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Magnetized Modified Holographic Ricci Dark Energy Cosmological Model in Saez-Ballester Theory of Gravitation

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Abstract

The main purpose of this paper is to explore the solutions of anisotropic Bianchi Type III space time with magnetized modified holographic Ricci dark energy in the framework of Saez-Ballester theory of gravitation (1).We have solved Saez- Ballester theory field equations using the hybrid expansion law proposed by Akarsu et al.(2).We have presented the cosmological model by the proposed expansion law with considering certain parameters and studied their evolutions.We have also discussed physical and kinematical properties of the model.

Keywords: Bianchi Type III, modified, holographic Ricci, dark energy, cosmological model, magnetic field, Saez-Ballester theory, gravitation.

1 Introduction

In modern cosmology the most attractive subject is the accelerated expansion of the universe which is based on the recent astronomical data (Riess et al.(3); Perlmutter et al. (4)) explaining the universe is spatially flat.The root cause for this phenomenon is still under extensive investigation.However,it is said an cosmic fluid called dark energy(DE) with a huge negative pressure is responsible for this expansions. This type of matter violates the strong energy conditions $\rho + 3p < 0$ where ρ is the energy density and p is the pressure of dark energy(DE).This concept of dark energy has attracted the attention of cosmologists all over the world to investigate suitable models of dark energy.As a result various dark energy models have come to lime light. The first and simple candidate for dark energy is the cosmological constant (Weinberg (5)) with the EoS parameter $\omega = -1$. But it suffers from the fine tuning and coincidence problem (Copeland et al.(6)).There are many other candidates for dark energy such as quintessence with EoS $\omega > -1$(Barreiro et al.(7)),Phantom with EoS $\omega < -1$ (Caldwell(8)), k-essence(Armendariz et al.(9)), Tachyon(Bagla et al.(10)),Chaplygin gas (Bento et al.(11)),Holographic dark energy(Li(12)) and so on.Padmanabhan(13) and Copeland et al.(6) presented a comprehensive review of DE and DE models.
It is well known that the holographic principle (Susskind(14)) plays a major role in the black hole and string theory,which is based on the fact that in quantum gravity,the entropy of a system scales not with its volume, but with its surface area L^2 . Inspired by the holographic principle,Cohen et al.(15) suggested that the vacuum energy density is proportional to the Hubble scale

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$l_H \approx H^{-1}$.In this model both the fine-tuning and coincidence problems can be alleviated.But it cannot explain the cosmic accelerated expansion because the effective EoS for such vacuum energy is zero.Li(12) proposed that the future event horizon of the universe was to be used as the characteristic length l. This holographic DE model not only presents a reasonable value for DE density, but also leads to a solution for the cosmic accelerated expansion. In fact, the choice of the characteristic length l is not unique for the holographic DE model. Gao et al.(16) assumed that the length l is given by the inverse of Ricci scalar curvature (i.e.,$|R|^{-1/2}$),which is the so called holographic Ricci DE model. It is argued that this model can solve the coincidence problem entirely. Thus,the properties of such holographic Ricci DE have been investigated widely Huang and Zhang(17, 18). Granda etal.(19) proposed a modified Ricci DE model in which the density of DE is a function of the Hubble parameter H and its derivative with respect to time ($\dot{H}$).Chen and Jing (20) presented a generalized DE model in which density of DE contains the second order derivative of Hubble's parameter with respect to time ($\ddot{H}$) and find that the age problem of the old objects above can be solved.The expression for energy density of this modified holographic Ricci DE (MHRDE) is defined by Chen and Jing (20) as

$$\rho_\wedge = 3M_p^2\left(\alpha_1 H^2 + \alpha_2 \dot{H} + \alpha_3 \ddot{H} H^{-1}\right), \tag{1.1}$$

where M_p^2 is the reduced Planck mass; and α_1 , α_2 and α_3 are three arbitrary dimensionless parameters.Sarkar and Mahanta (21), Sarkar(22), Adhav et al.(23),and Kiran et al.(24) investigated minimally interacting and interacting holographic DE Bianchi models in general relativity and scalartensor theories of gravitation.Santhi et al.(25) studied the locally rotationally symmetric Bianchi type-I generalized ghost pilgrim DE model in general relativity. Das and Sultana (26, 27) and Santhi et al.(28, 29) studied some Bianchi type anisotropic MHRDE cosmological models in general relativity and scalartensor theories.

In fact,to get a physically realistic description of the universe, one has to consider inhomogeneous models. In this case, the solutions of Einstein's field equations become more complicated or may be impossible. Therefore, many theoretical cosmologists are trying to use the spatially homogeneous and anisotropic Bianchi type models instead of inhomogeneous models.These types of space-times present a middle way between FRW models and inhomogeneous anisotropic universes,hence play an important role in modern cosmology.Bianchi type cosmological models are important in the sense that these are homogeneous and anisotropic from which the process of isotropization of the Universe is studied through the passage of time.The simplicity of field equations and relative case of solution made Bianchi type space times useful in constructing models of spatially homogeneous and anisotropic cosmologies.Spatially homogeneous and anisotropic cosmological models play a significant role in the description of large scale behavior of universe and such models have been widely studied in framework of General Relativity in search of a realistic picture of the universe in its early stages. Yadav(30) and Pradhan(31) have studied homogeneous and anisotropic Bianchi type-III space-time in context of massive strings. Yadav(32) has obtained Bianchi type-II anisotropic DE models with constant deceleration parameter.

