Article

Kaluza-Klein Bouncing Cosmological Model with Viscous Fluids

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Abstract

Bouncing behavior of Kaluza-Klein cosmological model has studied with viscous fluid by considering different forms of scale factors. Some physical properties of the fluids which realize them and the possibility to have acceleration after the bounce have discussed.

Keywords: Kaluza-Klein spacetime, scale factor, viscous fluid, bounce, energy conditions.

1. Introduction

According to the modern cosmological picture, our universe experiences the cosmic acceleration in the presence of strong repulsive force dubbed as dark energy (DE). The presence of this type of force has been predicted through various cosmological and astrological data. [19-20, 31-35]. However, its nature is still unknown which need further attention. The cosmological constant is the pioneer candidate of dark energy. The observations also indicate that the dark energy fluid in the universe is not a perfect fluid [21] and the viscosity plays a role in the evolution of the universe [5, 8, 13].

Various cosmological models for the evolution of the universe have been presented by number of researchers. They aim at explaining the acceleration era of the universe and they include future singularity phenomenon such as the dramatic Big Rip [11, 26-27], the other little Rip [4, 16-17], the pseudo Rip [18], The Quasi Rip [39], the bounce cosmology [1-2, 7, 10, 30]. Brevik *et al.* [6-7] has studied the little Rip and the pseudo Rip phenomenon, and also the bounce universe induced by an inhomogeneous dark fluid coupled with dark matter. Myrzakul *et al.*, Myrzakulov and Sevatiani and Elizalde *et al.* [15, 24-25] have studied cosmological models in modified theory of gravitation with viscous fluids.

In this paper, we discuss different bounce solutions and the features of the related viscous fluids, taking into account the necessity to have a cosmic (inflationary) acceleration after the bounce. The paper is organized as follows. In section 2, the formalism of inhomogeneous viscous fluid in

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Kaluza-Klein universe is presented. In section 3, we analyze the bounce solutions in fluids cosmology. In section 4, the same investigations are carried out for the bounce solutions, conclusions and remarks are given in section 5.

2. Metric and Field Equations

Five-dimensional Kaluza-Klein space-time is considered in the form

$$ds^2 = dt^2 - A^2(t)(dx^2 + dy^2 + dz^2) - B^2(t)d\psi^2 \quad , \tag{1}$$

where $A(t)$ and $B(t)$ are scale factors which are functions of cosmic time t. The fifth coordinate ψ is taken to be space-like.

The energy-momentum tensor for the viscous fluid is given by

$$T_{ij} = \rho u_i u_j + p(u_i u_j - g_{ij}) \, , \tag{2}$$

where u_i are the four co-moving velocity vectors, ρ is the energy density and g_{ij} is the metric tensor.

The fluid in the universe is inhomogeneous viscous fluid with equation of state [12, 28-29] given by

$$p = \gamma(\rho) - B(a, H, \dot{H}, ...) \, , \tag{3}$$

where the equation of state parameter γ may depend on ρ. B is a bulk viscosity which is general function of a, H and its derivatives. a is the average scale factor.

From the equations (2) and (3), the energy-momentum tensor for the viscous fluid is given by

$$T_{ij} = \rho u_i u_j + \gamma(\rho) - B(a, H, \dot{H}, ...)(u_i u_j - g_{ij}) \, . \tag{4}$$

The Einstein's field equations in general theory of gravitation are

$$R^i_j - \frac{1}{2} R g^i_j = -T^i_j \, . \tag{5}$$

With the help of equation (4) , the field equations (5) for the metric (1) are

$$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{B}}{AB} = \rho \quad , \tag{6}$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = -p \quad , \tag{7}$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} = -p \quad , \tag{8}$$

where an overhead dot $\left(\cdot \right)$ represents differentiation with respect to *t*.

The energy-conservation equation, which is the consequence of the field equations is given by,

$$T^{ij}_{;j} = 0 \, ,$$

where

$$T^{ij}_{;j} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^j}\left(T^{ij}\sqrt{-g}\right) + T^{jk}\Gamma^i_{jk}$$

which simplifies to

$$\dot{\rho} + 4H(\rho + p) = 0 \tag{9}$$

Using equation (3), equation (9) reduces to

$$\dot{\rho} + 4H(1+\gamma(\rho))\rho = 4HB(\rho,a,H,\dot{H},....) \ . \tag{10}$$

From the thermodynamic point of view, for positive entropy change in an irreversible process, the bulk viscosity must be a positive quantity [5, 8].

The cosmological parameter Ω is defined as:

$$\Omega = 1 + \frac{1}{a^2 H^2} \ . \tag{11}$$

The quantity Ω in general, may be different from.

By a bouncing universe, we mean a universe that undergoes a collapse, attains a minimum and then subsequently expands. For a successful bounce in Kaluza-Klein model, during contraction phase the scale factor $a(t)$ is decreasing i.e $(\dot{a}(t) < 0)$ and then in the expanding phase, the scale factor is increasing i.e $(\dot{a}(t) > 0)$. At the bounce point (i.e. at $t = t_b$), the minimal necessary conditions are

i) $\dot{a}(t_b) = 0$ and

ii) $\ddot{a} > 0$ for $t \in (t_b - \in, t_b)U(t_b, t_b + \in)$, for small $\in > 0$.

For non-singular bounce $\dot{a}(t) \neq 0$. These conditions may not be sufficient for a non-singular bounce. This is well explained by Singh et al. [36-38].

