Article

Holographic Dark Energy Model with Generalized Chaplygin Gas in Higher Dimensions

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Abstract

In this paper, we have studied a five dimensional FRW space-time filled with two minimally interacting fluids: matter and holographic dark energy in Saez-Ballester (1986) scalar tensor theory of gravitation. We have established a correspondence between the holographic dark energy models with the generalised Chaplygin gas. We also reconstructed the potential and dynamics of the scalar field which describes the Chaplygin cosmology. Also, we have discussed physical and geometrical properties of the model.

Keywords: FRW cosmology, Saez-Ballester theory, holographic dark energy, Chaplygin gas.

1. Introduction

Astronomical and cosmological observations, such as type Ia supernovae (SNe Ia) Perlmutter, Aldering, Della Valle 1998; Tegmark, Strauss, Blanton et al (2004).the cosmic microwave background (CMB) Miller, Caldwell, Devlin et al (1999), Bennet, Halpern, and Hinshaw et al., (2003), and Wilkinson Microwave Anisotropy Probe (WMAP) Spergel, Verde, and Peiris et al.(2003), indicate that the observable universe experiences an accelerated expansion. These observations suggest also that the universe is nearly flat and dominated by a non baryonic substratum. The source of this acceleration is usually attributed to an exotic type of fluid with negative pressure called commonly dark energy.

Various kinds of dark energy models have been proposed such as cosmological constant Peebles and Ratra(2003), quintessence Ratra and Peebles (1988), Caldwell, Dave, and Steinhardt (1998), Sami and Padmanabhan (2003), *k*-essence Armendariz-Picon and Mukhanov (2001), Chiba (2002), Scherrer (2004), Gibbons(2002), phantom Caldwell (2002), Elizalde (2004), Cline (2004), Chaplygin gas Kamenshchik (2001), quintom Feng (2006), holographic dark energy Hořava and Minic (2000), and extra dimensions Rogatko (2004)The nature of the dark sector of the universe (i.e., dark energy and dark matter) remains a mystery. An economical and attractive idea to unify the dark sector of the universe is to consider it as a single component that acts as

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both dark energy and dark matter. One way to achieve the unification of dark energy and dark matter is by using the so-called Chaplygin gas. The pure Chaplygin gas or generalized Chaplygin gas is a perfect fluid which behaves like a pressure less fluid at an early stage and a cosmological constant at a later stage. Among the different theories put forwarded in the literature in recent times, the single component fluid known as Chaplygin gas with an equation of state (EOS) $p = -B/\rho$ where ρ and p are the energy density and pressure respectively and B is a constant has attracted large interest in cosmology.

The above equation of state, however, has been conceived in studies of adiabatic fields. It was used to describe lifting forces on a plane wing in aero dynamics process. In cosmology, although it admits an accelerating universe, fails to address structure formation and cosmological perturbation power spectrum Gorini, Kamenshchik, Moschella and Pasquier (2004), Kamenshchik, Moschella and Pasquier (2001). Subsequently, a modified form of the equation of state $p = -B/\rho^{\alpha}$ with $0 \leq \alpha \leq 1$ was also considered to construct available cosmological model [Gorini, Kamenshchik and Moschella (2003), Alam, Sahni, Saini and Starobinsky (2003), Bento, Bertolami and Sen (2002), . Sahni, Saini, Starobinsky and Alam, (2003), which is known as generalized Chaplygin gas (GCG). It has two free parameters B and α respectively. The fluid behaves initially like dust for small size of the universe, but at a later epoch the fluid may be described by an equation of state $p = -\omega\rho$.

Recently holographic principle is incorporated in cosmology (Hsu 2004; Li 2004) to track the dark energy content of the universe. This principle was first put forward by G't Hooft (2009) in the context of black hole physics. According to the holographic principle, the entropy of a system scales not with its volume, but with its surface area. In the cosmological context, Fischler and Susskind (1998) have proposed a new version of the holographic principle, viz. at any time during cosmological evolution, the gravitational entropy within a closed surface should not be always larger than the particle entropy that passes through the past light-cone of that surface. In the context of the dark energy problem, though the holographic principle proposes a relation between the holographic dark energy density ρ_{Λ} and the Hubble parameter H as $\rho_{\Lambda} = H^2$, it does not contribute to the present accelerated expansion of the universe. Li (2004) has obtained an accelerating universe considering event horizon as the cosmological scale. The model is consistent with the cosmological observations. Granda and Oliveros (2008) proposed a holographic density of the form $\rho_{\Lambda} = \alpha H^2 + \beta\dot{H}$ where H is the Hubble parameter and α, β are constants which must satisfy the restrictions imposed by the current observational data. They showed that this new model of dark energy represents the accelerated expansion of the universe and is consistent with the current observational data.

