Article

Black Hole Complementarity as a Condition : on Pre and Post Selected String States

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Abstract

The holographic principle of black holes tells us the field theoretic information of strings on the event horizon is completely equivalent to field theoretic information in the spacetime one dimension larger outside. This physics is observed on a frame stationary with respect to the black hole. The question naturally arises; what physics is accessed by the observer falling through the event horizon on an inertial frame? This paper examines this and demonstrates a duality between the two perspectives. This question is important for the black hole small enough to exhibit fluctuations comparable to its scale. A sufficiently small quantum black hole will be composed of strings in a superposition of interior and exterior configurations or states

Holography and complementarity of black holes

The holographic principle determines field theoretic information on the boundary of a spacetime as equivalent to the theoretic information in the spacetime. This is constructed for a string interacting with black holes and the correspondence between the isometries of the boundary of an AdS spacetime and the conformal symmetries of quantum fields.

This paper addresses a more complete complementarity which includes the physical description of a string as detected by an observer commoving with the string falling in to a black hole. Observers outside and inside a black hole observe completely different physics, where the outside observer never receives a report from the interior measurements. Yet for a black hole sufficiently small or close to the Planck mass the uncertainty fluctuations in the event horizon places the two descriptions in a quantum superposition. Consideration of the interior state of a string may then be the next step in understanding quantum gravity according to holography and black hole complementarity.

The frozen string on the stretched horizon of a black hole is burned up by the radiation emitted by the black hole. The degrees of freedom of the string are then cancelled by modes which escape the black hole as Hawking radiation. These are pre and post selected states, where the inclusion of the interior states may order these in a closed timelike curve (CTC). If this CTC satisfies a chronology protection condition (CPC) then this permits a deeper black hole complementarity principle (BHCP). If this hypothesis is correct this implies a logic, if **CPC** then **BHCP**, which is equivalent to, if **not** − **BHCP** then **not** − **CPC**. So this is not a proof of the chronology protection conjecture, but if this hypothesis is a working theory the failure of the chronology protection conjecture is inconsistent with it.

String interaction with a black hole

The motion of a string onto a black hole approximates the dynamics of a string in a Rindler wedge. The Rindler wedge is defined by the frame of an accelerated observer, which is equivalent to the frame of a stationary observer near a black hole event horizon. The observer witnesses the final emission of radiation by the string just above the event horizon, where upon the string becomes frozen eternally on the particle horizon. Of course in the Rindler wedge case the string proceeds onwards on its geodesic or string world sheet with no apparent change due to this observed state of affairs. This is approximated as well with the black hole, where the string passes through the event horizon unaffected so long as the radius of curvature is much smaller than the string length. This picture persists until the string approaches the center or singularity of the black hole. At this point the Rindler wedge model departs from reality.

The interior perspective or the physics of a string as measured by an observer falling with the string, is outside the domain of the holographic principle. Once the string passes through the event horizon

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it evolves on a domain of causal support not included in the data set accessible to an observer on an accelerated frame stationary with respect to the black hole. The observer that falls through the event horizon observes the further evolution of the string beyond the frozen state the stationary observer finds as its final state. Further, the string evolves into a different state as it approaches the singularity. There the string will begin to experience a rapidly growing Weyl curvature. The stationary observer measures transverse modes of the string on the black hole horizon, while longitudinal coordinates are compressed to near the Planck length. The observer falling in with the string will witness the string distended by the growing Weyl curvature in the direction of motion. Consequently, this observer witnesses the extension of longitudinal extension of the string. The frozen state of the string measured by the exterior observer is cancelled by Hawking radiation which escapes later. The string is entangled with the black hole, and there is a superposition of configuration spaces for the string; the exterior and interior configuration variables. Hawking radiation emerges from the black hole and removes the superposition of the string configuration states (interior and exterior). The entanglement of the string with states on the stretched horizon is a dual superposition of the entanglement of the string interior to the black hole. However, the gauge conditions on transverse variables in the two configurations are not commensurate — or are in a quantum complementarity. The two configurations of the string have similarities to weak measurements [1], where pre- and post-selected states $|pre\rangle$ and $|post\rangle$ may be set to near to orthogonality and the value of an observable $\mathcal{O}$ may be weakly measured

$$\mathcal{O}_w ~=~ \frac{\langle pre|\mathcal{O}|post\rangle}{\langle pre|post\rangle}$$

so the value of this observable becomes large as $\langle pre|post\rangle ~\rightarrow~ 0$ [2]. The pre-selected states correspond to the Lorentz transformed transverse modes of seen by the stationary observer, while the post-selected state are near the black hole singularity. These two sets of string states are orthogonal as they correspond to different Hilbert spaces.

Black hole spacetime near an event horizon is similar to the case of a Rindler spacetime. Within light cone coordinates $X^\pm ~=~ (X^0 ~\pm~ X^1)/\sqrt{2} = ~\rho e^{\pm\tau}$, and the metric is

$$d\tau^2 ~=~ -\frac{dX^+}{X^+}\frac{dX^-}{X^-} ~+~ \frac{d\rho}{\rho} ~=~ -\frac{dX^+}{X^+}\frac{dX^-}{X^-} ~+~ (dX^i)^2, \tag{1}$$

for X^i contained in the plane of the horizon. A particle moving in a timelike direction is given by $X^+ ~-~ X^- ~= 2\rho sinh(\tau) ~=~ L$. So from the perspective of the accelerated observer the particle moves along a distance L with a larger boost angle as $X^- ~\rightarrow~ 0$ as the particle approaches the $X^+ ~\sim~ Le^\tau$ and $X^- ~\sim~ Le^{-\tau}$. These coordinates for a free particle obey the Lagrangian

$$L ~=~ -\frac{1}{2}\frac{dX^\mu}{d\sigma}\frac{dX_\mu}{d\sigma} ~-~ \frac{m^2}{2} \tag{2}$$