The bounce behavior of cosmological model is also realized using energy conditions as mentioned in Molina-Paris & Visser and Singh *et al.* [22, 38] . In terms of energy density ρ and pressure p, the energy conditions can be stated as:

Null Energy Condition (NEC) is satisfied when $\rho + p \geq 0$.

Weak Energy Condition (WEC) is satisfied when $\rho \geq 0$ and $\rho + p \geq 0$.

Dominant Energy Condition (DEC) is satisfied when $\rho \geq |p|$.

Strong Energy Condition (SEC) is satisfied when $\rho + p \geq 0$ and $\rho + 4p \geq 0$.

It is clear that violation of SEC will lead to a violation of other energy conditions.

3. Solution of the Field Equations

The field equations (6) to (8) are a system of three highly non-linear differential equations in four unknowns A, B, ρ and γ . The system is thus initially undetermined. We need one extra condition for solving the field equations completely. We assume that the expansion scalar (θ) is proportional to shear (σ). This condition leads to

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)=\alpha_0\left(3\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right),$$

which yields

$$\frac{\dot{B}}{B}=m\frac{\dot{A}}{A},$$

where α_0 and m are arbitrary constants.

Above equation on integration reduces to

$$B=\eta(\mathrm{A})^m,$$

where η is an integration constant.

Here, for simplicity and without loss generality, we assume that $\eta=1$.

Hence, we have

$$B=\mathrm{A}^m. \tag{12}$$

Collins *et al.* [14] have pointed out that for spatially homogeneous metric, the normal congruence to the homogenous expansion satisfies the condition $\frac{\sigma}{\theta}$ is constant.

4. CASE I: FLUID MODEL WITH $a(t)=\sqrt{a_0^2+\beta^2t^2}$

The bouncing cosmological model has obtained by choosing the average scale factor *a(t)* of the form [23]

$$a(t)=\sqrt{a_0^2+\beta^2t^2}, \tag{13}$$

where a_0, β are non-zero positive constants.

The above scale factor is the temporal analogue of the toy model traversable wormhole [23]. One may get phenomenological quintom bouncing model with proper renormalization of a_0, β [9].

The Hubble parameter is given by

$$H(t)=\frac{\dot{a}}{a}=\frac{\beta^2t}{a_0^2+\beta^2t^2}. \tag{14}$$

In terms of geometrical quantities, we have

$$\frac{\ddot{a}}{a}=\dot{H}+H^2=\frac{(\beta\,\mathrm{a}_0)^2}{(\mathrm{a}_0^2+\beta^2\,\mathrm{t}^2)^2}. \tag{15}$$

For the metric (1), the average scale factor is given by

$$a(t)=(\mathrm{A}^3\mathrm{B})^{\frac{1}{4}}. \tag{16}$$

From equations (13) and (16), we have

$$\Rightarrow \qquad A^3 B = (a_0^2 + \beta^2 t^2)^2 \ . \tag{17}$$

With the help of equation (12), it reduces to

$$A = (a_0^2 + \beta^2 t^2)^{\frac{2}{(m+3)}} \tag{18}$$

Using equation (18) in equation (12) therein

$$B = (a_0^2 + \beta^2 t^2)^{\frac{2m}{m+3}} \ . \tag{19}$$

With the help of equations (18) and (19), the metric (1) becomes

$$ds^2 = dt^2 - (a_0^2 + \beta^2 t^2)^{\frac{4}{m+3}} (dx^2 + dy^2 + dz^2) - (a_0^2 + \beta^2 t^2)^{\frac{4m}{m+3}} d\psi^2 \ . \tag{20}$$

Equation (20) represents Kaluza-Klein bouncing cosmological model with viscous fluid in general theory of relativity.

Some physical properties of the model

For the cosmological model (20), the physical quantities such as spatial volume V, Hubble parameter H, expansion scalar θ, mean anisotropy A_m, Shear scalar σ^2, energy density ρ are obtained as follows;

The spatial volume is in the form

$$V = a^4 = (a_0^2 + \beta^2 t^2)^2 \ . \tag{21}$$

The Hubble parameter is

$$H = \frac{1}{4}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = \frac{\beta^2 t}{(a_0^2 + \beta^2 t^2)} \ . \tag{22}$$

The expansion scalar is

$$\theta = 4H = \frac{4\beta^2 t}{(a_0^2 + \beta^2 t^2)} \ . \tag{23}$$

The mean anisotropy parameter A_m is

$$A_m = \frac{3(m-1)^2}{(m+3)^2} = \text{Constant} \neq 0 \ , \quad \text{for } m=1. \tag{24}$$

The shear scalar is

$$\sigma^2 = \frac{6(m-1)^2 \beta^4 t^2}{(m+3)^2 (a_0^2 + \beta^2 t^2)^2} \ . \tag{25}$$

It is observed that

$$\lim_{t \to \infty} \frac{\sigma^2}{\theta^2} = \frac{3(m-1)^2}{8(m+3)^2} \neq 0 \ , \text{ for } m \neq 1 \ . \tag{26}$$

The mean anisotropy parameter A_m and $\lim_{t \to \infty} \frac{\sigma^2}{\theta^2} = \text{constant} \neq 0$. Hence, the model is anisotropic throughout the evolution of the universe, except at $m=1$. (i.e. the model does not approach isotropy.)