The study of higher dimensional cosmology space-time is because of the underlying that the cosmos at its early stage of evolution of the universe might have had a higher dimensional era. The dimensionality of the world has long been a subject of discussion due to fact that our sense perceived only four dimensions, but there is nothing in the equation of relativity which restricts them to four dimensions. Witten (1984), Applequist et al. (1987), Chudos and Detweiler (1980) and Marciano (1984) are some of the authors who have initiated the discussion of higher dimensional cosmological models. A number of authors (Sahdev 1984; Chatterjee 1993; Emelyanov et al.1986) have studied physics of the universe in higher-dimensional space-time. These models are believed to be physical relevance possibly at the early times before the universe has undergone compactification transitions. Rao et al. (2012) have investigated Bianchi type-II, VIII and IX dark energy cosmological model in Saez - Ballester theory of gravitation, Rao et al. (2014) have obtained Five dimensional bulk viscous cosmological model with wet dark fluid in Saez-Ballester theory of gravitation, Recently Rao et al. (2015) have obtained Higher dimensional FRW holographic dark energy cosmological models in Brans-Dicke theory of gravitation.

Saez and Ballester (1986) formulated a scalar-tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field an antigravity regime appears. This theory also suggests a possible way to solve missing matter problem in non flat FRW cosmologies. The field equations given by Saez- Ballester (1986) for the combined scalar and tensor fields (using geometrized units with $c = 1$, $8\pi G = 1$) are

$$G_{ij} - \omega\phi^{n}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \tag{1.1}$$

and the scalar field ϕ satisfies the equation

$$2\phi^{n}\phi^{i}_{\ ,i} + r\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{1.2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}R\,g_{ij}$ is an Einstein tensor, R the scalar curvature, ω and n are constants, T_{ij} is the stress energy tensor of the matter.

The energy conservation equation is

$$T^{ij}_{\ \ ,j} = 0 \tag{1.3}$$

There has been a number of works done concerning their construction of the holographic scalar field models of dark energy. Setare studied the correspondence between the holographic dark energy and each one of tachyon (Setare 2007b), phantom (Setare 2007c), Chaplygin gas and generalised Chaplygin gas (Setare 2007e) in FRW universe. On the other hand, Setare (2007f) has shown a correspondence between the interacting generalised Chaplygin gas and phantom dark energy model in non-flat FRW universe. Setare and Vanegas (2009) have pointed out the cosmological dynamics of interacting holographic dark energy model.

Motivated by the above investigations, in this paper we have considered that the universe is filled with the normal matter and variable modified Chaplygin gas. Also we have considered the interaction between normal matter and variable modified Chaplygin gas in FRW universe with Saez-Ballester theory of gravitation. Then we have considered a correspondence between the holographic dark energy with generalized Chaplygin gas model.

2. Metric and Energy Momentum Tensor

We consider spatially homogeneous five dimensional FRW metric in the form

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right] + A^2(t)d\mu^2 \tag{2.1}$$

where $R(t)$ is the scale factor and $k = 0, -1\ or\ +1$ is the curvature parameter for flat, open and closed Universe, respectively. The fifth coordinate μ is also assumed to be space like coordinate.

The energy momentum tensors for matter and the holographic energy are defined as

$$T_{ij} = \rho_m u_i u_j \tag{2.2}$$

and

$$\overline{T}_{ij} = (\rho_\Lambda + p_\Lambda)u_i u_j - g_{ij}\rho_\Lambda \tag{2.3}$$

where ρ_m, ρ_Λ are energy densities of matter and holographic dark energy and p_Λ is the pressure of holographic dark energy.

In a co- moving coordinate system, we get

$$T_1^{\ 1} = T_2^{\ 2} = T_3^{\ 3} = 0,\ T_4^{\ 4} = \rho_m, T_5^{\ 5} = 0 \text{ and}$$

$$\overline{T}_1^{\ 1} = \overline{T}_2^{\ 2} = \overline{T}_3^{\ 3} = -p_\Lambda,\ \overline{T}_4^{\ 4} = \rho_\Lambda, \overline{T}_5^{\ 5} = p_\Lambda \tag{2.4}$$

where the quantities ρ_m, ρ_Λ and p_Λ are functions of 't' only.

