot of open questions in such approach. One of the fundamental problem is how to glue local DST's at least in the macroscopic "piece" of the Riemannian 4D spacetime. The second fundamental problem is the connection between tensorial charge, holomorphic sectional curvature and the unification of the interactions of different unitary fields. Some problems concerning PDE's look as merely technical but their solutions requires essential physical reinterpretation that will be discussed elsewhere.

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