The matter-energy density is given by

$$\rho = \frac{48(m+1)\beta^4 t^2}{(m+3)^2 (a_0^2 + \beta^2 t^2)} \ , \tag{27}$$

$$\rho + p = \frac{24(3m+1)\beta^4 t^2}{(m+3)^2 (a_0^2 + \beta^2 t^2)} - \frac{12\beta^2}{(m+3)(a_0^2 + \beta^2 t^2)} \ , \tag{28}$$

$$\rho - p = \frac{24\beta^4 t^2}{(m+3)(a_0^2 + \beta^2 t^2)^2} - \frac{12\beta^2}{(m+3)(a_0^2 + \beta^2 t^2)}, \tag{29}$$

$$\rho + 4p = \frac{48(3m-1)\beta^4 t^2}{(m+3)^2 (a_0^2 + \beta^2 t^2)} - \frac{48\beta^2}{(m+3)(a_0^2 + \beta^2 t^2)} \ . \tag{30}$$

The cosmological parameter for the closed universe takes the form,

$$\Omega = 1 + \frac{1}{a^2 H^2} \ .$$

$$\Omega = 1 + \frac{1}{\beta^2} + \frac{a_0^2}{\beta^4 t^2} . \tag{31}$$

Figures (4.1) (a), (b), (c), (d), (e), (f) represent the plots of time versus (a) Average scale factor (b) Hubble parameter (c) Energy density (ρ) (d) $\rho+p$ (e) $\rho-p$ and (f) $\rho+4p$ which are as shown below.

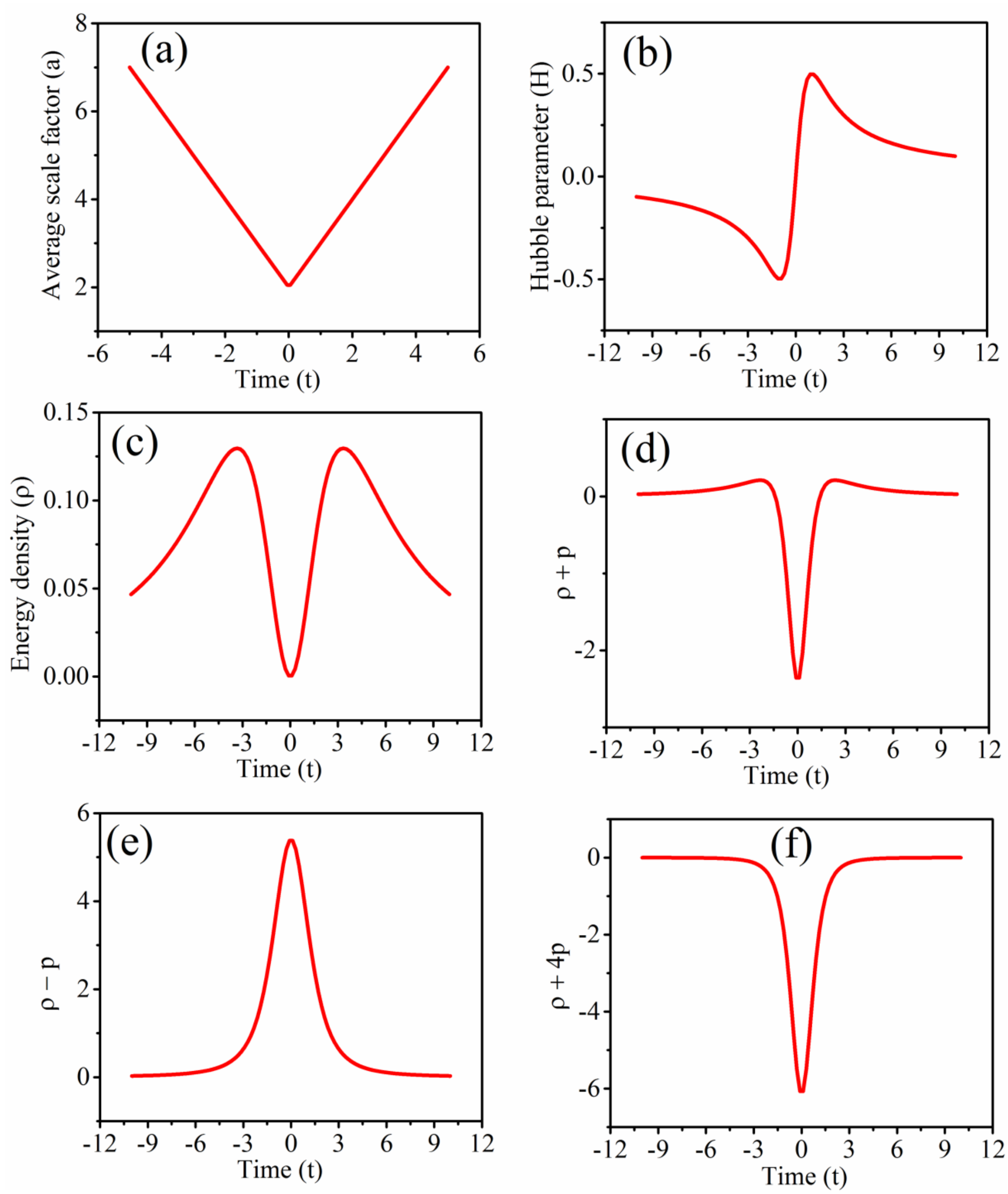

Fig. 4.1 Plots of time versus (a) Average scale factor (b) Hubble parameter (c) Energy density (ρ) (d) $\rho+p$ (e) $\rho-p$ (f) $\rho+4p$ for the values of constants $a_0=1$ and $\beta=1$.