The gauge $\sigma ~=~ X^-$ is chosen, as this variable acts as a boost, and is treated as a time variable. The Lagrangian

$$L ~=~ -\frac{1}{2}\Big(\frac{dX^i}{d\sigma}\frac{dX^i}{d\sigma} ~-~ 2\frac{dX^+}{d\sigma}\Big) ~-~ \frac{m^2}{2} \tag{3}$$

determines the momentum with $\dot X^a ~=~ dX^a/d\sigma$

$$P_+ ~=~ \frac{dL}{d\dot X^+} ~=~ 1,~ P_i ~=~ \frac{dL}{d\dot X^i} ~=~ \frac{dX^i}{d\sigma} ~=~ \dot X^i, \tag{4}$$

The particle may be sampled by the accelerated observer in a narrow range $X^- ~\in~ [-\delta,~ 0]$ as the particle appears frozen on the horizon. The value of $X^i(\sigma)$ evaluated in this interval

$$\bar X^i_\delta ~=~ \int_{-\delta}^0 \frac{dX^i}{d\sigma} ~\simeq~ \frac{1}{\delta}\int_{-\delta}^0 X^i(\sigma)d\sigma, \tag{5}$$

which indicates the probability for finding the particle in the limit $\delta ~\rightarrow~ 0$, where $\bar X^i_\delta = ~X^i(0) ~+~ P^i\delta/2$ becomes localized. However, this is a problem for the proper distance of the accelerated observer requires the horizon has a temperature $T \simeq g/2\pi$. The Rindler approximation holds more completely for the acceleration large, corresponding to a close distance to the black hole horizon. The difficulty with this analysis is the horizon with its thermal spectrum will spread the wave function across the horizon by thermal diffusion. This departs from the standard concept of quantum mechanics and complementarity, and is the foundation of the holographic principle

The Heisenberg uncertainty principle of quantum mechanics $\Delta E = \hbar c/\Delta x$ assumes an interesting property within the perspective of the holographic principle [3]. The spread in energy $\Delta E = E_{\Delta x}$ is the energy one must use to probe a region Δx. Let us assume that region is the radius of a Schwarzschild black hole, $R_s = 2GM/c^2 = 2GE_{\Delta x}/c^4$ and $\Delta x \simeq R_s$. This region is then inaccessible to observation. The energy of a Hawking radiation photon is $E \sim \hbar c/R_s$. A substitution into the equation for R_s gives $R_s = \sqrt{G\hbar/c^3}$, the Planck unit of length. Hence it is not possible to ascertain information on a smaller scale by scattering particles with transPlanckian energy. In fact we would at higher energy be probing larger scale physics. The uncertainty relationship with $E_{\delta x} \sim \hbar/\Delta t$ gives a space and time uncertainty relationship

$$\Delta x \Delta t \sim \frac{2G\hbar}{c^4}, \tag{6}$$

which is the Planck length in length-time units. If we observe a massive particle enter a black hole with a certain frequency, say the Compton frequency, that frequency is red shifted. Since this assumes the observer is watching the particle the time Δt is replaced by a proper time Δs, which changes by the Schwarzschild geometry by

$$\Delta s \sim GMe^{\Delta t/4GM}, \tag{7}$$

and the time-space uncertainty principle indicates the spread in the spatial direction increases

$$\Delta x = \frac{\ell_p^2}{\Delta s} \sim \frac{\ell_p^2}{GM}e^{\Delta t/4GM}. \tag{8}$$

This recovers a principal aspect of the holographic principle. The spread in the extent of a string will increase as it approaches the event horizon of a black hole. Further, it covers the surface of the black hole horizon exponentially fast.

The above Lagrangian gives dynamics of a string with transverse directions $X^i(\sigma,\ \tau)$, parameterized along a timelike direction τ and a spatial one σ. The same localization argument is made for $\bar X^i_\delta$, where the fluctuation $(\Delta X^i)^2 = \langle(\bar X^i_\delta)^2\rangle - \langle \bar X^i_\delta\rangle^2$, where with the average estimate above the divergence is logarithmic $(\Delta X^i)^2 \sim |log\delta|$. This spread is determined by $\delta \sim X^- = \tau$. This proper time is equated to equation 7 for small t and

$$(\Delta X)^2 \sim \frac{t}{4GM}. \tag{9}$$

Consider the physics on the frame of the infalling observer. The basic string action is generalized from the above as

$$S = -\int d\sigma^2\sqrt{h}h^{ab}g_{\mu\nu}\frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^a}, \tag{10}$$

for h^{ab} the metric on the string world sheet with parameters $\sigma^2 = (\tau,\ \sigma)$ and $g_{\mu\nu}$ the spacetime metric. The string is not parameterized by a variable with an exponential rapidity. A small variation in the string variable is $X^\mu(\delta\sigma) = X^\mu(0) + \delta X^\mu = X^\mu(0) + P^\mu\delta\sigma$, with the momentum determined by the covariant derivative in spacetime $P^\mu = DX^\mu/ds = \nabla_\alpha X^\mu(dx^\alpha/ds)$. The action may be expressed accordingly, where the term linear in P^μ is a boundary term and the second order term is expressed as

$$\frac{D}{ds}\frac{\partial X^\mu}{\partial\sigma^a}\frac{D}{ds}\frac{\partial X^\nu}{\partial\sigma^b} = \frac{D}{ds}\Big(\frac{D}{ds}\frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}\Big) - \frac{D^2}{ds^2}\frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}, \tag{11}$$

with the argument on the string set to $\delta\sigma = 0$. The over all covariant derivative terms is zero at the boundary, and we have

$$\frac{D^2}{ds^2}\frac{\partial X^\mu}{\partial\sigma^a} = R^\mu_{\alpha\sigma\beta}\frac{\partial X^\rho}{\partial\sigma^a}U^\alpha U^\beta. \tag{12}$$

Hence the action for the string becomes

$$S = -\int d\sigma^2\sqrt{h}h^{ab}g_{\mu\nu}\frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b} + \int d\sigma^2\sqrt{h}h^{ab}R_{\nu\alpha\rho\beta}\frac{\partial X^\rho}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}U^\alpha U^\beta. \tag{13}$$

The curvature is entirely derived from deviations in the string motion. In this case the region of spacetime of the string is source free and $R_{\mu\nu} = 0$. Consequently the Riemann curvature is entirely the Weyl curvature.