In the last few decades there has been much interest in alternative theories of gravitation, especially the scalar tensor theories proposed by Brans-Dicke (33), Nordvedt (34), Barber (35), Saez-Ballester (36), Lau and Prokhovnik (37). Brans-Dicke (33) scalar-tensor theory of gravitation introduces an additional scalar field φ beside the metric tensor g_{ij} and a dimensionless value coupling constant ω. This theory tends to general relativity for large value of the coupling constant . In Saez-Ballester theory, the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields in which an anti-gravity regime appears despite the dimensionless behavior of the scalar field.This theory suggests a possible way to solve the missing matter problem in non flat FRW models.Reddy et al.(38, 39), Rao et al.(40, 41, 42, 43),Pradhan et al.(44) are some of the authors who have investigated several aspects of Saez-Ballester theory of gravitation. Recently,Kiran et al.(24) have discussed stationary spherically symmetric one-kink model in Saez-Ballester theory of gravitation.

Magnetic field plays a vital role in the description of the energy distribution in the universe as it contains highly ionized matter.Large scale magnetic fields give rise to anisotropies in the universe. The magnetic field has the significant role in the dynamics of the universe depending on the direction of the field lines. Strong magnetic fields can be created due to adiabatic compression in cluster of galaxies. Primordial magnetic fields can have a significant impact on the cosmic microwave background anisotropy depending on the direction of field lines (Madsen and King (45, 46)). Many people have investigated the influence of magnetic field on the dynamics of the universe by analyzing anisotropic Bianchi models.Milaneschi et al.(47) studied the anisotropy and polarization properties of cosmic microwave background radiation in the homogeneous Bianchi I cosmological model.Sharif and Zubair(48, 49) investigated dynamics of Bianchi type universes with magnetized anisotropic DE.Recently,Santhi et al(50) have investigated anisotropic magnetized holographic Ricci dark energy cosmological models in general relativity.

With these motivations and following different techniques :In this paper, we would like to investigate the dynamics of anisotropic Bianchi type-III model in the presence of MHRDE and magnetic field. The paper has the following sections.In Section 2,we derived the Saez-Ballester theory field equations for anisotropic Bianchi type III metric with MHRDE. In Sect.3,we obtain solutions to the field equations.The physical and kinematical properties of the model discussed in Section 4 .Finally,in Section 5,we concluded the results.

2 Field Equations

We consider the spatially homogeneous and anisotropic Bianchi type-III spacetime described by the line element

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)e^{-2x}dy^2 - C^2(t)dz^2, \tag{2.1}$$

where A, B and C are functions of cosmic time t only.

We assume that the universe is filled with matter and magnetized anisotropic modified holographic Ricci dark energy (MHDE) fluid.Here we assume that the current is flowing along the x-axis so magnetic field is in the yz-plane.King and Coles (46), Jacobs (51),and Sharif and Zubair (48) used the magnetized perfect fluid energy momentum tensor to discuss the effects of magnetic field on the evolution of the universe.Here we take more general energymomentum tensors for matter and the magnetized anisotropic modified holographic Ricci DE fluid in the following form:

$$T_{ij} = diag\,(\rho_m, 0, 0, 0)$$

$$\bar{T}_{ij} = diag\,(\rho_\wedge + \rho_B, -\omega_\wedge\rho_\wedge + \rho_B, -(\omega_\wedge + \delta_y)\rho_\wedge - \rho_B, -(\omega_\wedge + \delta_z)\rho_\wedge - \rho_B) \tag{2.2}$$

where $\omega_\wedge = p_\wedge/\rho_\wedge$ is EoS parameter of DE;ρ_m is the energy density of the matter; $p_\wedge$ and $\rho_\wedge$ are pressure and energy density of DE,respectively;ρ_B stands for energy density of magnetic field,which can be obtained from Maxwell's equation (i.e.,$\left(F^{ij}\sqrt{-g}\right)_{ij} = 0$) ;$F_{ij}$ is the electro-magnetic field tensor and δ_y and δ_z are deviations from $\omega_\wedge$ in y and z directions,respectively.
The field equations given by Saez-Ballester theory of gravitation(1) for the combined scalar and tensor fields are given by

$$G_i^j - \omega\varphi^n(\varphi_{,i}\varphi^{,j} - \frac{1}{2}g_i^j\varphi_{,b}\varphi^{,b}) = -\left(T_i^j + \bar{T}_i^j\right) \tag{2.3}$$

and the scalar field φ equation satisfies the condition:

$$2\varphi^n\varphi^i_{;i} + n\varphi^{n-1}\varphi^{,b}\varphi_{,b} = 0 \tag{2.4}$$