Discussion: Fig. (4.1) (a) is the plot of time versus average scale factor. It is seen that, during contraction phase, the average scale factor $a(t)$ is decreasing (i.e. $\dot{a}(t)<0$) and then in the expanding phase $a(t)$ is increasing (i.e. $\dot{a}(t)>0$). Hence, the minimal necessary conditions (i) and (ii) for the bounce at time $t=0$ are satisfied [38].

(i) $\dot{a}(t)=0$ at $t=0$

and (ii) at $t=0$, $\ddot{a}(t)>0$ for $t\in(0-\in,0)\mathrm{U}(0,0+\in)$, where $\in$ is very small.

Fig. (4.1) (b) is the plot of time versus Hubble parameter. At $t=0$, $H=0$ and $a(0)=a_0$ with $\dot{H}>0$ in small neighborhood of $t=0$. Thus it satisfies the necessary conditions of bounce [36-37].

To realize the bounce in our model, let we obtain the values of ρ, $\rho+p$, $\rho-p$ and $\rho+4p$ for $m=2$.

$$\rho=(5.76)\beta^4t^2(a_0^2+\beta^2t^2)^{-2},$$

$$\rho+p=(6.72)\beta^4t^2(a_0^2+\beta^2t^2)^{-2}-(2.4)\beta^2(a_0^2+\beta^2t^2)^{-1}$$

$$\rho-p=(4.8)\beta^4t^2(a_0^2+\beta^2t^2)^{-2}+(2.4)\beta^2(a_0^2+\beta^2t^2)^{-1}$$

$$\rho+4p=(9.6)\beta^4t^2(a_0^2+\beta^2t^2)^{-2}-(9.6)\beta^2(a_0^2+\beta^2t^2)^{-1}$$

Figs. (4.1) (c), (d), (e) and (f) are the plots of time versus energy density $\left(\rho\right)$, $\rho+p$, $\rho-p$ and $\rho+3p$. It is observed that WEC, NEC and DEC energy conditions are satisfied at $t=0$ but SEC is violated at $t=0$. Hence, the bounce is realized in our model at $m=2$. The result is also true for $m=4,6$.

The bulk viscosity is

$$B(a,H,\dot{H},.....)=4H\zeta\quad. \tag{32}$$

In this specific case, equation (9), takes the form

$$\rho=-p+4H\zeta\quad. \tag{33}$$

From equation (33), we have

$$4H\zeta=\frac{24(3m+1)}{(m+3)^2}H^2-\frac{12\beta^2}{(m+3)\,\mathrm{a}^2} \tag{34}$$

Dividing by $4H$

$$\zeta(H,a)=\frac{6(3m+1)H}{(m+3)^2}-\frac{3\beta^2}{(m+3)Ha^2}\quad. \tag{35}$$

When the scale factor becomes large, $\zeta(\mathrm{H})\approx\frac{6(3m+1)H}{(\mathrm{m}+3)^2}$ and then we can treat it as a fluid with a bulk viscosity of $o(H)$. Therefore, with $\zeta(\mathrm{H,a})$ we can recognize bouncing universe with the scale factor in equation (13).

In another example, we take $\gamma(\rho)=\gamma$ (constant) and $B(\mathrm{a,H,\dot{H},......})=4\mathrm{H}\zeta$ with $\zeta=\in\rho$, where $\in$ is a constant.
From equation (9), we have

$$\dot{\rho}=-4H(\rho+\mathrm{p}).$$

Using equation (3), we get

$$\dot{\rho}=-4H\left(\gamma(\rho)\rho-B(a,H,\dot{H},......)\right).$$

Since, $\zeta=\in\rho$ and $B(\mathrm{a,H,\dot{H},......})=4\mathrm{H}\zeta$, we have $\rho=\frac{a_1}{(\mathrm{a}_0^2+\beta^2\,\mathrm{t}^2)^{\frac{3(1+\gamma-\in)}{2}}}$

$$\rho=\frac{a_1}{(a_0^2+\beta^2t^2)^{\frac{3(1+\gamma-\in)}{2}}} \tag{36}$$

where, a_1 is a constant of integration.
Also, from equation (3) we get

$$p=\gamma(\rho)\rho-B(a,H,\dot{H},.....)$$

Using $\gamma(p)=\gamma=$ constant and $B(\mathrm{a,H,\dot{H},......})=4\mathrm{H}\zeta$, we get

$$p=(\gamma-4\mathrm{H}\in)\rho.$$

The scenario with $\gamma=$ constant, $\zeta=\in\rho$ the energy density of the bouncing universe will decrease with increasing time (provided, $(1+\gamma>\in)$ and also the bulk viscosity. For the large time t,p will become negative.

5. CASE II: FLUID MODEL WITH $a(t)=a_1(\mathrm{e}^t+\mathrm{e}^{-t})$

The bouncing cosmological model has been obtained by choosing the average scale factor *a(t)* of the form [1]

$$a(t)=a_1(\mathrm{e}^t+\mathrm{e}^{-t}) \ , \tag{37}$$

where the a_1 , α are non-zero positive constants.
The Hubble parameter is given by

$$H(t)=\frac{\dot{a}}{a}=\frac{\alpha(\mathrm{e}^{\alpha t}-\mathrm{e}^{-\alpha t})}{(\mathrm{e}^{\alpha t}+\mathrm{e}^{-\alpha t})} \ . \tag{38}$$