3. Solutions of the Field equations

Now with the help of (2.2) to (2.4), the field equations (1.1) for the metric (2.1) can be written as

$$\frac{\ddot{A}}{A}+\frac{2\ddot{R}}{R}+\frac{2\dot{R}\dot{A}}{RA}+\frac{\dot{R}^2}{R^2}+\frac{k}{R^2}-\frac{1}{2}\omega\phi^n\dot{\phi}^2=-p_\Lambda \tag{3.1}$$

$$\frac{3\ddot{R}}{R}+\frac{3\dot{R}^2}{R^2}+\frac{3k}{R^2}-\frac{1}{2}\omega\phi^n\dot{\phi}^2=-p_\Lambda \tag{3.2}$$

$$\frac{3\dot{R}\dot{A}}{RA}+\frac{3\dot{R}^2}{R^2}+\frac{3k}{R^2}+\frac{1}{2}\omega\phi^n\dot{\phi}^2=\rho_m+\rho_\Lambda \tag{3.3}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{3\dot{R}}{R}+\frac{\dot{A}}{A}\right)+\frac{n}{2}\left(\frac{\dot{\phi}^2}{\phi^2}\right)=0 \tag{3.4}$$

$$\dot{\rho}_m+\dot{\rho}_\Lambda+\left[3\frac{\dot{R}}{R}+\frac{\dot{A}}{A}\right](\rho_m+\rho_\Lambda+p_\Lambda)=0 \tag{3.5}$$

Here the over head dot denotes differentiation with respect to't'.

The field equations (3.1) to (3.4) are only four independent equations with six unknowns $R, A, \rho_m, \rho_\Lambda, p_\Lambda$ & ϕ, which are functions of't'. Since these equations are non-linear in nature, in order to get a deterministic solution we take the following plausible physical condition:

The shear scalar σ is proportional to scalar expansion θ, so that we can take a linear relationship between the metric potentials R and S, i.e.,

$$A=R^m \quad , \tag{3.6}$$

where m is an arbitrary constant.

From equations (3.1), (3.2) & (3.6), we get

$$\frac{\ddot{R}}{R}+(m+2)\frac{\dot{R}^2}{R^2}=\frac{2k}{(m-1)R^2} \tag{3.7}$$

The continuity equation can be obtained as

$$\dot{\rho}_m+\dot{\rho}_\Lambda+\left(\frac{3\dot{R}}{R}+\frac{\dot{A}}{A}\right)(\rho_m+\rho_\Lambda+p_\Lambda)=0 \tag{3.8}$$

The continuity equation of the matter is

$$\dot{\rho}_m + \left(\frac{3\dot{R}}{R} + \frac{\dot{A}}{A}\right)\rho_m = 0 \tag{3.9}$$

The continuity equation of the holographic dark energy is

$$\dot{\rho}_\Lambda + \left(\frac{3\dot{R}}{R} + \frac{\dot{A}}{A}\right)(\rho_\Lambda + p_\Lambda) = 0 \tag{3.10}$$

The barotropic equation of state

$$p_\Lambda = \omega_\Lambda \rho_\Lambda \tag{3.11}$$

From equations (3.7), we get

$$R = (k_1 t + k_2) \ , \tag{3.12}$$

where $k_1 = \left[\frac{2k}{(m-1)(m+2)}\right]^{\frac{1}{2}}$ and k_2 is an integrating constant.

From equations (3.6) & (3.12), we get

$$A = (k_1 t + k_2)^m \tag{3.13}$$

The holographic dark energy density are given by

$$\rho_\Lambda = \frac{2}{\alpha - \beta}\left(H' + \frac{3\alpha}{2}H^2\right) \ , \tag{3.14}$$

where H is the Hubble parameter, α and β are constants which must satisfy the restrictions imposed by the current observational data.