where

$$G_i^j = R_i^j - \frac{1}{2}Rg_i^j$$

is the Einstein tensor; n is an arbitrary exponent;R_i^j is Ricci tensor, $R = g^{ij}R_{ij}$ is the Ricci scalar,ω is a dimensionless coupling constant, T_i^j is the energy-momentum tensor of matter and $\bar{T}_i^j$ is the energymomentum tensor of magnetized anisotropic modified holographic Ricci dark energy (MHRDE) fluid.
With the help of (2.2) and (2.4) ,the field equations (2.3) for the metric (2.1) in the comoving co-ordinate system take the following explicit form:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\omega\varphi^n\dot{\varphi}^2}{2} = -\omega_\wedge\rho_\wedge + \rho_B \tag{2.5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\omega\varphi^n\dot{\varphi}^2}{2} = -(\omega_\wedge + \delta_y)\rho_\wedge - \rho_B \tag{2.6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} - \frac{\omega\varphi^n\dot{\varphi}^2}{2} = -(\omega_\wedge + \delta_z)\rho_\wedge - \rho_B \tag{2.7}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} + \frac{\omega\varphi^n\dot{\varphi}^2}{2} = \rho_m + \rho_\wedge + \rho_B \tag{2.8}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \tag{2.9}$$

$$\ddot{\varphi} + (\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C})\dot{\varphi} + \frac{n\dot{\varphi}^2}{2\varphi} = 0 \tag{2.10}$$

The energymomentum conservation equation $\left(T_{ij} + \bar{T}_{ij}\right)_{;j} = 0$ leads to two equations for the anisotropic DE and magnetic field (King(46)) as

$$\dot{\rho_m} + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\rho_m + \dot{\rho_\wedge} + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)(1 + \omega_\wedge)\rho_\wedge + \left(\delta_y\frac{\dot{B}}{B} + \delta_z\frac{\dot{C}}{C}\right)\rho_\wedge = 0 \tag{2.11}$$

$$\rho_B = \frac{E}{B^2C^2} \qquad , \tag{2.12}$$

where E is constant.Here the overhead dot stands for ordinary differentiation with respect to time.

3 Solution of the field equations

From equation (2.9) ,we get

$$A = dB,$$

where d is an integration constant.Assuming the constant of integration $d = 1$,the above equation becomes

$$A = B \tag{3.1}$$

Using (3.1) in the field equation (2.5)-(2.11) reduces to a system of five equations with eight unknowns A,C,ρ_m ,δ_y,δ_z, $\omega_\wedge$, $\rho_\wedge$ and φ . Hence,more additional conditions relating these parameters are required to obtain explicit solution of the system. Hence, to get solution of the field equations we use the following possible conditions: (i). We assume that shear scalar of the model is proportional to expansion scalar, which leads to (Collins et al. (52))

$$A = C^k \tag{3.2}$$

(ii).We consider the MHRDE density given by (1.1) in Einstein's theory as

$$\rho_\wedge = 3\left(\alpha_1 H^2 + \alpha_2 \dot{H} + \alpha_3 \ddot{H} H^{-1}\right) \tag{3.3}$$

where $H = \frac{1}{3}\left(\frac{2\dot{A}}{A} + \frac{\dot{C}}{C}\right)$ is the mean Hubble parameter of the model. We know that the average scale factor is given by:

$$a(t) = (ABC)^{1/3}$$

(iii) The Hybrid Expansion Law
We consider the average scale factor as an increasing function of time (Akarsu et.al and Ram et.al (2, 53)) as follows:

$$a\left(t\right) = a_1 t^{\beta_1} e^{\beta_2 t} \tag{3.4}$$

where $a_1 > 0$,$\beta_1 \geq 0$ and $\beta_2 \geq 0$ are constants.They referred to this generalized form of scale factor as the hybrid expansion law, being the mixture of power law and exponential law cosmologies.We observe that (3.4) leads to the power law cosmology for $\beta_2 = 0$ and to exponential cosmology for $\beta_1 = 0$.Thus,the power law and exponential law cosmologies are special cases of hybrid expansion law cosmology.In hybrid cosmology,the universe exhibits a transition from deceleration to acceleration.Recently,Santhi et al.(28) discussed some Bianchi type generalized ghost pilgrim DE models in general relativity using hybrid expansion law. Now using (3.1) and (3.2) in (3.4) we obtain the expressions for metric potentials as

$$A = B = \left(a_1 t^{\beta_1} e^{\beta_2 t}\right)^{3k/2k+1}, C = \left(a_1 t^{\beta_1} e^{\beta_2 t}\right)^{3/2k+1} \tag{3.5}$$

Now the metric (2) can be written as

$$ds^2 = dt^2 - \left(a_1 t^{\beta_1} e^{\beta_2 t}\right)^{6k/2k+1} (dx^2 + e^{-2x} dy^2) - \left(a_1 t^{\beta_1} e^{\beta_2 t}\right)^{6/2k+1} dz^2 \tag{3.6}$$

4 Physical properties of the model

The following physical and kinematical parameters are very important for physical discussion of the model.From (3.3) and (3.5),we get the energy density of modified holographic Ricci dark energy (MHRDE) as

$$\rho_\wedge = 3\left[\alpha_1\left(\frac{\beta_1}{t}+\beta_2\right)^2 - \frac{\alpha_2\beta_1}{t^2} + \frac{2\beta_1\alpha_3}{t^2(\beta_1+\beta_2 t)}\right] \tag{4.1}$$

