Article

A Cosmological Model with Varying G & Λ

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Abstract

In this paper we study the Einstein's field equations with variable gravitational and cosmological "constants" for a spatially homogeneous and anisotropic Bianchi type-I space time. To study the transit behaviour of universe, we consider a law of variation of scale factor which yields a constant value of deceleration parameter comprising a class of models that depicts a transition of the universe from the early decelerated phase to the recent accelerating phase. Cosmological models admitting binary mixture of perfect fluid and dark energy. The physical significance of the cosmological models are also discussed.

Keywords: Anisotropic universe, dark energy, perfect fluid, variable cosmological term, chaplygin gas.

1. Introduction

The gravitational constant G and cosmological constant Λ are considered to be fundamental constants in Einstein's theory of gravity. The Newtonian constant of gravitation G plays an important role between geometry of space and matter in Einstein's field equations. The investigation of relativistic cosmological models usually has the energy momentum tensor of matter generated by a perfect fluid. High precision data from type Ia supernova, the cosmic microwave background and large scale structure which seems to hint the universe is presently dominated by an unknown form of energy, that is known as dark energy [1-7]. One of the obvious contender for the role of dark energy is Einstein's cosmological constant; whereas particle physics have failed to predict the correct density. Astrophysical observations indicate that the accelerated expansion of the universe is driven by exotic energy with large negative pressure which is known as dark energy [8].

Another candidate for the dark energy density has been used by Kamenshchik [9] purposed the use of perfect fluid with an exotic equation of state called as Chaplygin gas. Earlier researchers

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on this topic are Saha [10] , Setare [11] , Setare[12] , Malekjani[13] , Jamil , M.[14] , Sheykhi , A.[15] ,Setare [16] .Some of the recent discussions on the cosmological constant problems have been studied by Dolgov [17,18] , Tsagas and Maartens [19] , Sahani and Starobinsky [20] , Padmnabham [21,22] , Vishwakarma [23-26] and Pradhan et al [27-31]. Solving Einstein equations for a BI universe in the presence of dust, stiff matter and cosmological constant are reported in [32, 33]. A variety of cosmological models with varying Λ term have been tested by researchers to overcome these cosmological puzzles and to explore the accelerating behavior of the universe R.K. Tiwari et.al [34-39]. Among the anisotropic Bianchi Models, Bianchi type-I cosmological models are the simplest anisotropic universe models which are the generalization of FRW models.

In this paper we have investigated BI cosmological model with variable gravitational and cosmological constant in the presence of perfect fluid in an expanding universe. The Einstein field equation is solved in the homogeneous and anisotropic space time.

2. Solution of the field equation

The BI cosmological model is given as

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2, \tag{1}$$

where, A, B, C, are only functions of time, t.

The Einstein's field equations with G and Λ is

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi GT_{ij} + \Lambda g_{ij} \tag{2}$$

where R_{ij} is the Ricci tensor and R is the scalar curvature.

The distribution of matter in the space-time consists of a perfect fluid given by the energy momentum tensor

$$T_{ij} = (\rho + p)v_i v_j + pg_{ij} \tag{3}$$

where v_i is the four-velocity vector satisfying

$$v_i v^i = -1 \tag{4}$$

Here ρ is the total energy of perfect fluid and dark energy and p is the corresponding pressure. The Einstein field equations from the BI space time as

$$8\pi Gp - \Lambda = -\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC}, \tag{5}$$

$$8\pi Gp - \Lambda = -\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC}, \tag{6}$$

$$8\pi Gp - \Lambda = -\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} - \frac{\dot{A}\dot{B}}{AB}, \tag{7}$$

$$8\pi G\rho + \Lambda = \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{C}\dot{B}}{CB}, \tag{8}$$

where over-dot means differentiation with respect to t. We can obtain the Hubble parameter as

$$H = \frac{1}{3}\theta = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{\dot{a}}{a} \tag{9}$$

where $a = (ABC)^{1/3}$ and a is the scale factor and $\theta = u^{j}_{;j}$ is the scalar expansion. The deceleration parameter (q) is related to H as

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -1 - \frac{\dot{H}}{H^2} \tag{10}$$

The deceleration parameter is negative for an accelerating and positive for decelerating phase of universe. It is defined the shear scalar σ_{ij} as [35]

$$\sigma_{ij} = \nu_{i,j} + \frac{1}{2}\left(\nu_{i;k}\nu^{k}\nu_{j} + \nu_{j;k}\nu^{k}\nu_{i}\right) + \frac{1}{3}\theta\left(g_{ij} + \nu_{i}\nu_{j}\right), \tag{11}$$