From equations (3.12)-(3.15), we get the holographic dark energy density

$$8\pi\rho_\Lambda = \frac{\pi\, k\,(m+3)[3\alpha(m+3)-8]}{(\alpha-\beta)(m-1)(m+2)(k_1 t + k_2)^2} \tag{3.15}$$

From equations (3.3), (3.12), (3.13), & (3.15), we get the matter energy density

$$8\pi\rho_m = \frac{k\,(m+3)[24m(\alpha-\beta) - 3\alpha(m+3)+8]}{8(\alpha-\beta)(m-1)(m+2)(k_1 t + k_2)^2} + \frac{1}{2}\omega\, l \tag{3.16}$$

From equations (3.2), (3.12) & (3.13), we get the pressure of holographic dark energy

$$8\pi\, p_\Lambda = \frac{1}{2}\omega\, l - \frac{3\,k\,[(m-1)(m+2)-2]}{(m-1)(m+2)(k_1 t + k_2)^2} \tag{3.17}$$

From equations (3.11), (3.15) & (3.17), we get

$$w_\Lambda = \frac{p_\Lambda}{\rho\Lambda} = \frac{\dfrac{\pi\, k\,(m+3)[3\alpha(m+3)-8]}{(\alpha-\beta)(m-1)(m+2)(k_1 t+k_2)^2}}{\dfrac{1}{2}\omega\, l - \dfrac{3\,k\,[(m-1)(m+2)-2]}{(m-1)(m+2)(k_1 t+k_2)^2}} \tag{3.18}$$

The coincident parameter is

$$r = \frac{\rho_\Lambda}{\rho_m} = \frac{\dfrac{\pi\, k\,(m+3)[3\alpha(m+3)-8]}{(\alpha-\beta)(m-1)(m+2)(k_1 t+k_2)^2}}{\dfrac{k\,(m+3)[24m(\alpha-\beta)-3\alpha(m+3)+8]}{8(\alpha-\beta)(m-1)(m+2)(k_1 t+k_2)^2}+\dfrac{1}{2}\omega l} \tag{3.19}$$

Correspondence between the Holographic and Generalised Chaplygin gas model of dark energy:

To establish the correspondence between the holographic dark energy with Generalised Chaplygin gas dark energy model, we compare the EOS and the dark energy density for the corresponding models of dark energy. The pressure and the density of the Generalised Chaplygin gas is given by

$$p_{ch} = -\frac{A}{\rho_{ch}^{l}} \tag{3.21}$$

$$\rho_{ch} = -\frac{[B+Aa^{3(1+l)}]^{\frac{1}{1+l}}}{a^3} = \left[A+\frac{B}{a^{3(1+l)}}\right]^{\frac{1}{1+l}} \tag{3.22}$$

where a is the average scale factor of the universe and A, B, l are positive constants with $0 < l \leq 1$.

Now following Setare (2007d) we assume that the origin of the dark energy is a scalar field φ, so

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \left[A+\frac{B}{a^{4(1+l)}}\right]^{\frac{1}{1+l}} \tag{3.23}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) = \frac{-A}{\left[A+\dfrac{B}{a^{4(1+l)}}\right]^{\frac{l}{1+l}}} \tag{3.24}$$

$$w_{ch} = \frac{p_{ch}}{\rho_{ch}} = \frac{-A}{\rho_{ch}^{1+l}} = \frac{-A}{A+\dfrac{B}{a^{4(1+l)}}} \tag{3.25}$$

Now adding (3.23) and (3.24), we get

$$\dot{\phi}^2 = \left[A + \frac{B}{a^{4(1+l)}}\right]^{\frac{1}{1+l}} - \frac{A}{\left[A + \frac{B}{a^{4(1+l)}}\right]^{\frac{l}{1+l}}} \tag{3.26}$$

Again subtracting (3.24) from (3.23), we get

$$V(\phi) = \frac{1}{2}\left[A + \frac{B}{a^{4(1+l)}}\right]^{\frac{1}{1+l}} + \frac{A}{2\left[A + \frac{B}{a^{4(1+l)}}\right]^{\frac{l}{1+l}}} \tag{3.27}$$

Now we assume that the holographic dark energy density is equivalent to the Generalised Chaplygin gas energy density. Therefore using equations (3.15) and (3.23), we get

$$B = a^{4(1+l)}\left\{\left[\frac{\pi k(m+3)[3\alpha(m+3)-8]}{(\alpha-\beta)(m-1)(m+2)(k_1 t+k_2)^2}\right]^{1+l} - A\right\} \tag{3.28}$$