From (2.12) and (3.5),we have the energy density of the magnetic field as

$$\rho_B = \frac{E}{\left(a_1 t^{\beta_1} e^{\beta_2 t}\right)^{6(k+1)/2k+1}}, \tag{4.2}$$

where E is constant.
From equation (2.10),we get scalar field as follows:

$$\varphi = \left(\frac{n+2}{2}\right)\left(\int \frac{C_1 dt}{(a_1 t^{\beta_1} e^{\beta_2 t})^3}\right)^{\frac{2}{n+2}} \tag{4.3}$$

where C_1 is constant.
Now from field equations (2.5)-(2.8), (2.12), (3.3), (3.5) and (4.1) we get the EoS parameter of MHRDE as

$$\omega_\wedge = \frac{\frac{-9}{(2k+1)^2}\left(\frac{\beta_1}{t}+\beta_2\right)^2(2k^2+1) + 3\frac{(k+1)\beta_1}{(2k+1)t^2} + \frac{\omega C_1^2\left(a_1 t^{\beta_1} e^{\beta_2 t}\right)^{k+1/2k+1}+E}{2\left(a_1 t^{\beta_1} e^{\beta_2 t}\right)^{6(k+1)/2k+1}}}{3\left(\alpha_1(\frac{\beta_1}{t}+\beta_2)^2 - \frac{\beta_1\alpha_2}{t^2} + \frac{2\alpha_3\beta_1}{t^2(\beta_1+\beta_2 t)}\right)} \tag{4.4}$$

the deviations from EoS parameter as

$$\delta_y = \frac{2E}{\left(a_1 t^{\beta_1} e^{\beta_2 t}\right)^{6(k+1)/2k+1} 3\left[-\alpha_1\left(\frac{\beta_1}{t}+\beta_2\right)^2 + \frac{\alpha_2\beta_1}{t^2} - \frac{2\alpha_1\beta_3}{3t^2(\beta_1+\beta_2 t)}\right]} \tag{4.5}$$

$$\delta_z = \frac{\frac{-3(k-1)(\frac{\beta_1\beta_2}{t}+\beta_2^2)}{2k+1} + \frac{1}{(a_1 t^{\beta_1} e^{\beta_2 t})^{6k/2k+1}}}{3\left[\alpha_1\left(\frac{\beta_1}{t}+\beta_2\right)^2 - \frac{\alpha_2\beta_1}{t^2} + \frac{2\alpha_1\beta_3}{3t^2(\beta_1+\beta_2 t)}\right]} + \frac{2E}{\left(a_1 t^{\beta_1} e^{\beta_2 t}\right)^{6(k+1)/2k+1} 3\left[-\alpha_1\left(\frac{\beta_1}{t}+\beta_2\right)^2 + \frac{\alpha_2\beta_1}{t^2} - \frac{2\alpha_1\beta_3}{3t^2(\beta_1+\beta_2 t)}\right]} \tag{4.6}$$

and the energy density of matter can be found as

$$\begin{aligned}\rho_m = \frac{9k(k+2)}{(2k+1)^2}\left(\frac{\beta_1}{t}+\beta_2\right)^2 - \Bigg(&\frac{E + \left(a_1 t^{\beta_1} e^{\beta_2 t}\right)^{6/2k+1} - \omega C_1^2\left(a_1 t^{\beta_1} e^{\beta_2 t}\right)^{(k+1)/2k+1}}{2\left(a_1 t^{\beta_1} e^{\beta_2 t}\right)^{6(k+1)/2k+1}}\Bigg) \\ &-3\left[\alpha_1\left(\frac{\beta_1}{t}+\beta_2\right) - \frac{\alpha_2\beta_1}{t^2} + \frac{2\alpha_1\beta_3}{3t^2(\beta_1+\beta_2 t)}\right]\end{aligned} \tag{4.7}$$

Thus the metric(3.6) together with(3.5)(4.4) constitutes Bianchi type-III MHRDE cosmological model in Saez-Ballester theory with hybrid expansion law. The proper volume of the model is defined by

$$V = ABC = \left(a_1 t^{\beta_1} e^{\beta_2 t}\right)^3 \tag{4.8}$$

Hubble's parameter

$$H = \frac{\dot{a}}{a} = \frac{\beta_1}{t} + \beta_2 \tag{4.9}$$

The scalar expansion is

$$\theta = 3H = 3\left(\frac{\beta_1}{t} + \beta_2\right) \tag{4.10}$$

The shear scalar is

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right)^2 = \frac{3(k-1)^2}{(2k+1)^2}\left(\frac{\beta_1}{t} + \beta_2\right)^2 \tag{4.11}$$

The deceleration parameter is

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = -1 + \frac{\beta_1}{(\beta_1 + \beta_2 t)^2} \tag{4.12}$$

The anisotropic parameter is

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{6(k-1)^2}{(2k+1)^2} \tag{4.13}$$

where $H_1 = H_2 = \frac{\dot{A}}{A}$ and $H_3 == \frac{\dot{C}}{C}$ are directional Hubble's parameters.
The state finder parameters are

$$r = \frac{\dddot{a}}{aH^3} = 1 + \frac{2\beta_1}{(\beta_1 + t\beta_2)^3} - \frac{3\beta_1}{(\beta_1 + t\beta_2)^2} \tag{4.14}$$

$$s = \frac{r-1}{3(q-1/2)} = \frac{2\beta_1\left[2 - 3(\beta_1 + \beta_2 t)\right]}{3(\beta_1 + \beta_2 t)\left(\beta_1 - 3(\beta_1 + \beta_2 t)^2\right)} \tag{4.15}$$

The illustrations of physical parameters using graphical representations are shown below as the followings.