A black hole with radius $r_s = 2M >> \ell$ has negligible Weyl curvature near the event horizon. The observer at a stationary position, the fiducial observer or FIDO, observes the string physics largely independent of tidal forces. The stationary observer watches the transverse coordinates of the string according to a gauge choice on the longitudinal coordinate, so that $X^+(\sigma)$ is not an independent degree of freedom. This is the light cone coordinate condition, with $dX^-/d\tau = 1$. The two-form of the string field through a two volume $d\sigma \wedge d\tau$ in the string world sheet parameter is the form

$$\underline{\omega} = \Big(\frac{\partial X^-}{\partial \sigma}\frac{\partial X^+}{\partial \tau} - \frac{\partial X^i}{\partial \sigma}\frac{\partial X^i}{\partial \tau}\Big) d\sigma \wedge d\tau. \tag{14}$$

This two-form is invariant under the reparameterization of the string by $\sigma \rightarrow \sigma + \delta\sigma$, with

$$X \rightarrow X + \frac{\partial X}{\partial \sigma}. \tag{15}$$

The two-form is a topological charge $\int \underline{\omega} = k$, where for this charge zero in this gauge condition

$$\int \underline{\omega} = \int d\sigma \wedge d\tau \frac{\partial X^+}{\partial \tau} - \int d\sigma \wedge d\tau \frac{\partial X^i}{\partial \sigma}\frac{\partial X^i}{\partial \tau} = 0, \tag{16}$$

which gives the equation

$$\frac{\partial X^+}{\partial \tau} = \frac{\partial X^i}{\partial \sigma}\frac{\partial X^i}{\partial \tau}. \tag{17}$$

This equation is strange, for with X^i has 0 unit dimension the left hand side has units of 1, while the right hand side has units of 2. Consequently, the longitudinal coordinate $X^+(\sigma)$ must have units of 1 as well. Thus the fluctuation length in $X^+(\sigma)$ is $\Delta X^+(\sigma) \sim \ell_s^2/\delta$, for $\Delta X^-(\sigma) \sim \delta$ and there is the coordinate uncertainty principle [3]

$$\Delta X^+(\sigma)\Delta X^-(\sigma) \sim \ell^2. \tag{18}$$

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Newtonian gravity, general relativity, and the FREFO string

To first examine physics on the FREFO this we return to Newtonian gravity and evolution of a unit volume as it follows a freely falling path. It is possible to visualize this volume as defined by a cloud of test mass particles or a fluid of negligible mass. The fluid is then replaced by a probability distribution of a quantum particle, or the density matrix of that particle. The wave function is squeezed, similar to the action of a sapphire crystal on photon states. The squeezing of these states is similar to the reduction of the X^- and the expansion of X^+ is the case above with the string on the horizon, but instead with a reduction in X^+ and an expansion of X^-. In this case the process is governed by the Weyl curvature.

Consider the spatial part of a volume, and consider it falling in a gravity field. Let a unit of this volume be $V = x \wedge y \wedge z$, which is a parallelogram in three dimensions. Now compute a variation in this volume in a nonrelativistic (classical) setting

$$\delta V = \delta x_1 \wedge x_2 \wedge x_3 + x_1 \wedge \delta x_2 \wedge x_3 + x_1 \wedge x_2 \wedge \delta x_3. \tag{19}$$

This variation is expanded as

$$\delta x_i = \delta t \frac{dx_i}{dt} + \frac{1}{2}\delta t^2 \frac{d^2 x_i}{dt^2}. \tag{20}$$

Now compute this in the case for the observer falling with this volume. In that case the Newtonian gravity force has been eliminated, so $\vec{F}_{grav} = 0$ and $\vec{F} = \nabla\Phi$, for Φ the gravitational potential. Since we are falling with the volume the first order term is zero, momentum constant, but the second order term (force) across the volume is

$$\vec{F} = \vec{x}\nabla \cdot \vec{F}_{grav}, \tag{21}$$

and so the variation in the volume time a unit mass is

$$m\delta V ~=~ \frac{1}{2}V\delta t^2\nabla\cdot\vec{F}_{grav} ~=~ \frac{1}{2}V\delta t^2\nabla^2\Phi, \tag{22}$$

and the Poisson equation for this matter free volume is $\nabla^2\Phi ~=~ 0$. Thus the volume measure is preserved, though it is distended into a prolate ellipsoid. The antipodal points on the prolate ellipsoid accelerate away from each other by

$$a ~=~ \frac{2GM\Delta x}{R^3}, \tag{23}$$

where $a ~=~ d^2\Delta x/dt^2$. For a small change in distance from the gravitating mass $\Delta x ~\simeq~ exp(\sqrt{2GM/R^3}t)$ or $\Delta x ~\simeq~ X(1 ~+~ \kappa)$ for $\kappa ~=~ \sqrt{2GM/R^3}t)$and X the initial distance between the antipodal points.