For the metric (1), the average scale factor is given by

$$a(t)=(\mathrm{A}^3B)^{\frac{1}{4}} \ . \tag{39}$$

From equations (37) and (39) we have

$$A^3B=a_1^4(e^{\alpha t}+e^{-\alpha t})^4 \tag{40}$$

With the help of equation (12), equation (40) reduce to

$$A = a_1^{\frac{4}{m+3}} (e^{\alpha t} + e^{-\alpha t})^{\frac{4}{m+3}}. \tag{41}$$

Using equation (41), equation (12) leads to

$$\Rightarrow \qquad B = a_1^{\frac{4m}{m+3}} (e^{\alpha t} + e^{-\alpha t})^{\frac{4m}{m+3}} . \tag{42}$$

With the help of equations (4) and(42), the metric (1) can be written as

$$ds^2 = dt^2 - a_1^{\frac{8}{m+3}} (e^{\alpha t} + e^{-\alpha t})^{\frac{8}{m+3}} (dx^2 + dy^2 + dz^2) - a_1^{\frac{8m}{m+3}} (e^{\alpha t} + e^{-\alpha t})^{\frac{8m}{m+3}} d\psi^2. \tag{43}$$

The equation (43) represents Kaluza-Klein viscous fluid cosmological model in general theory of relativity.

SOME PHYSICAL PROPERTIES OF THE MODEL

For the cosmological model (43), the physical quantity such as spatial volume V, Hubble parameter H, expansion scalar θ, mean anisotropy A_m shear scalar σ^2, energy density ρ are obtained as follows.

The spatial volume is in the form

$$V = a_1^4 (e^{\alpha t} + e^{-\alpha t})^4. \tag{44}$$

The Hubble parameter is

$$H = \frac{1}{4}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = \frac{\alpha(e^{\alpha t} - e^{-\alpha t})}{(e^{\alpha t} + e^{-\alpha t})}. \tag{45}$$

The expansion scalar is given by

$$\theta = 4H = \frac{4\alpha(e^{\alpha t} - e^{-\alpha t})}{(e^{\alpha t} + e^{-\alpha t})}. \tag{46}$$

The mean anisotropy parameter is given by

$$A_m = \frac{3(m-1)^2}{(m+3)^2} = \text{Constant} \neq 0 \text{, for } m \neq 1. \tag{47}$$

The shear scalar is

$$\sigma^2 = \frac{6\alpha^2 (m-1)^2}{(m+3)^2} \frac{(e^{\alpha t} - e^{-\alpha t})^2}{(e^{\alpha t} + e^{-\alpha t})^2}. \tag{48}$$

It is observed that

$$\lim_{t \to \infty} \frac{\sigma^2}{\theta^2} = \frac{3(m-1)^2}{8(m+3)^2} = \text{Constant} \neq 0 \text{, for } m \neq 1. \tag{49}$$

The mean anisotropy parameter A_m is constant and $\lim_{t \to \infty} \frac{\sigma^2}{\theta^2} \neq 0$ is also constant. Hence the model is anisotropic throughout the evolution of the universe, except at $m = 1$i.e. the model does not approach isotropy.

The matter-energy density is given by

$$\rho = \frac{48\alpha^2 (m+1)(e^{\alpha t} - e^{-\alpha t})^2}{(m+3)^2 (e^{\alpha t} + e^{-\alpha t})^2}, \tag{50}$$

$$\rho + p = \frac{12\alpha^2 (5m-1)(e^{\alpha t} - e^{-\alpha t})^2}{(m+3)^2 (e^{\alpha t} + e^{-\alpha t})^2} - \frac{12\alpha^2}{(m+3)}, \tag{51}$$

$$\rho - p = \frac{36\alpha^2 (e^{\alpha t} - e^{-\alpha t})^2}{(m+3)(e^{\alpha t} + e^{-\alpha t})^2} + \frac{12\alpha^2}{(m+3)}, \tag{52}$$

$$\rho + 4p = \frac{96\alpha^2 (m-1)(e^{\alpha t} - e^{-\alpha t})^2}{(m+3)^2 (e^{\alpha t} + e^{-\alpha t})^2} - \frac{48\alpha^2}{(m+3)}. \tag{53}$$

The cosmological parameter for the closed universe takes the form:

$$\Omega = 1 + \frac{1}{a^2 H^2}.$$

Using (37) and (38), it simplifies to

$$\Omega = 1 + \frac{1}{a_1^2 \alpha^2 (e^{\alpha t} + e^{-\alpha t})^2}. \tag{54}$$

As t increases, after the bounce Ω decreases due to the acceleration of the universe.

Figures (5.2) (a), (b), (c), (d), (e), (f) represents the plots of time versus (a) Average scale factor (b) Hubble parameter (c) Energy density (ρ) (d) $\rho + p$ (e) $\rho - p$ (f) $\rho + 4p$ which are as shown below.

Discussion: Fig. (5.2) (a) is the plot of time versus average scale factor. It is seen that, during contraction phase, the average scale factor $a(t)$ is decreasing (i.e. $\dot{a}(t) < 0$) and then in the expanding phase $a(t)$ is increasing (i.e. $\dot{a}(t) > 0$). Hence, the minimal necessary conditions (i) and (ii) for the bounce at time $t = 0$ are satisfied [38].

(i) $\dot{a}(t) = 0$ at $t = 0$

and (ii) at $t = 0$, $\ddot{a}(t) > 0$ for $t \in (0 - \in, 0) \mathrm{U} (0, 0 + \in)$, where $\in$ is very small.