Figure 1 indicates the behavior of EoS parameter for the MHRDE model, which shows that the EoS parameter starts from lower quintessence region and goes to the higher phantom region for the values of $\beta_1 = 0.2$ and $\beta_2 = 0.8$ as time t increases.However,the EoS parameter remains in the phantom region as time t goes to infinity.Hence,in these cases we observe that the EoS parameter favors the pilgrim DE phenomenon for these two values of β's .
From Figure.2, we conclude that the present universe with hybrid expansion law evolves with variable deceleration parameter and transition from deceleration to acceleration takes place at

$$t = \frac{\sqrt{\beta_1} - \beta_1}{\beta_2}.$$

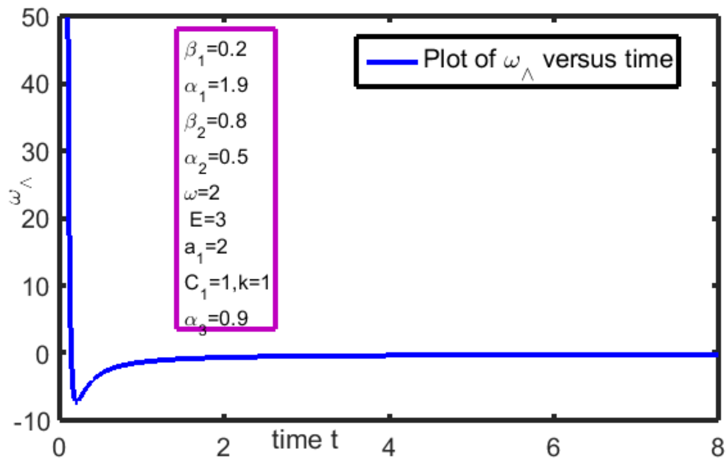

Figure 1: **P**lot of $\omega_\wedge$ versus time t.

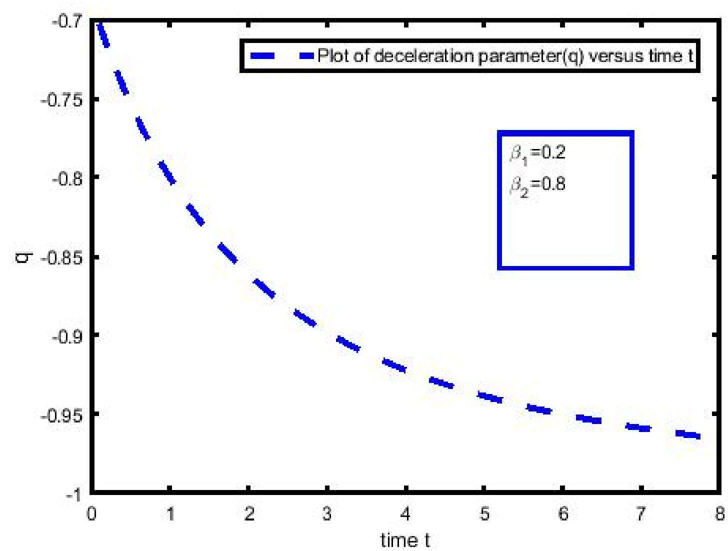

Figure 2: **P**lot of deceleration parameter versus time t.

It is also clear from Figure.2 that for $\beta_1 \geq 1$,the model is evolving only in the accelerating phase whereas for $\beta_1 < 1$,the model is evolving from the early decelerated phase to the present accelerating phase.As time t goes to infinity ,then the deceleration parameter q approaches to -1.That means for large values of time t, deceleration parameter(q) becomes constant and shift to accelerated universe. From equation (4.12),we find that $q \to \infty$ as $t \to 0$ and $q \to -1$ as $t \to \infty$, which implies that the model is not steady state.In Figure.3,graphical plot of time verses

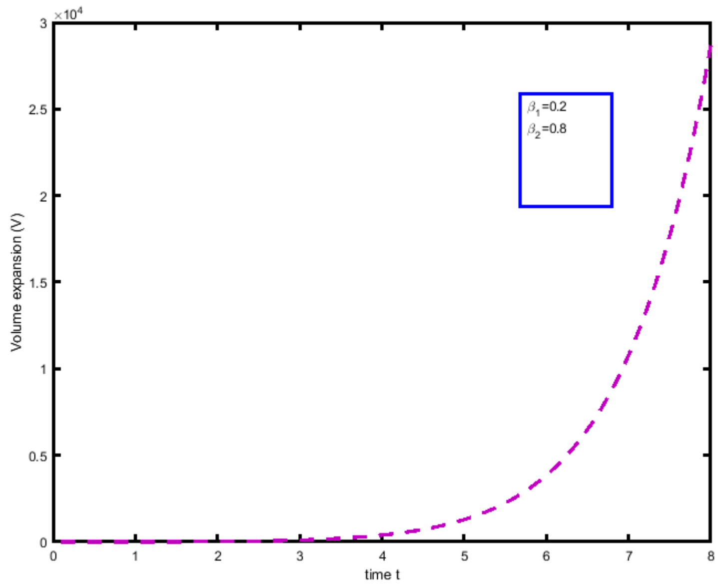

Figure 3: **P**lot of volume expansion versus time t.