Now replace this volume by the quantum density operator ρ. For x the center of a quantum wave function falling in a gravity field, its value shifted off of that position is $\psi(x ~\pm~ y)$. The wave function describes the Wigner quasi-probability function [4]

$$W(p,~x) ~=~ \frac{1}{\pi\hbar}\int_{-\infty}^{\infty}\psi^*(x ~+~ y)\psi(x ~-~ y)e^{2ipy/\hbar}. \tag{24}$$

For $\psi^*(x ~+~ y)\psi(x ~-~ y) ~=~ \langle x ~+~ y|\hat\rho|x ~-~ y\rangle$ the time evolution of the Wigner function is

$$\dot W ~=~ \{H,~W\}_{MB}, \tag{25}$$

where this last bracket is the Moyal bracket [5],

$$\{H,~W\}_{MB} ~=~ \{H,~W\}_{PB} ~+~ \sum_{n=1}^{\infty}\frac{(-1)^n\hbar^{2n}}{2^{2n}(2n ~+~ 1)!}\partial_x^{2n+1}H\partial_p^{2n+1}W. \tag{26}$$

The first bracket is the Poisson bracket for Liouville phase space flows in classical mechanics. The additional terms are quantum corrections to $O(\hbar^{2n})$ for $n ~\rightarrow~ \infty$ The Moyal bracket is also written according to classical flows by

$$\{H,~W\} ~=~ -\frac{i}{\hbar}sin\big(i\hbar\{H,~W\}_{PB}\big). \tag{27}$$

The Wigner function for the wave function $\psi(x) ~=~ Aexp(ipx/\hbar)$ is

$$W(p,~x) ~=~ \frac{\hbar}{2ip}e^{2ipx/\hbar} ~\rightarrow~ \frac{\hbar}{p}sin(2px/\hbar). \tag{28}$$

For simplicity we consider a one dimensional wave function along the radius of the gravity field. We then have the tidal acceleration on the dummy variable $y' ~=~ y(1 ~+~ \kappa)$, which changes the Wigner function to $W(p(1 ~+~ \kappa),~x)$. The classical flows of the Wigner function is the Poisson bracket

$$\{H,~W\}_{PB} ~=~ \frac{\partial W}{\partial x}\frac{\partial H}{\partial p} ~-~ \frac{\partial W}{\partial p}\frac{\partial H}{\partial x} ~=~ 2ip(1 ~+~ \kappa)W\partial_pH ~-~ \Big(\frac{ix}{\hbar} ~-~ \frac{1}{p}\Big)W\partial_xH \tag{29}$$

For a free particle falling in the gravity field $\partial_xH ~=~ 0$ and $\partial_pH ~=~ p$. This then gives the time evolution of the Wigner function and Moyal bracket as

$$\dot W ~=~ \{H,~W\}_{MB} ~=~ sin(-2\hbar p^2(1 ~+~ \kappa)W). \tag{30a}$$

The evolution of the Wigner function is an oscillation with increasing frequency, which is a chirped wave function. This type of function in acoustics is a classic case where the function is squeezed, as the frequency is a monotonic function of time. The Fourier transform of the Wigner function is

$$\mathcal{F}W ~\sim~ \frac{1}{\sqrt{2\pi}}\frac{1}{p(1 ~+~ \kappa)}\delta(k ~-~ p(1 ~+~ \kappa)). \tag{30b}$$

This squashing of the phase space volume by the gravitational tidal force may reduce the transverse direction size of the phase space volume below the unit of action $\Delta x \sim \sqrt{\hbar}$, and expand the associated momentum uncertainty beyond $\Delta p \sim \sqrt{n\hbar}$.

This illustrates how gravitation may influence a quantum system by tidal acceleration. Gravitation can adjust the uncertainties in conjugate observable so they are off quadrature. The adjustment in the uncertainty is due to an operator which acts on raising and lowering operators by Bogoliubov transformations. The off quadrature uncertainty configuration of the wave function is then not related to other configurations by unitary dynamics of the system. As the longitudinal coordinate direction expands enormously the momentum becomes completely specified, which mimics the measurement process. The gravitational squeezing of the density matrix puts the quantum system in a configuration not described by a unitary transformation on the initial conditions.

This is now extended to general relativity and the infalling string as measured by the FREFO frame. The Riemann curvature is defined by the antisymmetric covariant derivatives of a vector field U_a

$$U_{\alpha;\beta\gamma} - U_{\alpha;\gamma\beta} = R^{\delta}{}_{\alpha\beta\gamma}U_{\delta}, \tag{31}$$

and by the deviation between geodesics with tangent velocities U_α separated by a vector V_b is

$$V^{\alpha}{}_{;\beta\gamma}U^{\beta}U^{\gamma} = R^{\alpha}{}_{\beta\gamma\delta}U^{\beta}U^{\gamma}V^{\delta}. \tag{32}$$

The four-vector in general has a first covariant derivative defined by acceleration, a rotation tensor, a shear tensor, and volume expansion in the Raychaudhuri equation [6]. For $U_{\alpha;\beta}$ symmetric and $A_\alpha U^\alpha = 0$ the first derivative of a four vector is given by

$$U_{\alpha;\beta} = \frac{1}{2}(U_{\alpha;\beta} + U_{\beta;\alpha}) = \sigma_{\alpha\beta}, \tag{33}$$

which is the shear tensor. This is appropriate for the motion of particles, or a cloud, in a source free region, where the curvature of spacetime defines the shear tensor

$$\sigma_{\alpha\beta;\gamma} - \sigma_{\alpha\gamma;\beta} = R_{\delta\alpha\beta\gamma}U^{\delta}. \tag{34}$$

Then geodesic deviation for a cloud of geodesic particles is then

$$V_{\alpha;\beta\gamma}U^{\alpha}U^{\beta}U^{\gamma} = (\sigma_{\beta;\gamma\alpha} - \sigma_{\beta;\alpha\gamma})U^{\beta}U^{\gamma}V^{\alpha}. \tag{35}$$