By using equations (1) and (11), we obtain

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{1}{6}\theta^2 \tag{12}$$

Using equations (5) - (8), we find

$$3H^2 = 8\pi G\rho + \sigma^2 + \Lambda \tag{13}$$

$$H^2(2q-1) = 8\pi Gp + \sigma^2 - \Lambda \tag{14}$$

Eliminating p and Λ from equations (5) - (7), we find

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{C}}{C}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0 \tag{15}$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0 \tag{16}$$

and by integrating equations (15) and (16), we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{l_1}{ABC} \tag{17}$$

and
$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{l_2}{ABC} \tag{18}$$

where l_1 and l_2 are the integration constants.

On account of equations (17), (18) and (12), σ^2 is derived as

$$\sigma^2 = \frac{l_1^2 + l_2^2 + l_1 l_2}{3a^6} = \frac{b^2}{3a^6} \tag{19}$$

where $b^2 = l_1^2 + l_2^2 + l_1 l_2$. Equation (19) implies that

$$\dot{\sigma} + 3H\sigma = 0. \tag{20}$$

From equation (13), we obtain

$$\frac{\sigma^2}{3H^2} = 1 - \frac{8\pi G\rho}{3H^2} - \frac{\Lambda}{3H^2} = 1 - \Omega - \Omega_\lambda \tag{21}$$

where $\Omega = \rho/\rho_c$ and $\Omega_\lambda = \rho_\Lambda/\rho_c$ are the total density parameter and cosmological constant density parameter. Here $\rho_c = 3H^2/8\pi G$ and $\rho_\Lambda = \Lambda/8\pi G$ are the critical density and cosmological constant density. For p=0, equation (14) gives

$$\frac{\Lambda}{H^2} = 1 - 2q + \frac{\sigma^2}{H^2} \tag{22}$$

Putting equation (22) into equation (21) gives

$$\Omega = \frac{2}{3}\left(q + 1 - \frac{\sigma^2}{H^2}\right) \tag{23}$$

If the space time is isotropic, $\sigma = 0$, we observe that in an anisotropic space time value of total density parameter Ω is smaller in comparison to its value in the isotropic space. The deceleration parameter is constant

i.e., q=k (24)

where k is a constant.

Let us consider a universe with power law

$$a = \left[k_1(k+1)t + k_2\right]^{1/k+1}$$

Put, $\left[k_1(k+1)t + k_2\right] = T$, we get

$$a = T^{1/k+1} \tag{25}$$

Hubble parameter H and shear scalar σ , for this model are given by

$$H = \frac{\dot{a}}{a} = \frac{1}{(k+1)T} \tag{26}$$

$$\sigma^2 = \frac{b^2}{3(T)^{6/k+1}} \tag{27}$$

we see at the beginning of the universe σ^2 is very large which indicates that the universe is very anisotropic at the early time and when time is growing the shear tensor tends to zero which shows that the anisotropic parameter is disappeared and the universe is isotropic at late time.

3. General non Linear Equation of State

The equation of state is defined as

$$p = \omega\rho - A\rho^{\eta} \tag{28}$$

where A>0, $\omega \neq -1$ and η are the constants. The conventional energy conservation separated equation (3) into

$$\dot{\rho} + 3\frac{\dot{a}}{a}\rho\left(1 + \omega - A\rho^{\eta-1}\right) = 0 \tag{29}$$

Inserting equation (25) into equation (29), we get

$$\rho = \left(\frac{1+\omega}{A + C(1+\omega)T^{3(1+\omega)(\eta-1)/k+1}}\right)^{\frac{1}{\eta-1}} \tag{30}$$

Substituting equation (25) - (27) and (28), (30), (31) into equation (13) and (14) the gravitational parameter G and cosmological constant Λ are given by

$$G = \frac{\dfrac{1}{(k+1)T^2} - \dfrac{b^2}{3T^{6/k+1}}}{4\pi\left[(1+\omega)\left\{\dfrac{1+\omega}{A + C(1+\omega)T^{3(1+\omega)(\eta-1)/k+1}}\right\}^{1/\eta-1} - A\left\{\dfrac{1+\omega}{A + C(1+\omega)T^{3(1+\omega)(\eta-1)/k+1}}\right\}^{\eta/\eta-1}\right]} \tag{31}$$