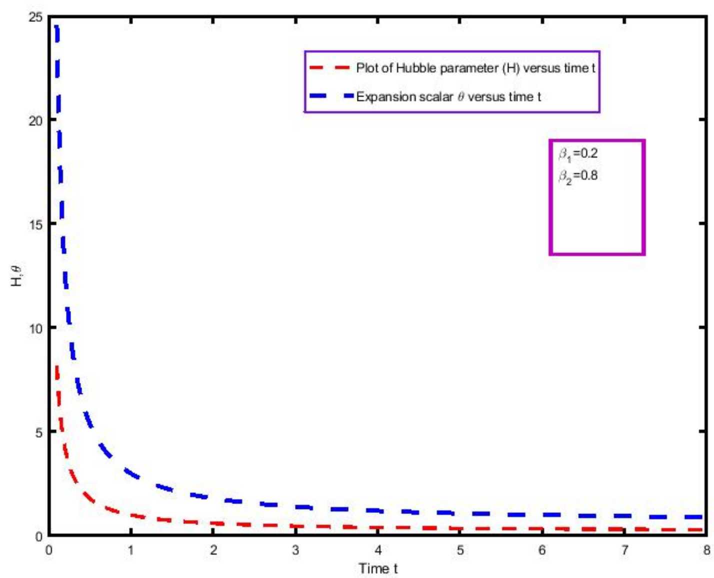

Figure 4: **P**lot of Hubble parameter and Expansion scalar versus time t.

spatial volume, from (4.8),we find that $V = 0$ at $t = 0$ and gradually increases as $t \to \infty$. This shows that the universe starts evolving with zero volume at $t = 0$ and expands with the increase of the age of the universe.The volume expansion of the model has point type singularity

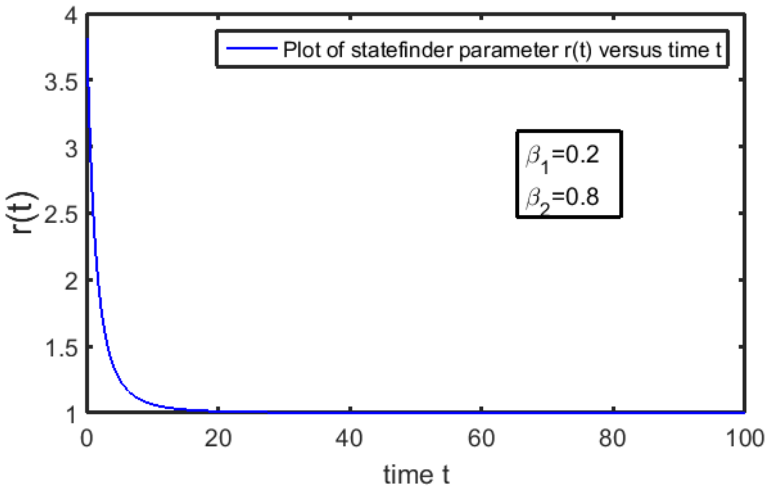

Figure 5: **P**lot of state finder parameter r(t)versus time t.

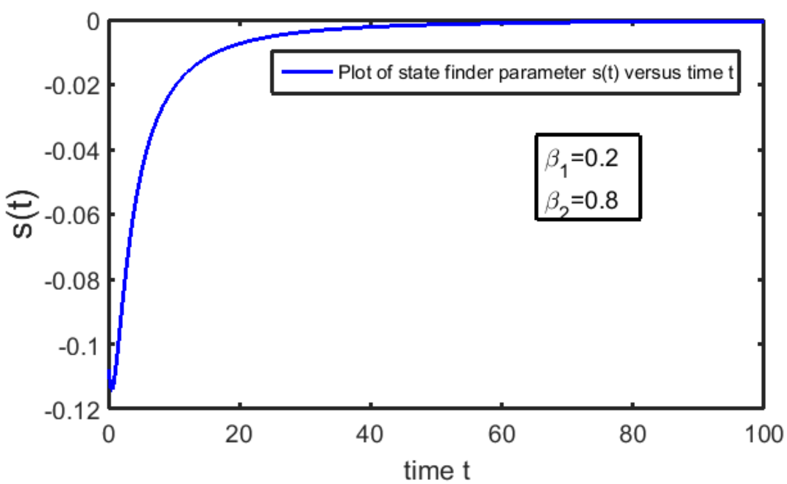

Figure 6: **P**lot of state finder parameter s(t)versus time t.

$t = 0$ and it increases as time t increases. From Figure.4,we observed that both the average Hubble parameter and expansion scalar of the model are decreasing functions as t increases.From (4.9) and (4.10) as time t increases both the expansion scalar and the average Hubble parameter approaches constant value.However,when t approaches to zero,both H and θ tends to infinite. From Figure.5,it was observed that the state finder parameter $r(t)$ versus t with the $\beta_1 = 0.2$ and $\beta_2 = 0.8$ decreases as time increases .

