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Five-Dimensional Spherically Symmetric Perfect Fluid Cosmological Model in Lyra Manifold

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Abstract

The main aim of this paper is to obtain five-dimensional spherically symmetric models in the presence of perfect fluid source in the gravitational theory in the frame work of Lyra (Math. Z., 54: 52, 1951) manifold. Solving the gravitational field equations using a relation between metric potentials and a power law between the displacement vector of the manifold and the average scale factor we have presented an exact solution of the field equations. It is interesting to observe that our solution represents a stiff fluid or Zeldovich fluid model in five dimensional Lyra manifold. We have also determined the kinematical and physical parameters of the model and discussed their physical behavior in cosmology.

Keywords: Lyra manifold, perfect fluid, five-dimensional, spherically symmetric.

1. Introduction

Unification of all fundamental interactions, including gravitation, has been the subject of investigation in general relativity. Higher dimensional space-times are very important for this purpose. In particular five dimensional cosmological models play a vital role in studying the cosmos at its early stages of evolution of the universe [1-2]. As the universe evolves, the extra dimensions are not observable due to dynamical contraction and compactification with the passage of time and it ultimately reduces to four dimensional continuum [3]. There is an ample literature on higher dimensional cosmological models in general relativity [4-8].

Einstein [9] has geometrized gravitational field in his general theory of relativity. This has inspired several researchers to geometrize the other physical fields. Weyl [10] has formulated a unified theory to geometrize gravitation and electromagnetism. Lyra [11] proposed a modification of Riemannian geometry by introducing an additional gauge function into the structure less manifold as a result of which the cosmological constant arises naturally from the geometry. In this theory both the scalar and tensor fields have geometrical significance. Subsequently Sen [12] and Sen and Dunn [13] suggested a new scalar tensor theory of gravitation. They modified the Einstein's field equations based on Lyra's manifold in normal gauge as

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2} \varphi_i \varphi_j - \frac{3}{4} g_{ij} \varphi_k \varphi^k = -T_{ij} \tag{1}$$

where φ_i is a displacement field and the other symbols have their usual meaning as in Riemannian geometry (Here we have chosen gravitational units so that $8\pi G = c = 1$) . We assume φ_i to be time-like so that

$$\varphi_i = (\beta, 0, 0, 0, 0) \tag{2}$$

where β is a function of cosmic time t. Jeavons et al.[14] pointed out that the field equations proposed by Sen and Dunn are heuristically useful even though they are not derived from the usual variational principle. A brief note on Lyra's geometry is given by Singh and Singh [15]

It has been shown by Halford [16] that the energy conservation law does not hold in the cosmological theory based on Lyra's geometry. Halford [17] has also shown that the scalar tensor theory of gravitation in Lyra manifold gives same effects, within observational limits, as in the Einstein theory. Soleng [18] pointed out that the constant gauge vector φ_i in Lyra's geometry together with a creation field becomes Hoyle's [19] creation field cosmology or contains a special vacuum field, which together with the gauge vector may be considered as a cosmological term. Further Soleng [20] showed that for matter with zero spin the field equations of his scalar tensor theory reduce to those of Brans-Dicke theory. Bhamra [21], Kalyanshetty and Waghmode [22], Reddy and Innaiah [23], Beesham [24], Reddy and Venkateswarlu [25], and Singh and Desikan [26] are some of the authors who have investigated various aspects of the four dimensional cosmological models in Lyra's manifold. Reddy et al. [27] have investigated Bianchi type-I cosmological model with extra dimensions in Lyra manifold while Mohanty et al. [28-29] showed the non existence of five dimensional perfect fluid cosmological model in this manifold. and obtained the exact solutions of the field equations for empty universe. Also, Mohanty et al. [30] showed that in a five dimensional space-time the general perfect fluid distribution does not survive but degenerates into stiff fluid distribution in this particular manifold. Kaluza-Klein FRW cosmological models have been constructed by Mohanty et al. [31] in Lyra geometry. Higher dimensional cosmological models, in this geometry, have also been discussed by Rahaman [32] and Rahaman et al. [33-34].