$$\Lambda = \frac{3}{(k+1)^2T^2} - \frac{b^2}{3T^{6/k+1}} - \frac{2\left\{\dfrac{1}{(k+1)T^2} - \dfrac{b^2}{3T^{6/k+1}}\right\}}{\left\{(1+\omega) - A\left(\dfrac{1+\omega}{A + C(1+\omega)T^{3(1+\omega)(\eta-1)/k+1}}\right)\right\}^{\eta}} \tag{32}$$

and

$$\rho_c = \frac{\frac{3}{(k+1)^2T^2}\left[(1+\omega)\left\{\frac{1+\omega}{A+C(1+\omega)T^{3(1+\omega)(\eta-1)/k+1}}\right\}^{1/\eta-1} - A\left\{\frac{1+\omega}{A+C(1+\omega)T^{3(1+\omega)(\eta-1)/k+1}}\right\}^{\eta/\eta-1}\right]}{\frac{2}{(k+1)T^2} - \frac{2}{3}\frac{b^2}{T^{6/k+1}}} \tag{33}$$

and then we can obtain

$$\Omega = \frac{\frac{2}{(k+1)T^2} - \frac{2b^2}{3T^{6/k+1}}}{\frac{3}{(k+1)^2T^2}\left[(1+\omega) - A\left\{\frac{1+\omega}{A+C(1+\omega)T^{3(1+\omega)(\eta-1)/k+1}}\right\}^{\eta}\right]} \tag{34}$$

$$\Omega_\Lambda = 1 - \frac{\left\{\frac{2}{(k+1)T^2} - \frac{2b^2}{3T^{6/k+1}}\right\}}{\frac{3}{(k+1)^2T^2}\left[(1+\omega) - A\left\{\frac{1+\omega}{A+C(1+\omega)T^{\frac{3(1+\omega)(\eta-1)}{k+1}}}\right\}^{\eta}\right]} - \frac{b^2(k+1)^2}{T^{4-2k}} \tag{35}$$

4. Typical Example

4.1: For $\eta = 2,$ the equation of state is written as

$$p = \omega\rho - A\rho^2 \tag{36}$$

$$\rho = \left(\frac{1+\omega}{A+C(1+\omega)T^{3(1+\omega)/k+1}}\right) \tag{37}$$

where C is the constant. From eq. (37) it can be seen that at the earlier time ρ tends to finite, for $\omega > -1$ give $\rho = (1+\omega)/A + C(1+\omega)$

The gravitational constant G and cosmological constant Λ are given by

$$G=\frac{\frac{1}{(k+1)T^{2}}-\frac{b^{2}}{3T^{6/k+1}}}{4\pi\left[(1+\omega)\left\{\frac{1+\omega}{A+C(1+\omega)T^{3(1+\omega)/k+1}}\right\}-A\left\{\frac{1+\omega}{A+C(1+\omega)T^{3(1+\omega)/k+1}}\right\}^{2}\right]} \tag{38}$$

and

$$\Lambda=\frac{3}{(k+1)^{2}T^{2}}-\frac{b^{2}}{3T^{6/k+1}}-\frac{2\left\{\frac{1}{(k+1)T^{2}}-\frac{b^{2}}{3T^{6/k+1}}\right\}}{\left\{(1+\omega)-A\left(\frac{1+\omega}{A+C(1+\omega)T^{3(1+\omega)/k+1}}\right)\right\}^{2}} \tag{39}$$

For $0<\frac{1}{k+1}<\frac{1}{3}$, we have $G\to\infty$ when $T\to 0$ and for $\frac{1}{k+1}=\frac{1}{3}$ and $b^{2}<1$, we have $G\to+\infty$ and $\Lambda\to\infty$ at $T=0$. For $-1<\omega<1$, G and Λ tends to zero as $T\to\infty$. The critical density

$$\rho_{c}=\frac{\frac{3}{(k+1)^{2}T^{2}}\left[(1+\omega)\left\{\frac{1+\omega}{A+C(1+\omega)T^{3(1+\omega)/k+1}}\right\}-A\left\{\frac{1+\omega}{A+C(1+\omega)T^{3(1+\omega)/k+1}}\right\}^{2}\right]}{\frac{2}{(k+1)T^{2}}-\frac{2}{3}\frac{b^{2}}{T^{6/k+1}}} \tag{40}$$