This implies that in the accelerated universe,in magnetized modified holographic Ricci dark energy the Eos parameter becomes from quintessence region to Phantom region. Figure.6 shows the plot of state finder parameter $s(t)$ versus time t. As show in the figure as time t increases, state finder parameter $s(t)$ increases.As time t increases ,it goes from low quintessence region to

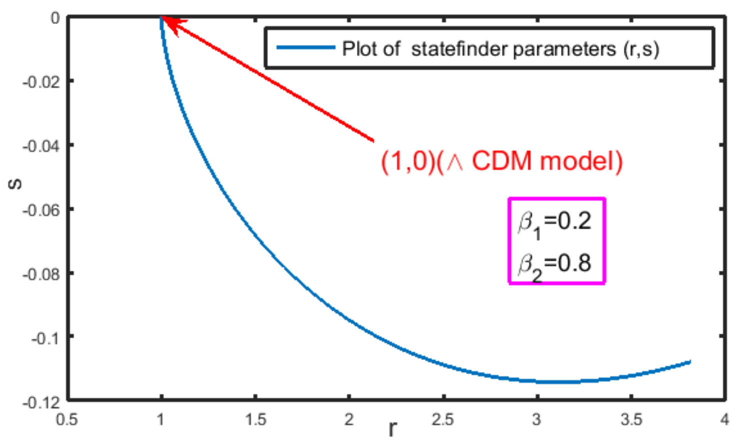

Figure 7: **P**lot of state finder parameter of s versus r.

high quintessence region.From Figure.7 indicates that at the cosmic time state finder parameter curve meets the point $(r, s) = (1, 0)$,which shows to $\wedge$CDM model.The r, s plane corresponding to $\beta' s$ parameters,also provides the region Chaplygin gas model($s < 0$ and $r > 1$). From (4.3), we find that the scalar field is found to be diverges with respect to time at $t = 0$ and gradually increases.But as $t \to \infty$,then the scalar field will be decreases to zero in the course of evolution.

Further we find that

$$\frac{\sigma^2}{\theta^2} \neq 0$$

as $t \to \infty$ which implies that the model does not approach isotropy with respect to the increase of the age of the universe.Also it is found that the mean anisotropy parameter $\Delta \neq 0$ for $k \neq 1$. Thus our universe is anisotropic at all time for $k \neq 1$.

5 Conclusion

In this paper, we have investigated anisotropic Bianchi Type III magnetized modified holographic Ricci dark energy cosmological model in the frame work of Saez-Ballester theory of gravitation.Also we assumed the scale factor which has the hybrid expansion form.Hence the following results were obtained:we find that the model starts with the big-bang at $t = 0$. We also find that for $t \to \infty$,the energy density of magnetic field decreases and approaches to zero. Further we find that the scalar field decreases and become constant as time t tends to infinity.The physical properties of the model obtained by using hybrid expansion law provides a very nice description of the transition from the early deceleration to present cosmic acceleration, which is an essential feature for evolution of the universe. We observe that the spatial volume of the model tends to zero at $t = 0$.For the model, the spatial volume is constant at $t = 0$ and increases exponentially with time. This shows that at the initial epoch,the universe starts with constant volume and expands exponentially approaching infinite volume. Therefore, the model has point-type singularity at $t = 0$. At this epoch, all the physical and kinematical parameters diverge.

As $t \to \infty$, spatial volume becomes infinite. As $t \to \infty$, Hubbles parameter H is constant hence the universe expands forever with a constant rate. Hubbles parameter(H) and expansion scalar(θ) are infinite at $t = 0$ whereas, they are finite as $t \to \infty$,which indicates inflationary scenario. From Fig.2,we conclude that our models represent early decelerated phase to present accelerating phase.Recent observations of type Ia supernovae,expose that the present universe is accelerating and the value of the deceleration parameter lies in the range of $-1 \leq q < 0$. It follows that in our model,one can choose values of deceleration parameter consistent with recent observations.
The behavior of EoS parameter of the model is shown in Fig. 1, we observe that the EoS parameter starts with a comparatively high value of phantom and always remains in that region throughout the evolution of the universe.This behavior resembles the pilgrim DE.The contribution of magnetic field is exhibited in the expression of the physical parameters $\omega_\wedge$,ρ_m and skewness parameters.If $E = 0$,the effect of magnetic field vanishes. The rs plane which is shown in Fig.7 indicates that the trajectories of the rs plane for the model corresponds to the $\wedge$CDM model. Also,the trajectory coincides with some well-known DE model corresponding to Chaplygin gas.Thus, the inflationary scenario exists.It is observed that the Hubble parameter (H),expansion scalar (θ),mean anisotropic parameter of the expansion (Δ) ,shear scalar(σ),magnetized DE density (ρ_B) and energy density of modified holographic Ricci dark ($\rho_\wedge$) are decreasing functions of time .