Fig. (5.2) (b) is the plot of time versus Hubble parameter. At time $t = 0$, $H = 0$ and $a(0) = 2a_1$ with $\dot{H}(t) > 0$ in small neighborhood of $t = 0$. Therefore, the above scale factor satisfies necessary conditions for a non-singular bounce. With the above scale factor, we have $\ddot{a} > 0$ before and after the bounce. [36-37].

To realize the bounce in our model, let we obtain the values of ρ, $\rho + p$, $\rho - p$ and $\rho + 4p$ for $m = 2$.

$$\rho = (5.76)\alpha^2 \frac{(e^{\alpha t} - e^{-\alpha t})^2}{(e^{\alpha t} + e^{-\alpha t})^2},$$

$$\rho + p = (4.32)\alpha^2 \frac{(e^{\alpha t} - e^{-\alpha t})^2}{(e^{\alpha t} + e^{-\alpha t})^2} - (2.5)\alpha^2,$$

$$\rho - p = (7.2)\alpha^2 \frac{(e^{\alpha t} - e^{-\alpha t})^2}{(e^{\alpha t} + e^{-\alpha t})^2} + (2.4)\alpha^2,$$

$$\rho + 4p = (3.84)\alpha^2 \frac{(e^{\alpha t} - e^{-\alpha t})^2}{(e^{\alpha t} + e^{-\alpha t})^2} - (9.6)\alpha^2.$$

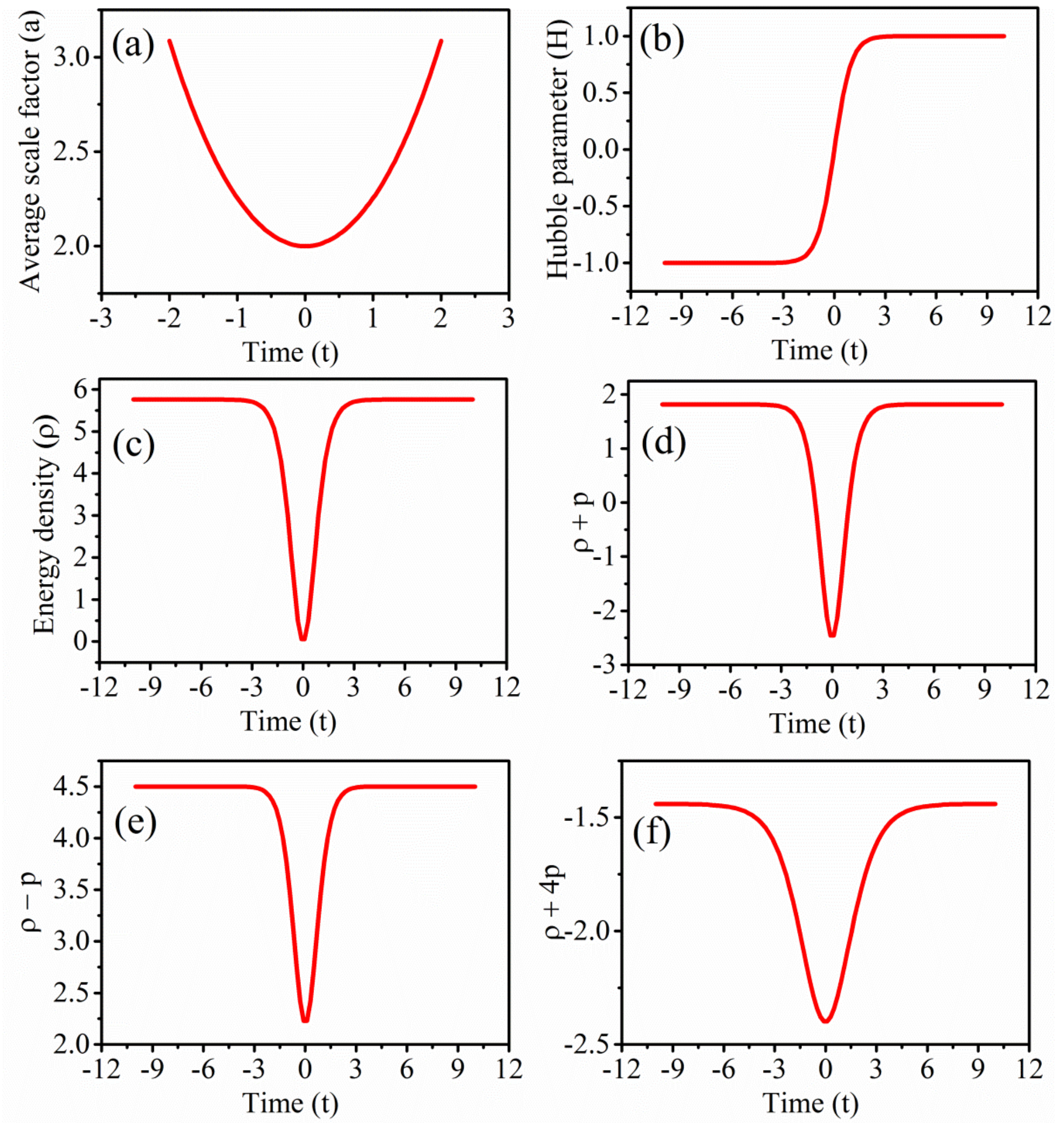

Fig. 5.2 (a), (b), (c), (d), (e), (f) represents the plots of time versus (a) Average scale factor (b) Hubble parameter (c) Energy density (ρ) (d) $\rho + p$ (e) $\rho - p$ (f) $\rho + 4p$ for the values of constants $\alpha = 0.5$ and $a_1 = 1$.