The total density parameter

$$\Omega=\frac{\frac{2}{(k+1)T^{2}}-\frac{2b^{2}}{3T^{6/k+1}}}{\frac{3}{(k+1)^{2}T^{2}}\left[(1+\omega)-A\left\{\frac{1+\omega}{A+C(1+\omega)T^{3(1+\omega)/k+1}}\right\}^{2}\right]} \tag{41}$$

and

$$\Omega_\Lambda = 1 - \frac{\left\{ \frac{2}{(k+1)T^2} - \frac{2b^2}{3T^{6/k+1}} \right\}}{\frac{3}{(k+1)^2 T^2}\left[(1+\omega) - A\left\{ \frac{1+\omega}{A + C(1+\omega)T^{\frac{3(1+\omega)(\eta-1)}{k+1}}} \right\}^2 \right]} - \frac{b^2(k+1)^2}{T^{4-2k}} \tag{42}$$

For $-1 < \omega < 1$ and $\frac{1}{k+1} = \frac{1}{3}$ we get $\Omega \to \infty$ and $\Omega_\Lambda \to -\infty$ as $T \to 0$. For $\frac{1}{k+1} = \frac{1}{3}$ inserting the value of Ω and Ω_Λ, equation (21) becomes $\sigma/H = \sqrt{3}$, the anisotropy does not die out asymptotically. Also for $-1 < \omega < 1$ and $\frac{1}{k+1} = \frac{1}{3}$ we get $\Omega \to 0$ and $\Omega_\Lambda \to 1$ as $T \to \infty$.

4.2: For $\eta = 1 + \frac{1}{n}$, the equation of state is written as

$$p = \omega\rho - A\rho^{1+\frac{1}{n}} \tag{43}$$

where A and n are the polytropic constant and the polytropic index respectively. Here $n > 0$ and $-1 < \omega < 1$, in this case

$$\rho = \left(\frac{1+\omega}{A + C(1+\omega)T^{3(1+\omega)/n(k+1)}} \right)^n \tag{44}$$

where C is the integration constant and energy density is positive for an arbitrary number of η. In the case of $C(1+\omega)T^{3(1+\omega)/n(k+1)} = -A$, we have $\rho \to \infty$ and therefore the polytropic gas has finite time singularity at $T = -\left(A/C(1+\omega)\right)^{n(k+1)/3(1+\omega)}$. In this type of singularity, at characteristic time T, the energy density $\rho \to \infty$ and the pressure density $|p| \to \infty$, is indicated by type III singularity.

The gravitational parameter G and cosmological constant Λ are given by

$$G = \frac{\frac{1}{(k+1)T^2} - \frac{b^2}{3T^{6/k+1}}}{4\pi\left[(1+\omega)\left\{\frac{1+\omega}{A + C(1+\omega)T^{3(1+\omega)/n(k+1)}}\right\}^{\eta} - A\left\{\frac{1+\omega}{A + C(1+\omega)T^{3(1+\omega)/n(k+1)}}\right\}^{n+1}\right]} \tag{45}$$

$$\Lambda = \frac{3}{(k+1)^2 T^2} - \frac{b^2}{3T^{6/k+1}} - \frac{2\left\{\frac{1}{(k+1)T^2} - \frac{b^2}{3T^{6/k+1}}\right\}}{\left\{(1+\omega) - A\left(\frac{1+\omega}{A + C(1+\omega)T^{3(1+\omega)/n(k+1)}}\right)\right\}^{\frac{n+1}{n}}} \tag{46}$$

and

$$\rho_c = \frac{\frac{3}{(k+1)^2 T^2}\left[(1+\omega)\left\{\frac{1+\omega}{A + C(1+\omega)T^{3(1+\omega)/n(k+1)}}\right\} - A\left\{\frac{1+\omega}{A + C(1+\omega)T^{3(1+\omega)/n(k+1)}}\right\}^{n+1}\right]}{\frac{2}{(k+1)T^2} - \frac{2}{3}\frac{b^2}{T^{6/k+1}}} \tag{47}$$

and then we can obtain

$$\Omega = \frac{\frac{2}{(k+1)T^2} - \frac{2b^2}{3T^{6/k+1}}}{\frac{3}{(k+1)^2 T^2}\left[(1+\omega) - A\left\{\frac{1+\omega}{A + C(1+\omega)T^{3(1+\omega)/n(k+1)}}\right\}^{\frac{n+1}{n}}\right]} \tag{48}$$

$$\Omega_\Lambda = 1 - \frac{\left\{\frac{2}{(k+1)T^2} - \frac{2b^2}{3T^{6/k+1}}\right\}}{\frac{3}{(k+1)^2 T^2}\left[(1+\omega) - A\left\{\frac{1+\omega}{A + C(1+\omega)T^{\frac{3(1+\omega)(n-1)}{k+1}}}\right\}^{\frac{n+1}{n}}\right]} - \frac{b^2(k+1)^2}{T^{4-2k}}$$