From equations (3.18) and (3.25), we get

$$w = \frac{p_\Lambda}{\rho\Lambda} = \frac{\dfrac{\pi k(m+3)[3\alpha(m+3)-8]}{(\alpha-\beta)(m-1)(m+2)(k_1 t+k_2)^2}}{\dfrac{1}{2}\omega l - \dfrac{3k[(m-1)(m+2)-2]}{(m-1)(m+2)(k_1 t+k_2)^2}} = \frac{-A}{\rho_{ch}^{1+l}} = \frac{-A}{A + \dfrac{B}{a^{4(1+l)}}} \tag{3.29}$$

From equation (3.29), we get

$$A = \left[\frac{\pi k(m+3)[3\alpha(m+3)-8]}{(\alpha-\beta)(m-1)(m+2)(k_1 t+k_2)^2}\right]^{2+l}\left[\frac{3k[(m-1)(m+2)-2]}{(m-1)(m+2)(k_1 t+k_2)^2} - \frac{1}{2}\omega l\right]^{-1} \tag{3.30}$$

Using equation (3.30) in (3.28), we get

$$B = a^{4(1+l)}\left[\frac{\pi k(m+3)[3\alpha(m+3)-8]}{(\alpha-\beta)(m-1)(m+2)(k_1 t+k_2)^2}\right]^{1+l}\left\{1 - \left[\frac{\dfrac{\pi k(m+3)[3\alpha(m+3)-8]}{(\alpha-\beta)(m-1)(m+2)(k_1 t+k_2)^2}}{\dfrac{3k[(m-1)(m+2)-2]}{(m-1)(m+2)(k_1 t+k_2)^2} - \dfrac{1}{2}\omega l}\right]\right\} \tag{3.31}$$

Using the values of A and B in equations (3.26) and (3.27), we get the potential and dynamics of the scalar field as

$$\phi=\int\left\{\frac{\pi k(m+3)[3\alpha(m+3)-8]}{(\alpha-\beta)(m-1)(m+2)(k_1t+k_2)^2}\left[1-\left[\frac{\pi k(m+3)[3\alpha(m+3)-8]}{(\alpha-\beta)(m-1)(m+2)(k_1t+k_2)^2}\right]^l\left[\frac{3k[(m-1)(m+2)-2]}{(m-1)(m+2)(k_1t+k_2)^2}-\frac{1}{2}\omega l\right]^{-1}\right]\right\}^{\frac{1}{2}}dt \qquad (3.32)$$

$$V(\phi)=\frac{1}{2}\left[\frac{\pi k(m+3)[3\alpha(m+3)-8]}{(\alpha-\beta)(m-1)(m+2)(k_1t+k_2)^2}\right]+\frac{1}{2}\left[\frac{\left(\frac{\pi k(m+3)[3\alpha(m+3)-8]}{(\alpha-\beta)(m-1)(m+2)(k_1t+k_2)^2}\right)^{l+1}}{\frac{3k[(m-1)(m+2)-2]}{(m-1)(m+2)(k_1t+k_2)^2}-\frac{1}{2}\omega l}\right] \qquad (3.33)$$

The metric (2.1), in this case, can be written as

$$ds^2=-dt^2+(k_1t+k_2)^2\left[\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2\sin^2\theta d\phi^2\right]+(k_1t+k_2)^{2m}d\mu^2 \qquad (3.34)$$

Thus (3.34) together with (3.15) - (3.19) & (3.28) – (3.33) constitutes a five dimensional FRW Holographic dark energy model with generalized Chaplygin gas in Saez-Ballester (1986) theory of gravitation.

4. Some other important properties of the models

The spatial volume for the model is

$$V=(-g)^{\frac{1}{2}}=(k_1t+k_2)^{(m+3)} \qquad (4.1)$$

The average scale factor for the model is

$$a(t)=V^{\frac{1}{4}}=(k_1t+k_2)^{\frac{(m+3)}{4}} \qquad (4.2)$$

The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta=u^i{}_{,i}=(m+3)\frac{k_1}{(k_1t+k_2)} \qquad (4.3)$$

and the shear scalar σ is given by

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{7}{18}(m+3)^2 \frac{k_1^2}{(k_1 t + k_2)^2} \tag{4.4}$$

The deceleration parameter q is given by

$$q = (-3\theta^{-2})(\theta_{,i}u^i + \frac{1}{3}\theta^2) = \frac{-m}{m+3} \tag{4.5}$$

The deceleration parameter appears with negative sign implies accelerating expansion of the universe, which is consistent with the present day observations.