Samanta and Dhal [35] found a new class of higher dimensional cosmological models of the early universe filled with perfect fluid source in the frame work of f(R,T) gravity [36]with the help of five dimensional spherically symmetric metric. Recently Rao and Jayasudha [37-38] obtained five dimensional spherically symmetric perfect fluid models in Saez-Ballester [39] and Brans-Dicke [40] scalar-tensor theories of gravitation. Here our main motive is to discuss five dimensional spherically symmetric perfect fluid cosmological models in the theory of gravitation formulated in Lyra geometry. These models are significant because of the fact that they give us apicture of evolution of the universe at its early stages.

The plan of this paper is the following: In Section-2, we have derived the gravitational field equations in the presence of perfect fluid matter distribution in Lyra manifold when the displacement vector of the manifold is a function of cosmic time. In Section-3 we have solved thefield equations and presented perfect fluid model which turns out to be stiff fluid or Zeldovich universe. Section-4 is devoted to the summary and conclusions.

2. Field equations in Lyra manifold

Spherically symmetric five dimensional metric can be written in the form

$$ds^2 = -e^{\lambda}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) - e^{\mu} d\psi^2 + dt^2 \tag{3}$$

where λ and μ are cosmic scale factors which are functions of cosmic time t.

We consider the energy momentum tensor for matter source as

$$T_{ij} = (\rho + p) u_i u_j - p g_{ij} \tag{4}$$

where u^i=(1,0,0,0,0) is the five-velocity in co moving coordinate system which satisfies the conditions

$$u^i u_i = 1; u^i u_j = 0 \tag{5}$$

Here ρ is the energy density and p is the isotropic pressure of matter source.

Now using Eqs. (1),(2), (4) and (5) the gravitational field equations in Lyra manifold can be, explicitly written in the form

$$\frac{3}{4}\left(\dot{\lambda}^2 + \dot{\lambda}\dot{\mu}\right) - \frac{3}{4}\beta^2 = \rho \tag{6}$$

$$\ddot{\lambda} + \frac{3}{4}\dot{\lambda}^2 + \frac{1}{2}\ddot{\mu} + \frac{1}{4}\dot{\mu}^2 + \frac{1}{2}\dot{\lambda}\dot{\mu} + \frac{3}{4}\beta^2 = -p \tag{7}$$

$$\frac{3}{2}\left(\ddot{\lambda} + \dot{\lambda}^2\right) + \frac{3}{4}\beta^2 = -p \tag{8}$$

where an overhead dot indicates derivative with respect to cosmic time t.

3. Solution and the model

We observe that the Eqs.(6)-(8) are a set of three independent equations in five unknowns λ, μ, ρ, p and β .Hence to find a determinate solution we use the following two significant conditions:

(i) a relation between metric potentials given by [35]

$$\lambda = \alpha\mu \tag{9}$$

where $\alpha \neq 0$ is a proportionality constant .

(ii) the power law relation between the scalar field β and the average scale factor $a(t)$ given by

$$\beta = \beta_0 a^k \tag{10}$$

where β_0 and k is are positive constants. This we have taken following Johri and Sudharsan [41] and Johri and Desikan [42]who have studied the evolution of the universe in Brans-Dicke (BD) scalar-tensor theory by assuming a power law relation between BD scalar field and scale factor of the universe in the form $\varphi \propto a^k$.

Now from Eqs. (6)-(9) it is a simple matter to get

$$\ddot{\mu} + \left(\frac{3\alpha+1}{2}\right)\dot{\mu}^2 = 0 \tag{11}$$

Integrating Eq.(10) and using Eq.(9) we obtain

$$e^{\mu} = \left[\left(\frac{3\alpha+1}{2}\right)(\alpha_0 t + t_0)\right]^{\frac{2}{3\alpha+1}} \tag{12}$$

$$e^{\lambda} = \left[\left(\frac{3\alpha+1}{2}\right)(\alpha_0 t + t_0)\right]^{\frac{2\alpha}{3\alpha+1}} \tag{13}$$

Here α_0 and t_0 are constants of integration. Choosing $\alpha_0 = 1, t_0 = 0$ and using Eqs. (12) in Eq.(3) we can write the model, in this case as

$$ds^2 = dt^2 - \left[\left(\frac{3\alpha+1}{2}\right)t\right]^{\frac{2\alpha}{3\alpha+1}}\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] - \left[\left(\frac{3\alpha+1}{2}\right)t\right]^{\frac{2}{3\alpha+1}} d\psi^2 \tag{14}$$

4. Physical discussion of the model

Here we discuss the physical and kinematical properties of the model (14). We find the following: The average scale factor a(t), volume scale factor V are given by

$$V = a^4 = e^{\frac{3\lambda+\mu}{2}} = \left(\frac{3\alpha+1}{2}\right)t \tag{15}$$