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References

[1] D.Saez,V.J.Ballester, : Phys. Lett. A **133**,477(1986)

[2] . Akarsu, S. Kumar, R. Myrzakulov, M. Sami, and L. Xu. JCAP **01**, 022 (2014)

[3] A.G. Riess, A.V. Filippenko, P. Challis, et al. Astron. J.**116**, 1009 (1998)

[4] S. Perlmutter, G. Aldering, G. Goldhaber, et al. Astrophys. J.**517**, 565 (1999)

[5] S.Weinberg, : Rev. Mod. Phys.**61**, 1 (1989)

[6] E.J. Copeland, M. Sami, and S. Tsujikawa. Int. J. Mod. Phys. D, **15**, 1753(2006)

[7] T. Barreiro, et al.: Phys. Rev. D **61**, 12730 (2000)

[8] R.R.Caldwell,: Phys. Lett. B **23**, 545 (2002)

[9] C.Armendariz,et al.: Phys.Rev.D**63**, 103570(2001)

[10] J.S.Bagla,et al.:Phys.Rev.D **67**, 063504 (2003)

[11] M.C.Bento,O.Bertolami,and A.A. Sen. Phys. Rev. D **66**, 043507 (2002)

[12] M. Li. Phys. Lett. B **603**, 1 (2004)

[13] T.Padmanabhan. Phys. Rep.**380**, 235 (2003)

[14] L.Susskind. J. Math. Phys. **36**, 6377 (1995)

[15] A.G.Cohen,D.B.Kaplan,and A.E. Nelson. Phys. Rev. Lett. **82**, 4971 (1999)

[16] C. Gao, F. Wu, X. Chen, and Y. Shen. Phys. Rev. D, **79**, 043511 (2009)

[17] Q.G. Huang and M. Li. JCAP, **0408**, 013(2004)

[18] X. Zhang and F.Q. Wu. Phys. Rev. D, **72**, 043524 (2005)

[19] L.N. Granda and A. Oliveros. Phys. Lett. B, **669**,275 (2008)

[20] S.Chen and J. Jing. Phys. Lett. B, **679**, 144 (2009)

[21] S. Sarkar and C.R. Mahanta. Int. J. Theor. Phys. **52**, 1482 (2013)

[22] S.Sarkar.Astrophys.SpaceSci.**349**,985(2014)

[23] K.S. Adhav, S.L. Munde, G.B. Tayade, and V.D. Bokey. Astrophys. Space Sci. **359**, 24 (2015)

[24] M. Kiran, D. R. K. Reddy, V. U. M. Rao andM. P. V. V. Baskara Rao, Astrophys. Space Sci.**356**, 137 (2014).

[25] M.V. Santhi, V.U.M. Rao, and Y. Aditya. Int. J. Theor. Phys. (2016)

[26] K.Das and T. Sultana. Astrophys. Space Sci. **360**, 4 (2015)

[27] K.Das an T.Sultana.Astrophys.SpaceSci.361,53(2016)

[28] M.V. Santhi, V.U.M. Rao, and Y. Aditya. Prespacetime J. **7**, 1379 (2016)

[29] M.V. Santhi, V.U.M. Rao, and Y. Aditya. Can. J. Phys. **95**, 179 (2016)

[30] M.K.Yadav,Rai, A., Pradhan, A.: Int. J. Theor. Phys. **46**, 2677 (2007)

[31] A.Pradhan, Amirhashchi, H., Zainuddin, H.: Astrophys. Space Sci. (2010)

[32] A.K.Yadav,L.Yadav : :1007.1411 [gr-qc] (2010)

[33] C. Brans and R.H. Dicke Phys. Rev. **124**, 925(1961)

[34] K. Nordvedt Astrophys. J. **161**, 1059(1970)

[35] G.A. Barber Gen. Relativ. Gravit. **14**, 117(1982)

[36] D. Saez and V.J. Ballester Phys. Lett. **A113**, 467(1985)

[37] Y.K. Lau and S.J. Prokhovnik Aust. J. Phys. **39**, 339(1986)

[38] D. R. K. Reddy, R. L. Naidu and V. U. M.Rao, Astrophys. Space Sci. **306** , 185 (2006).

[39] D. R. K. Reddy and R. L. Naidu, Astrophys.Space Sci. **307** , 395 (2007).

[40] V. U. M. Rao, T. Vinutha and M. Vijaya Santhi, Astrophys. Space Sci. **312** , 189 (2007).

[41] V. U. M. Rao, M. Vijaya Santhi and T. Vinutha, Astrophys. Space Sci. **317**, 27 (2008).

[42] V. U. M. Rao, G. S. D. Kumari and K. V.S. Sireesha, Astrophys. Space Sci. **335**,635(2011).

[43] V. U. M. Rao, K. V. S. Sireesha and D. Neelima, ISRN Astron. and Astrophys.**2013** ,924834 (2013).

[44] A.Pradhan, A. K. Singh and D. S. Chouhan,Int. J. Theor. Phys. **52** , 266 (2013)

[45] M.S. Madsen. Mon. Not. R. Astron. Soc. **237**, 109 (1989)

[46] E.J. King and P. Coles. Class. Quant. Grav. **24**, 2061 (2007)

[47] E. Milaneschi and R. Fabbri. Astron. Astrophys. **151**, 7 (1985)

[48] M. Sharif and M. Zubair. Astrophys. Space Sci. **330**, 399 (2010)

[49] M. Sharif and M. Zubair. Astrophys. Space Sci. **339**, 45 (2012)

[50] M.V. Santhi, V.U.M. Rao, and Y. Aditya. Can. J. Phys.**95**, 381(2017)

[51] K.C. Jacobs. Astrophys. J. **155**, 379 (1969)

[52] C.B. Collins, et al. Gen. Relativ. Gravit. **12**, 805 (1983)

[53] S. Ram and S. Chandel. Astrophys. Space Sci. **355**, 195 (2015)