The gravitation part of the action in equation 13 in a source free region may be rewritten with the Riemann curvature replaced by the Weyl curvature. The Lagrangian in $\pm$ coordinates is

$$L = \sqrt{h}h^{ab}C_{+\alpha-\beta}\frac{\partial X^-}{\partial\sigma^a}\frac{\partial X^+}{\partial\sigma^b}U^{\alpha}U^{\beta}. \tag{36}$$

The observer on the FREFO frame operates in a different gauge, one where $X^+ \rightarrow 0$ and $X^- \rightarrow \infty$ due to tidal forces. The squeezing of states determines us that $X^- \simeq \tau$, which is the FREFO gauge. This gauge is opposite of the case above. Now express the shear according to the string velocity $\sigma_{a\alpha\beta} = (\partial_a X_\alpha)_{;\beta} = (1/2)((\partial_a X_\alpha)_{;\beta} + (\partial_a X_\beta)_{;\alpha})$ so that

$$\sigma_{\alpha\beta;\gamma} - \sigma_{\alpha\gamma;\beta} = (\partial_a X_\alpha)_{;[\beta\gamma]} = C_{\delta\alpha\beta\gamma}\partial_a X^{\delta}, \tag{37}$$

which leads to the equations

$$\Big(\frac{\partial X_\pm}{\partial\sigma^a}\Big)_{;[\pm\mp]}\frac{\partial X^\mp}{\partial\sigma^b} = C_{\mp\pm\mp\pm}\frac{\partial X^\mp}{\partial\sigma^a}\frac{\partial X^\mp}{\partial\sigma^b}. \tag{38}$$

The Weyl tensor satisfies the Bel criteria for a type D solution for a black hole [7]. Given the independent vectors V^α and V^γ then

$$C_{\alpha\beta\gamma\delta}V^{\alpha}V^{\gamma} = \kappa V_{\beta}V_{\delta},\ *C_{\alpha\beta\gamma\delta}V^{\alpha}V^{\gamma} = \kappa V_{\beta}V_{\delta}. \tag{39}$$

For $V_a^{\pm} = \partial_a X^{\pm}$ then

$$C_{\pm\mp\pm\mp}\frac{\partial X^{\pm}}{\partial\sigma^a}\frac{\partial X^{\pm}}{\partial\sigma^b} = \kappa\frac{\partial X^{\mp}}{\partial\sigma^a}\frac{\partial X^{\mp}}{\partial\sigma^b}, \tag{40}$$

for κ a constant. A multiplication of equation 40 by $\partial_a X^{\mp}\partial_b X^{\mp}$ results in κ on the right hand side. This determines the condition that $\Delta X^+\Delta X^- = constant$, which is a version of equation 18. This gives $\Delta X^- \simeq \ell_s^2/\delta$ for the localization given by the vanishing of ΔX^+. This recovers the same equation for the noncommutative geometry as seen by the FIDO, but now on the FREFO reference frame.

This condition on the longitudinal variable is opposite that on the FIDO frame. From a quantum field perspective these amount to two types of squeezing conditions on a string state. These two conditions are two different types of gauge conditions on the system, which are not determined by the dynamics of the string along the transverse directions. This is analysed with respect to the S-matrix for the string observed on the two frames. The S-matrix in these two configurations have different domains of causal support as well.

S-matrix causal domains and path integrals

The connection between FIDO and FREFO observations of a string leads to a path integral realization of field propagations on different causal domains. For $r_0 = 2M$ the Schwarzschild metric

$$ds^2 = (1 - r_0/r)dt^2 - (1 - r_0/r)^{-1}dr^2 - r^2 d\Omega^2 \tag{40}$$

determines the proper distance for an observer close to the $d\rho = \int\sqrt{g_{rr}(r)}dr$ with the limits $[r_0, r]$

$$\rho = r_0\, ln(r - r_0) + r. \tag{41}$$

The causal domain $[r_0, r]$ is such that r may range to ∞. The FREFRO causal domain is given by $d\tau = \int\sqrt{g_{tt}(r)}dt$. For $dr/dt = \sqrt{g_{tt}/g_{rr}}\sqrt{r_0/r}$ and the distance of the FREFO path is

$$\tau = \int\sqrt{\frac{r_0}{g_{tt}}}dr = 2\sqrt{r_0(r - r_0)}, \tag{42}$$

which holds on the domain $[0, \infty]$. The imaginary condition suggests the signature change, and may be removed as $\tau = 2\sqrt{|r_0(r - r_0)|}$. The states of the system on the two causal domains are computed according to separate S-matrices.

The infalling string as observed on the FIFO frame is quickly frozen, with its transverse extent covering the horizon. This state is observed upon reaching within a few Planck lengths of the horizon, which is the stretched horizon. The frozen state of the string is determined by the FIDO observer at a time $t << T_{bh}$ for T_{bh} the lifetime of the black hole. On the FREFO frame the string approach the center of the black hole in a short period of time. The string becomes entangled with the black hole interior and later emerges at time $t \sim T_{bh}$. These states annihilate the string states on the horizon. The FIDO frame states on the horizon are then a set of p-selected states determined by the p-selected states in the distant future. The prefix p- may be interpreted equally as pre and post, for the interior states have no observable time direction from the FIDO frame. The random fluctuations of quanta in the distant future then fix the states of the string in the past as observed on a FIFO frame. This is an ambiguity with respect to the meaning of a time and time ordering if the string has a single S-matrix which satisfied the FIDO and FREFO frame observations.