The Hubble parameter H is found to be

$$H = \frac{\dot{a}}{a} = \frac{1}{4t} \tag{16}$$

The scalar expansion of the model is

$$\theta = u^i_i = \frac{3}{2}\dot{\lambda} + \frac{1}{2}\dot{\mu} = \frac{1}{t} \tag{17}$$

The shear scalar of the model is

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{3}{8}\dot{\lambda}^2 + \frac{1}{8}\dot{\mu}^2 - \frac{1}{2}\dot{\lambda} - \frac{1}{6}\dot{\mu} + \frac{2}{9} = \frac{3\alpha^2+1}{2(3\alpha+1)^2 t^2} - \frac{3\alpha+1}{2(3\alpha+1)t} + \frac{9}{2} \tag{18}$$

The average anisotropy parameter of the model is

$$A_h = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{\Delta H_i}{H}\right) = 0 \tag{19}$$

where $\Delta H_i = H_i$ - H (i=1,2,3,4) represent the directional Hubble parameters.

Now from the Eqs.(10) and (14), we obtain the displacement vector of the manifold as

$$\beta = \left[\left(\frac{3\alpha+1}{2}\right)t\right]^{\frac{k}{4}} \tag{20}$$

From Eqs. (14), (20) and (6) the energy density in our model is obtained as

$$\rho = \frac{3\alpha(\alpha+1)}{(3\alpha+1)^2 t^2} - \frac{3}{4}\left(\frac{(3\alpha+1)}{2}t\right)^{\frac{k}{2}} \tag{21}$$

From Eqs. (14),(20) and (8) the isotropic pressure in the model is given by

$$p = \frac{3\alpha(\alpha+1)}{(3\alpha+1)^2 t^2} - \frac{3}{4}\left(\frac{(3\alpha+1)}{2}t\right)^{\frac{k}{2}} \tag{22}$$

From Eqs. (21) and (22) it can be observed that

$$\rho = p \tag{23}$$

for our model which shows that the model (14) represents five dimensional stiff fluid or Zeldovich universe in Lyra manifold.

Using Eq. (16), the deceleration parameter in the universe is given by

$$q = -1 + \frac{d}{dt}(H^{-1}) = 3 \tag{24}$$

The scalar curvature R for our model is

$$R = 3\ddot{\lambda} + \ddot{\mu} + 3\dot{\lambda}^2 + \frac{1}{2}\dot{\mu}^2 + \frac{3}{2}\dot{\lambda}\dot{\mu} = \frac{3\alpha(\alpha-1)}{(3\alpha+1)^2 t^2} \tag{25}$$

It can be seen that our model (14) is quite similar to the model obtained by Rao and Jayasudha [37] in Saez-Ballester theory of gravitation . However they differ in the physical behavior. The following is the physical significance of our model. It can be seen that the volume V=0 and the scalar curvature R is infinite at t=0 which show that the universe evolves from zero volume and infinite curvature. The Hubble parameter, H, and the scalar expansion θ diverse at t=0 and vanish as t approaches infinity while the shear scalar σ^2 diverse at t=0 and becomes constant at infinite time. The pressure p and the energy density ρ remain infinite throughout the evolution of the universe. The displacement vector field of the manifold vanishes at the initial epoch and becomes infinite as $t \to \infty$.It is well known that when q>0, the universe decelerates in the standard way and it accelerates when q<0. In this case the universe decelerates in the standard way. Also, from Eq.(19) we conclude that there is an isotropic expansion for all times so that the universe will accelerate due to 'cosmic re-collapse'[43]. This confirms the observations of modern cosmology which advocate the recent scenario of accelerated expansion of the universe [44-45].

5. Summary and conclusions

The present work deals with the theory of gravitation in Lyra geometry in five dimensional spherically symmetric space-time with perfect fluid distribution as a source, The gravitational field equations of this theory are solved using a relation between metric potentials. The exact solution obtained, here, shows that we arrive at a stiff fluid model in five dimensions. It is interesting to note that the models is free from initial singularity and evolves from zero volume and infinite curvature, the pressure and energy density being infinite throughout the evolution. The displacement vector of the manifold increases with time. Since the universe exhibits isotropic expansion throughout the evolution it will accelerate at late times.

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