The Hubble's parameter H is given by

$$H = \frac{(m+3)}{4}\frac{k_1}{(k_1 t + k_2)} \tag{4.6}$$

The mean anisotropy parameter $A_m$is given by

$$A_m = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 = \frac{10m^2 - 36m + 90}{4(m+3)^2}, \quad \text{where } \Delta H_i = H_i - H \quad (i = 1,2,3,4) \tag{4.7}$$

The jerk parameter is given by

$$j = \frac{m^2 - 3m}{(m+3)^2} \tag{4.8}$$

Look-back time-red shift: The look-back time, $\Delta t = t_0 - t(z)$ is the difference between the age of the universe at present time (z=0) and the age of the universe when a particular light ray at red shift z, the expansion scalar of the universe $a(t_z)$ is related to a_0 by $1 + z = \frac{a_0}{a}$, where $a_0$is the present scale factor. Therefore from (4.2), we get

$$1 + z = \frac{a_0}{a} = \left(\frac{k_1 t_0 + k_2}{k_1 t + k_2}\right)^{(m+3)/4} \tag{4.9}$$

This equation can also be expressed as

$$H_0 \Delta t = \frac{(m+3)}{4}\left[1 - (1+z)^{4/(m+3)}\right] \tag{4.10}$$

where H_0 is the Hubble's constant.

Luminosity distance:

Luminosity distance is defined as the distance which will preserve the validity of the inverse law for the fall of intensity and, is given by

$$d_L = r_1 (1+z) a_0 \tag{4.11}$$

where r_1 is the radial coordinate distance of the object at light emission and is given by

$$r_1 = \int_T^{T_0} \frac{1}{a} dT = \frac{4(2-n)}{5(1-n)} [(2-n)(k_3 T_0 + k_4)]^{\frac{5(1-n)}{4(2-n)}} \left[1 - (1+z)^{5(n-1)/(3+n)} \right] \tag{4.12}$$

From equations (4.9) - (4.12), we get the luminosity distance

$$d_L = \frac{4(2-n)}{5(1-n) k_3} a_0 (1+z) [(2-n)(k_3 T_0 + k_4)]^{\frac{5(1-n)}{4(2-n)}} \left[1 - (1+z)^{5(n-1)/(3+n)} \right] \tag{4.13}$$

The distance modulus (D) is given by

$$D(z) = 5 \log d_L + 25 \tag{4.14}$$

where d_L stands for the luminosity distance.

From equations (4.13) and (4.14), we get the distance modulus

$$D(z) = 5 \log \left\{ \frac{4(2-n)}{5(1-n) k_3} a_0 (1+z) [(2-n)(k_3 T_0 + k_4)]^{\frac{5(1-n)}{4(2-n)}} \left[1 - (1+z)^{5(n-1)/(3+n)} \right] \right\} + 25 \tag{4.15}$$

The tensor of rotation $w_{ij} = u_{i,j} - u_{j,i}$ is identically zero and hence this universe is non-rotational.

5. Discussion and Conclusions

In this paper we have presented spatially homogeneous anisotropic five dimensional FRW Holographic dark energy model with generalized Chaplygin gas in a scalar tensor theory of gravitation proposed by Saez and Ballester (1986).

The following are the observations and conclusions:

- The model (3.34) has singularity at $t = \frac{-k_2}{k_1}$ for $m < 0$.
- The spatial volume increases with the increase of time $'t'$.
- At $t = \frac{-k_2}{k_1}$, the expansion scalar θ, shear scalar σ and the Hubble parameter H decreases with the increase of time.
- From (4.7), we can observe that $A_m \neq 0$ and this indicates that this universe is anisotropic.
- The matter energy density, the holographic dark energy density and the pressure of holographic dark energy are decreases with the increase of time' t'.
- The deceleration parameter appears with negative sign implies accelerating expansion of the universe, which is consistent with the present day observations.
- We have obtained expressions for look-back time ΔT, distance modulus $D(z)$ and luminosity distance d_L versus red shift and discussed their significance.
- We have also reconstructed the potentials and the dynamics of the scalar field for this anisotropic accelerating model of the universe.
- All the models presented here are anisotropic, non-rotating, expanding and also accelerating. Hence they represent not only the early stage of evolution but also the present universe.

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