The S-matrix is the most general unitary description for the scattering of states between states on asymptotic regions defined as in and out.[8] The S-matrix is defined by a set of in and out states which are defined on different Hilbert spaces. Different sets of operators $\{a_i^\dagger(k),\ k \in [0, \infty]\}$ and $\{a_o^\dagger(k),\ k \in [0, \infty]\}$ define the Hilbert spaces

$$\begin{aligned}\mathcal{H}_i &= \{|, I\ k_1,\ k_2,\ \dots\ k_n\rangle_i = a^\dagger(k_1)a^\dagger(k_2)\ \dots a^\dagger(k_n)|I,\ 0\rangle\}\\ \mathcal{H}_o &= \{|O,\ k_1,\ k_2,\ \dots\ k_n\rangle_i = b^\dagger(k_1)b^\dagger(k_2)\ \dots b^\dagger(k_n)|O,\ 0\rangle_i\},\end{aligned} \tag{43}$$

as spans of vector spaces given by the raising operators. The left hand sides of the equation are eigenstates of a momentum operator P^ν.

The "in" state and "out" states are related to each other according to an S-matrix, so $|in\rangle = S|out\rangle$. An expectation of a field ϕ_o is then $\langle out|\phi_o|out\rangle = \langle in|S\phi_o S^\dagger|in\rangle$, where unitarity requires $\langle out|\phi_o|out\rangle = \langle in|\phi_i|in\rangle$

$$\phi_o = S^\dagger\phi_i S. \tag{44}$$

In the Heisenberg interpretation the fields evolve and the states are time independent. An initial state is expressed according to the final states as

$$|I\ ,\{k_n\}\rangle = \sum_{n=0}^{\infty}\int d^4k_1\dots d^4k'_n C(\alpha\{k_n\})|O,\ \alpha\{k_n\}\rangle, \tag{45}$$

where $C(\{k_n\})$ is the probability amplitude for the transition $|I,\ \{k_n\}\rangle \rightarrow\ |O,\ \{k_n\}\rangle$ and $\{k_n\} = k_1,\ k_2,\ \dots,\ k_n\ = \alpha$. The expectation of the S-matrix with respect to initial states is

$$\langle I,\ \alpha|S|O,\ \beta\rangle = S_{\alpha\beta}. \tag{46}$$

The S-matrix describes the evolution of fields from a region out to asymptotic infinity. The evolution of the operators $a(k,\ t) = U^{-1}(t)a(k,\ 0)U(t)$ gives $a(k,\ \infty)$ according to $U(\infty)$. A field transforms by the S-mtarix as $\phi_o = S^{-1}\phi_i S$, which gives the assignment $S = e^{i\theta}U(-\infty)$ for θ a phase angle. This phase angle is determined by the action of the S-matrix on a state, such as $S|0\rangle = |0\rangle$ and $S|k\rangle = |k\rangle$. Hence, momentum states are eigenstates of the S-matrix as the unit, and the S-matrix is nonsingular for the conservation of momentum. The use of completeness relationships indicates that $e^{i\theta} = \langle 0|U|0\rangle^{-1}$, which is nonzero. Then for the unitary operator generated by a Hamiltonian H the S-matrix is

$$S = \langle 0|U|0\rangle^{-1}exp\Big(-i\int^{\infty} dtH(t)\Big). \tag{47}$$

This now permits us to examine a path integral for the transition amplitude for the black hole case. The progression from an initial state $|I\rangle$ and a final state $|F\rangle$, essentially the same as the out state above, is under a Fourier transform of the S-matrix to a position representation a sum over paths with the S-matrix configuration.. The initial state in the position representation is represented as

$$|I,\ x\rangle = \prod_n\frac{1}{\sqrt{(2\pi)n}}\int d^4k_n e^{ik_nx}|I\ ,\{k_n\}\rangle$$

$$= \prod_n\frac{1}{\sqrt{(2\pi)n}}\sum_{n=0}^{\infty}\int d^4k_n e^{ik_nx}\int^{\infty} d^4k_1\dots d^4k'_n C(\alpha\{k_n\})|O,\ \alpha\{k_n\}\rangle, \tag{48}$$

where all of this complicated mathematics is written as $|I,\ x\rangle = \int dx\sigma(x)|x\rangle$, where $\sigma(x)$ is a symbol for the half of S-matrix Fourier transformed to a spatial coordinate representation. The outstates are written according to a summation over positions so $\langle F| = \langle y\int dy\tilde\sigma(y)$. The transition amplitude between $|I,\ x\rangle$ and $|F,\ x\rangle$ is a path integral over all paths determined by the action $S = \int_0^t dtL(x,\ \dot x)$

$$\langle F,\ x|e^{-iHt/\hbar}|I,\ x\rangle = \int_{-\infty}^{\infty} dxdy\sigma(x)\tilde\sigma^\dagger(y)\int_x^y\mathcal{D}[x(t)]e^{iS/\hbar}. \tag{49}$$

where the scattering from x to y are given by an S-matrix or the product $\sigma(x)\tilde\sigma^\dagger(y)$.

Now include the two descriptions by the FIDO and FREFO frames. The two frames together form a loop. At the terminal phase of the black hole the FIDO frame observes the frozen string annihilated by the field emerging from the black hole. The FIDO concludes there is some internal path of string fields. This would be particularly the case if the black hole is a quantum black hole and the horizon exhibits some measure of uncertainty fluctuation. With a quantum horizon the path observed on the FREFO frame has some quantum superposition with the path observed outside on the FIDO frame. The state of the string on the horizon is fixed very quickly according to the FIDO frame. The string remains in this state for the duration of the black hole. This is a form of pre-selection of states of the radiation which emerges later from the black hole. Similarly, the FREFO frame observes the continued evolution of the string as it proceeds to the singularity. The evolution of states on the singularity is not possible to track. All causal information proceeds to the singular region, and the infalling observer is unable to access information about the string on the singularity. Further this observer is absorbed as well. The state of the string in this frame becomes entangled with the singularity and might be considered a pre-selection for the state of the string state at the end of the black hole's duration. However, the singularity cant be regarded as having any temporal meaning. Therefore, we could just as well say the string near the singularity is post-selected by the string on the horizon in the distant future as observed on the FIDO frame.