Figs. (5.2) (c), (d), (e) and (f) are the plots of time versus energy density (ρ), $\rho+p$, $\rho-p$ and $\rho+3p$. It is observed that WEC, NEC and DEC energy conditions are satisfied at $t=0$ but SEC is violated at $t=0$. Hence, the bounce is realized in our model at $m=2$.

As an example of viscous fluid realizing bounce scenario, we take $\gamma=-1$ and bulk viscosity is given by

$B(\mathrm{a},\mathrm{H},\dot{\mathrm{H}},......)=4\mathrm{H}\zeta$, ($\zeta$ being coefficient of bulk viscosity), we have

$$p=-\rho+4H\zeta \ , \tag{55}$$

From equation (51), we have

$$4H\zeta=\frac{12(5m-1)H^2}{(m+3)}-\frac{12\alpha^2}{(m+3)}$$

Dividing by $4H$

$$\zeta(\mathrm{H})=\frac{3(5m-1)H}{(m+3)^2}-\frac{3\alpha^2}{(m+3)H} \ . \tag{56}$$

As another example, we take $\gamma(\rho)=\gamma=$(constant) and $B(\mathrm{a},\mathrm{H},\dot{\mathrm{H}},......)=4\mathrm{H}\zeta$ with $\zeta=\in\rho$, where $\in$ is a constant.

From equation (9) , we have

$$\dot{\rho}=-4H(\rho+p) \ .$$

Using equation (3) we get,

$$\dot{\rho}=-4H\left(\gamma(\rho)\rho-\mathrm{B}(\mathrm{a},\mathrm{H},\dot{\mathrm{H}},.....)\right).$$

Since, $\zeta=\in\rho$ and $B(\mathrm{a},\mathrm{H},\dot{\mathrm{H}},......)=4\mathrm{H}\zeta$, we have

$$\rho=\frac{a_1}{(a_0^2+\beta^2t^2)^{\frac{3(1+\gamma-\in)}{2}}} \ , \tag{57}$$

where, ρ_0 is a constant.

Also, from equation (3) we get

$$p=\gamma(\rho)\rho-B(\mathrm{a},\mathrm{H},\dot{\mathrm{H}},......) \ . \tag{58}$$

Using $\gamma(\rho)=\gamma=$ constant and $B(\mathrm{a},\mathrm{H},\dot{\mathrm{H}},......)=4\mathrm{H}\zeta$, we get

$$p=(\gamma-4H\in)\rho \ .$$

Therefore, the scenario with $\gamma=$ constant, $\zeta=\in\rho$, the energy density of bouncing universe will decrease with increasing time (provided $1+\gamma>\in$).

6. Conclusion

Kaluza-Klein cosmological model has investigated with viscous fluid by considering two specific forms of the scale factors proposed by Morris & Thorne [22] and Bamba *et al.* [1]. In both the cases, we have realized the bounce behavior of the model. The bounce behavior has been proposed as an alternatives scenario for the Big Bang. Our investigation has taken into consideration necessity to have an acceleration after the bounce in the context inflation.

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References

[1] Bamba, K., Maharenko, A. N., Myagky, A. N., Nojiri, S., Odintsov, S. D., "Bounce cosmology from f(R) gravity and f(R) bi-gravity", 2013; arXiv:1309.3748v1/hep-th.

[2] Brandenberger, R. H., "Alternatives to cosmological inflation", Int. J. Mod. Phys. Conf. Ser. 01, 67 2011 ; arXiv:0902.4731/hep-th.

[3] Brevik, I., Timoshkin, A., "Inhomogeneous dark fluid and dark matter leading to a bouncing cosmology universe", 1, 24, 2015; arXiv:1503.02916/gr-qc.

[4] Brevik, I., Elizalde, E. Nojiri, S., Odintsov, S. D. , "Viscous little rip cosmology", Phys. Rev. D 84, 103508, 2011; .arXiv. 1107.4642/hep-th.

[5] Brevik, I., Gorbunova, O., "Dark Energy & Viscous cosmology", Gen. Rel. Grav. 37,2039-2045(2005)

[6] Brevik, I., Gron, φ., "Relativistic universe models. in: Recent Advances in cosmology", P.97. Nova sci. pub., New York, 2013, arXiv:1409.8561/gr-qc.

[7] Brevik, I., Obukhov, V. V., Timoshkin. A. V., "Bounce universe induced by an inhomogeneous dark fluid coupled with dark matter", Mod. Phys. Lett. A29, 1450078, 2014.

[8] Brevik. I., Gorbunava, O. Shaido, Y. A., "Viscous FRW Cosmology in Modified Gravity", Int. J. Mod. Phys. D14, 1899-1906, 2005.

[9] Cai, Y. F., Qiu, T., Zhang, X., Piao, Y., Li, M., "Bouncing Universe with Quintom Matter", J. High Energy Phys. 10, 071, 2007.

[10] Cai, Y-F., Chen, S. H., Dent, J. B., Dutta, S., Saridakis, E. N., "Matter bounce cosmology with the f(T) gravity", Class. Quant. Grav. 28, 215011, 2011, arXiv:1104.4349/astro-ph.co.

[11] Caldwell, R. R. Kamionkowski, M.,Weinberg, N. N., "Phantom energy and cosmic doomsday", Phys. Rev. Lett. 91, 071301, 2003, arXiv; astro-ph0302505.

[12] Capozziello, S., Cardone. V. f., Elizalde, E., Nojiri, S., Odinstov, S. D., "Observational Constraints on Dark Energy with Generalized Equation of State", Phys. Rev. D73,043512, 2006.