(49)

For $-1 < \omega < 1$ and $\frac{1}{k+1} = \frac{1}{3}$ we get $\Omega \to \infty$ and $\Omega_\Lambda \to -\infty$ as $T \to 0$. For $\frac{1}{k+1} = \frac{1}{3}$ inserting the value of Ω and Ω_Λ, equation (21) becomes $\sigma/H = \sqrt{3}$, this anisotropy decrease as the universe expands and this explains the present isotropy of the universe. Also for $-1 < \omega < 1$ and $\frac{1}{k+1} = \frac{1}{3}$ we get $\Omega \to 0$ and $\Omega_\Lambda \to 1$ as $T \to \infty$.

4.3: For $\eta = -\alpha,$ the equation of state is written as

$$p = \omega\rho - \frac{A}{\rho^\alpha} \tag{50}$$

$$\rho = \left(\frac{1+\omega}{A + C(1+\omega)T^{-3(1+\omega)(\alpha+1)/(k+1)}}\right)^{-\frac{1}{(\alpha+1)}} \tag{51}$$

The gravitational parameter G and cosmological constant Λ are given by

$$G = \frac{\frac{1}{(k+1)T^2} - \frac{b^2}{3T^{6/k+1}}}{4\pi\left[(1+\omega)\left\{\frac{1+\omega}{A + C(1+\omega)T^{-3(1+\omega)(\alpha+1)/(k+1)}}\right\}^{-\frac{1}{(\alpha+1)}} - A\left\{\frac{1+\omega}{A + C(1+\omega)T^{-3(1+\omega)(\alpha+1)/(k+1)}}\right\}^{\frac{\alpha}{\alpha+1}}\right]} \tag{52}$$

$$\Lambda = \frac{3}{(k+1)^2 T^2} - \frac{b^2}{3T^{6/k+1}} - \frac{2\left\{\frac{1}{(k+1)T^2} - \frac{b^2}{3T^{6/k+1}}\right\}}{\left\{(1+\omega) - A\left(\frac{1+\omega}{A + C(1+\omega)T^{-3(1+\omega)(\alpha+1)/(k+1)}}\right)\right\}^{-\alpha}} \tag{53}$$

and

$$\rho_c = \frac{\frac{3}{(k+1)^2 T^2}\left[(1+\omega)\left\{\frac{1+\omega}{A + C(1+\omega)T^{-3(1+\omega)(\alpha+1)/(k+1)}}\right\}^{-\frac{1}{\alpha+1}} - A\left\{\frac{1+\omega}{A + C(1+\omega)T^{-3(1+\omega)(\alpha+1)/(k+1)}}\right\}^{\frac{\alpha}{\alpha+1}}\right]}{\frac{2}{(k+1)T^2} - \frac{2}{3}\frac{b^2}{T^{6/k+1}}} \tag{54}$$

and then we can obtain

$$\Omega = \frac{\frac{2}{(k+1)T^2} - \frac{2b^2}{3T^{6/k+1}}}{\frac{3}{(k+1)^2 T^2}\left[(1+\omega) - A\left\{\frac{1+\omega}{A + C(1+\omega)T^{-3(1+\omega)(\alpha+1)/(k+1)}}\right\}^{-\alpha}\right]} \tag{55}$$

and

$$\Omega_\Lambda = 1 - \frac{\left\{\frac{2}{(k+1)T^2} - \frac{2b^2}{3T^{6/k+1}}\right\}}{\frac{3}{(k+1)^2 T^2}\left[(1+\omega) - A\left\{\frac{1+\omega}{A + C(1+\omega)T^{\frac{-3(1+\omega)(\alpha+1)}{k+1}}}\right\}^{-\alpha}\right]} - \frac{b^2(k+1)^2}{T^{4-2k}} \tag{56}$$

For $b^2 < 1$ and $\frac{1}{k+1} = \frac{1}{3}$, we obtain G and $\Lambda \to \infty$ as $T \to 0$. At $T \to \infty$, G and Λ tend to zero. Also $\Omega \to \infty$ and $\Omega_\Lambda \to -\infty$ as $T \to 0$ $\Omega \to 0$. For $-1 < \omega < 1$ and $\frac{1}{k+1} = \frac{1}{3}$ we get $\Omega \to 0$ and $\Omega_\Lambda \to 1$ as $T \to \infty$.