The FIDO path for the string is along the horizon, while the FREFO path is some entanglement with the interior of the black hole. Together, these form a closed loop, and the relationship between the p-selected states is such that this is a closed path. This means the evolution of the initial and final states includes a closed loop so that

$$\langle F_c|\langle F|e^{-iHt/\hbar}|I\rangle|I_c\rangle \;=\; \int_{-\infty}^{\infty} dxdydxdy\sigma(x)\sigma(x)\tilde\sigma^\dagger(y)\tilde\sigma^\dagger(y)\int_{x,x}^{yy}\mathcal{D}[x(t)]e^{iS/\hbar}. \tag{50}$$

Now consider the following ansatz: The relationship between pre-and post-selected states with respect to time is irrelevant [9]. In other words the information content of the loop is constant and there is no backwards time causal influence on $\langle F|e^{-iHt/\hbar}|I\rangle$. This requires that

$$\int_{-\infty}^{\infty} dxdy\sigma(x)\tilde\sigma^\dagger(y) \;=\; \delta(x\;-\;y) \tag{51}$$

so the identification of x with y results in no causal propagation of information. The associated S-matrix product $\sigma(x)\tilde\sigma^\dagger(y)$ is given by the partial isometries

$$\sigma(x)\;\rightarrow\; S_1^{1/2} \;=\; e^{i\theta}exp\Big(-i\int^{\infty}dtH(t)\Big),\; \sigma(y)\;\rightarrow\; S_2^{1/2} \;=\; e^{i\theta}exp\Big(-i\int^{\infty}dtH(t)\Big) \tag{52}$$

Now consider the two Hamiltonians where they pertain to the two cases of different causal domains of support. The first Hamiltonian in light cone string theory is just

$$H \;=\; \int_0^{2\pi} d\sigma\Big(\frac{1}{2}|P_i(\sigma)|^2 \;+\; \frac{1}{2}\Big(\frac{\partial X_i}{\partial\sigma}\Big)^2\Big), \tag{53}$$

which has a target map to spacetime. This is the dynamics of the string to the event horizon as measured on the FIDO frame. The Hamiltonian for the interior region requires the Weyl tensor analysis above

$$H \;=\; \int_0^{2\pi} d\sigma C^{\pm}{}_{\mp\pm\mp}\Big(\frac{1}{2}|P^{\mp}(\sigma)^2| \;+\; \frac{1}{2}\Big(\frac{\partial X^{\mp}}{\partial\sigma}\Big)^2\Big) \;+\; H_{sing}. \tag{54}$$

The term H_{sing} is the unknown dynamics of the singularity. The product $\sigma(x)\tilde\sigma^\dagger(y)$ implies an S-matrix product of the form $\langle I,\;\alpha|\sigma\sigma|O,\;\beta\rangle$. The S-matrix applies to states in different squeezed states, where σ applies for $\Delta X^+\;\rightarrow\;0$ and σ for states with $\Delta X^-\;\rightarrow\;0$. The two basis of states are opposite in being off quadrature, so that σ and σ apply in nearly orthogonal states. Consequently the completeness sums $\sum_\gamma|O,\;\gamma\rangle\langle O,\;\gamma|$ and $\sum_\gamma|I,\;\gamma\rangle\langle I,\;\gamma|$ in the appropriate squeezed states gives

$$\langle I,\;\alpha|\sigma\sigma|O,\;\beta\rangle \;=\; \sum_{\gamma\gamma}\langle I,\;\alpha|\sigma|O,\;\gamma\rangle\langle O,\;\gamma|I,\;\gamma\rangle\langle I,\;\gamma|\sigma|O,\;\beta\rangle. \tag{55}$$

It is then necessary to interpret the product $\sigma(x)\tilde\sigma^\dagger(y)$ under a weak measurement. In the limit the two sets of states are squeezed to near orthogonality $\langle O,\;\gamma|I,\;\gamma\rangle\;\rightarrow\;0$, where the various terms under orthogonality are approximately equal. The weak measurement result is approximately

$$(\sigma\sigma)_{\alpha\beta}\;\simeq\;\frac{\langle I,\;\alpha|\sigma\sigma|O,\;\beta\rangle}{\sum_{\gamma\gamma}\langle O,\;\gamma|I,\;\gamma\rangle}, \tag{56}$$

which approaches unity. The nature of H_{sing} may then be estimated.

Gauge conditions and extra large dimensions

There is a curious development here, for the FIDO frame has one gauge condition and the FRERO another. The two gauge conditions are not compatible, though they exist in causally separate domains. The two gauge conditions $X^\pm = \tau$ apply in regions separated by an event horizon which prevent any closed timelike propagation of information which permit the information accessible on the FREFO frame from being communicated to the outside to the FIDO frame. There do not exist closed timelike curves (CTCs) which communicates any information tied to either gauge across the horizon in both directions. However, if the black hole is quantum mechanical, or has large uncertainties in its horizon an exterior observer may then perform a measurement on quantum states in a superposition of exterior and interior states.