[13] Cataldo. M. Cntz, N., Lepe, S., "Viscous Dark Energy and Phantom evaluation", Phys. Lett. B619, 5, 2005.

[14] Collins, C. B., Glass, E. N., Wilkinson, D. A., "Exact spatially homogeneous cosmologies", Gen. Rel. Grav., 12 (10), 805-823, 1980.

[15] Elizalde, E., Obukhov, V. V., Timoshkin. A. V., "Inhomogeneous viscous dark fluid coupled with dark matter in the FRW universe", Mod. Phys. Lett. A29,1450132, 2014, arXiv: 1406.7653/ gr-qc.

[16] Frampton, H., Ludwick, K. J., Scherrer, R. J., "The little rip", Phys. Rev. D84, 063003, 2011, arXiv:1106.4996 v1/ astro-ph.co.

[17] Frampton, P. H., Ludwick, K. J., Nojiri S., Odinstov, S. D., Schrrer R. J., "Models for little rip dark energy", Phys. Lett. B. 708, 204, 2012a.

[18] Frampton, P. H., Ludwick, K. J., Schener, R. J., "Pseudo-rip cosmological models intermediate between the cosmological constant and the little rip", Phys. Rev. D85, 083001, 2012b, arXiv. 1112.2964 v2 /astro-ph.co.

[19] Garnavich, P. M. , *et al*., " Constraints on cosmological models from Hubble space telescope observations of high z-supernovae", Astrophys. J. 493, L53 , 1998.

[20] Hinshaw, G. F., et al., "Nine Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results", Astrophys. J. Suppt. Ser.208, 19, 2013.

[21] Jaffe, T. R., Banday, A. J., Eriksen, H. K., Gorski, K. M. Hensen, F. K., "Evidence of vorticity shear at large angular scales in the WMAP data : A Violation of cosmological isotropy?", Astrophys. J. 629.LI-L4, 2005.

[22] Molina-Paris, C., Visser, M., "Minimal conditions for the creation of a Friedman-Robertson-Walker universe from a bounce", Phys. Lett. B455,90, 1999.

[23] Morris, M. S., Thorne, K. S., "Wormholes in space-time and their use for interstellar travel:A tool for teaching general relativity", Am. J. Phys.56,395-412, 1988. DOI:10.1119/1.15620.

[24] Myrzabulov, R., Sebastiani. L., "Bounce solutions in viscous fluid cosmology", 2014b, arXiv:1403.0681v1/gr-qc.

[25] Myrazakul, S., Myrzabulov, R., Sebastiani L., "Inhomogeneous viscous fluids in FRW universe and finite future time singularities", 2013. arXiv:1311.6939 v1/gr-qc.

[26] Nojiri, S., Odinstov S. D., "Final state and thermodynamics of a dark energy universe", Phys. Rev. D70,103522, 2004.

[27] Nojiri, S., Odintsov S.D., " Quatum-de-sitter cosmology and phantom matter", Phys. Lett. B 562, 147, 2003.

[28] Nojiri, S., Odintsov, S. D., Tsujikawa, S., "Properties of singularities in the (phantom) dark energy universe", Phys. Rev. D, 71, 063004, 2005.

[29] Nojiri, S., Odintsov, S. D., "The new form of the equation of state for dark energy fluid and accelerating universe", Phys. Lett. B, 639, 144-150, 2006.

[30] Novello, M., Perez Bergliatta, S. E., "Bouncing cosmology", Phys. Rep. 463,127, 2008.

[31] Peebles, P. J., Ratra, B., "The Cosmological constant and Dark energy", Rev. Mod. Phys. 75, 559-606, 2003. DOI:10.1103/RevModPhys.75.559.

[32] Perlmutter S, Aldering G, Goldhaber G *et al.*, "Measurement X and K from 42 redshift Supernovae", The Astrophys J. 517(2):565–586, 1999. doi:10.1086/307221

[33] Riess A.G., Filippenko A. V., Challis P. *et al.*, "Observational evidence from supernovae for an accelerating Universe and a cosmological constant", Astron J. 116:1009–1038, 1998.

[34] Riess A. G., Sirolger L. G., Tonry J. *et al.* , " Type-Ia Supernovae discoveries at z[1 from the hule space telescope: evidence for past deceleration and constraints on dark energy evolution", The Astrophys J. 607:665–687, 2004. doi:10.1086/383612

[35] Spergel D. N., Verde L., Peiris H.V., Komatsu E. *et al.* , "First year Wilkinson microwave probe (WMAP) observations: determination of cosmological parameters", Astrophys J. Suppl. 148:175–194, 2003.

[36] Singh, T., Chaubey, R., Singh, A., “Bounce conditions for FRW models in modified gravity theories”, Eur. Phys. J. Plus, 130(2), p. 31, 2015a.

[37] Singh, T., Chaubey, R., Singh, A., “Bounce Conditions in Kantowski-Sachs and Bianchi Cosmologies in Modified Gravity Theories”, Int. J. Mod. Phy. A, 30, 1550073, 2015b.

[38] Singh, T. Chaubey, R., Singh, A., “Bouncing cosmologies with viscous fluids”, Astro. Spa. Sci., 361, 106, 2016.

[39] Wei, H., Wang, L. F., Guo, X. J., “Quasi-rip: a new type of rip model without cosmic dooms days”, Rev. D86, 083003, 2012, arXiv. 1207.2898 v1/gr-qc.