5. Conclusion

We have studied Einstein's field equations for Bianchi type I cosmological models with variable G and Λ in presence of perfect fluid and dark energy. We have derived power law and energy density solution, all of which isotropize at late time. The EoS parameters of the chaplygin gas in anisotropic universe were obtained. It was observed that if $b^2 < 0$, total density parameter and cosmological constant density parameter in anisotropic universe are larger than the Ω and Ω_Λ in FRW space time, while the vice versa needs $0 < b^2 < 1$. We calculated the deceleration parameter (q), obtained the decelerated and an accelerated phases of the expansion of the universe in Chaplygin gas model and shows that this model corresponds to an accelerated universe.

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References

[1] A G Riess, et al.: J. Astrophys. 116, 1009 (1998).
[2] S Perlmutter, et al.: J. Astrophys. 517, 565 (1999).
[3] J L Tonry, et al.: J. Astrophys. 594, 1 (2003).
[4] C L Bennett, et al.: J. Astrophys. Suppl. Ser. 148, 1 (2003).
[5] C B Netterfield, et al.: J.Astrophys. 571, 604 (2002).
[6] N W Halverson, et al.: J. Astrophys. 568, 38 (2002).
[7] C Deffayet, et al.: Phys. Rev. D, Part. Fields 65, 044023 (2002).
[8] J P Singh, Astrophys. Space Sci. 318, 103-107 (2008)
[9] A Yu Kamenshchik, U. Moschella, V. Pasquier,: Phys. Lett. B 511,265 (2001).
[10] B Saha, : Chin. J. Phys. 43, 1035-1043 (2005).
[11] M R Setare, : Phys. Lett. B 654, 1-6 (2007).
[12] M R Setare, : Phys. Lett. B 642, 421 (2006).
[13] M Malekjani, A Khodam-Mohamadi, N Nazari-Pooya,: Astrophys. Space Sci. 334,193 (2011).
[14] M. Jamil, E.N. Saridakis, M.R. Setare, : Phys. Lett. B 679, 172 (2009).
[15] A Sheykhi, M R Setare, : Int. J. Theor. Phys. 49, 2777 (2010)
[16] M R Setare, : Int. J. Mod. Phys. D 18, 419 (2009).
[17] A D Dolgov, Phys. Rev. D 48, 2499 (1993)
[18] A D Dolgov, Phys. Rev. D 55, 588 (1997)
[19] C G Tsagas, R Maartens, gr-qc/9912022 (2000)
[20] V Sahni, A Starobinsky, Int. J. Mod. Phys. D q, 373 (2000)
[21] T Padmnabhan, Phys. Rep. 380, 235; hep-th/02122990 (2003)
[22] T Padmnabhan, S M Chitre, Phys. Lett. A. 120, 43 (1987)
[23] R G Vishwakarma, Abdussattar, Phys. Rev. D 60, 063507 (1999)
[24] R G Vishwakarma, Class. Quant. Grav. 17, 3833 (2000)
[25] R G Vishwakarma, Gen. Rel. Grav. 33, 1973 (2001)
[26] R G Vishwakarma, Class. Quant. Grav., **19**, 474 (2002)
[27] A Pradhan, V K Yadav, I Chakraborty, Int. J. Mod. Phys. D 10, 339 (2001)
[28] A Pradhan, V K Yadav, N N Saste, Int. J. Mod. Phys. D 11, 857 (2002)
[29] A Pradhan, K Yadav, Int. J. Mod. Phys. D 11, 839 (2002)
[30] A Pradhan, H R Pandey, Int. J. Mod. Phys. D 12, 941(2003)
[31] A Pradhan, A K Singh, S Otarod, Rom. Journ. Phys. 52, 445 (2007)
[32] I M Khalatnikov, A Y Kamenshchik, : Phys. Lett. B 553, 119 (2003).
[33] J P Singh, R K Tiwari,: Pramana 70, 565 (2008).
[34] R K Tiwari, Astrophys. Astron. 10, 4 (2010)
[35] R K Tiwari, Astrophys. Space Sci. 318, 243 (2008)
[36] R K Tiwari, R Singh,: Eur. Phys. J. Plus **129**, 253 (2014)
[37] R K Tiwari, R Singh,: Int. J. Theor. Phys. **54**, 1417 (2015)
[38] R K Tiwari, R Singh,: Int. J. Theor. Phys. **54**, 2652 (2015)
[39] R K Tiwari, R Singh,: Astrophys. Space Sci. **357**, 130 (2015)