The two gauge conditions are then locally determined by some additional field. This additional field is a manifestation of additional dimensions with a parameter that sets the $X^\pm$ gauge at two different parameter values. As a toy model consider a five dimensional spacetime plus R space. This fifth dimension is a space with a gauge connection or potential $A = \phi$ which defines a force $F = -d\phi/dx_5$, for x_5 a parameter on this fifth dimension. The gauge condition on $X^\pm$ is a function $G(\phi)$ which sets at $\phi = 0, \pi$ $X^- = \tau$, $X^+ = \tau$ respectively. The exterior and interior states of the string are local gauge conditions as a superposition of amplitudes

$$\chi(\phi) = sin(\phi/2)X^+ + cos(\phi/2)X^-. \tag{57}$$

An obvious simple case for a field in a one dimensional chord of length L would be where $\phi = \pi x_5/L$ as a model similar to a constant gravity on Earth. This means the gauge conditions for spacetime on the FIDO and FREFO frames are set by a gauge condition on an internal symmetry. Local gauge transformations of this internal gauge field then determine the FIDO and FREFO gauge conditions as coordinate choices: on coordinate fixed at a constant proper distance from the black hole horizon and the other freely falling into the black hole.

This simple toy model suggests for a more complete theory. The spacetime metric near the singularity is $ds^2 \simeq -(1/r)dt^2 + rdr^2 + r^2d\Omega^2$, which is the metric for $AdS \times S^2$. The symmetry of this system is $SL(2, R)$ of conformal quantum mechanics. This projective $PSL(2, R)$ with a linear fibration over $R \cup \{\infty\}$. The Killing form on $SL(2, R)$ with signature $(2, 1)$ induces an isomorphism between $PSL(2, R)$ and the Lorentz group $SO^+(2, 1)$. This action of $PSL(2, R)$ on Minkowski space defines isometries on the hyperboloid plane. This is in five dimensions a fibration over the Lorentizian part of $M^{2,1} \times S^2$ as this fifth dimension.

This is curiously dual to the model of a black hole in the AdS_4 spacetime with a black hole. There is a decomposition $AdS_4 \rightarrow AdS_2 \times S^2$ near the horizon of the black hole, which induces a local phase change with respect to a $U(1)$ gauge field. Solid state physics analogues are known to exist and correlated well with the onset of superconductivity at a quantum critical point [10].

Chronology protection and the character of physical law

This hypothesis requires the delta function condition on the closed curve in spacetime, which is equivalent to stating there is no causal information propagated on any CTC. This holds here for black holes, and presumably holds for all spacetimes. The chronology condition is then a consistency requirement, for if this hypothesis fails to hold, or in association with any superseding or better hypothesis, the chronology condition is false.

This argument then has some parallels with the character Salviati Galileo penned in his *Dialogue Concerning the Two Chief World Systems.* Salviati proposed an argument for how the falure of the Aristotilean system lead to an inconsistency:

If then we take two bodies whose natural speeds are different, it is clear that on uniting the two, the more rapid one will be partly retarded by the slower, and the slower will be somewhat hastened by the swifter. Do you not agree with me in this opinion?

This argument illustrates an inconsistency with the then prevailing Aristotelian theory of motion. This argument was modified for a number of alternative cases, in particular on a ship, and the inconsistency remained. This is similar to what is argued here. Without some superseding theory, one which is more general or more physically correct, the falsehood of this theory leads to the falsehood of the chronology protection conjecture.

The chronology protection conjecture might of course simply be false, but this does lead to troubles with understanding how nature is self-consistent. This stems from the standard time machine problem, or the grandfather paradox. There have been some proposals for working around this problem, but they tend to involve very exceptional requirements. So for the sake of simplicity is has generally been regarded that the chronology protection conjecture is a safe physical axiom.

This implies an equivalency of sorts between inertial and accelerated frames. The physics observed on the FIDO and FREFO frames are equivalent under an internal symmetry. There is then a gauge potential of some sort which parameterizes observed conditions on an infinite number of frames in a Schwarzschild spacetime. Consequently the physics observed properly on a freely falling frame is equivalent to physics on any accelerated frame, either inside or outside the black hole.

The extension of the black hole complementarity principle, and further generalizations with AdS spacetimes, leads to a framework for understanding quantum gravity further. This generalization is most important for the case where the black hole is so small that the horizon exhibits quantum uncertainty fluctuations. In this case the squeezing of the observables $X^{\pm}$ according to $\Delta X^{+} \rightarrow 0$ or $\Delta X^{-} \rightarrow 0$ are outcomes of experiments on a superposition of the interior and exterior state of the black hole. The experimenter couples auxiliary states to the black hole to form an entanglement, and the outcome recorded is the so called collapse.

This is a duality between state in interior and exterior regions of the black hole, with a reciprocal relationship between scales. T-duality is an exchange between quantum modes and the winding number of a string. This pertains to compactified dimensions which are much larger than the string scale, and the dual $R \rightarrow \alpha/R$ on a much smaller scale, with the exchange of mode and winding number $n \leftrightarrow w$. This is an exchange symmetry between type IIA strings of massless nonchiral fermions with type IIB strings of chiral fermions. This is also an exchange between $SO(32)$ and heterotic string $E_8 \times E_8$ [11],[12].

This system has S-duality structure as well. A complex valued scalar field constructed from the axion field, χ and the diliton field ϕ as $\xi = \chi + ie^{-\phi}$ is conformal and transforms by $SL(2,\ R)$ [12]. The discrete winding reduces this to the discrete structure $SL(2,Z)$ and the field transforms under the linear fractional transformations of $PSL(2,R)$. Hence the charge of a black hole, its mass has a duality with its magnetic monopole analogue, or NUT parameter. This indicates a duality with a Taub-NUT spacetime, which has a correspondence with AdS.

This duality in noncommutative geometry of the S-matrices for FIDO and FREFO frames is then a gauged U-duality. On the two limiting cases for a classical black hole, this internal gauge (the fifth dimension parameter) is set differently on two causally separated charts, which induces a spacetime gauge which appears global. Yet for a quantum black hole these spacetime gauge conditions are blurred into each other and become less distinct. This suggests spacetime geometry becomes subsumed into a greater geometry, or that it completely dissolves and is replaced by another type of physics.

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