Article

Dynamics of Bianchi Type-VI$_0$ Universe with Magnetized Anisotropic Dark Energy in Lyra Geometry

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Abstract

obtain a spatially homogeneous Bianchi type-VI$_0$ cosmological model in the presence of magnetized anisotropic dark energy within the framework of Lyra geometry. The energy-momentum tensor consists of an anisotropic fluid with anisotropic equation of state and a uniform magnetic field of energy density ρ_b. Exact solutions of the field equations are obtained by utilizing the hybrid expansion law for the average factor of the model. The physical and kinematical behaviors of the cosmological model are discussed. We conclude that the universe model as well as anisotropic fluid approach isotropy through the evolution of the universe in consistent with the recent observations of the present-day universe.

Keywords: Bianchi-VI0 model, anisotropic dark energy, magnetic field, Lyra geometry, hybrid expansion law.

1 Introduction

The recent developments in the modern cosmology have attracted cosmologists towards the study of cosmological models with dark energy (DE) in general relativity as well as in modified theories of gravitation. The astronomical observations of luminosity-distance and redshift relation of type I$_a$ supernovae (Perlmutter et al. [1, 2, 3]; Riess et al. [4, 5]), Cosmic Microwave Background Radiation (Spergel et al. [6]), the Galaxy Power Spectrum (Tegmark [7]) etc have given convincing indication that the universe is not only expanding but is also accelerating. This accelerating expansion of the universe is driven by a mysterious energy with negative pressure, known as DE, acting as antigravity responsible for gearing up the universe. There are several candidates for DE such as cosmological constant [8], quintessence [9], phantom field [10], techyon field [11], chaplygin gas [12] models, which are extensively studied to construct cosmological models by many cosmologists for describing the late-time acceleration of the universe.

The presence of magnetic fields in galactic and intergalactic spaces is evident from the recent observations (Maartens [13]; Grasso and Rubinstein [14]). The large scale magnetic fields can be

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detected by observing their effects on CMB radiation. Many cosmologists have discussed the effect of magnetic field on the dynamics of the universe by anisotropic Bianchi space-times. Jacob [15] studied the effects of a uniform magnetic field in Bianchi type-I cosmological model. King and Coles [16] discussed the dynamics of the magnetized axisymmetric Bianchi type-I universe with vacuum energy and discussed the behaviour of the scale factors perpendicular and parallel to the field lines. Roy et al. [17], Bali et al. [18], Katore and Sancheti [19] etc. studied the effects of magnetic field on the dynamics of the anisotropic Bianchi type-VI$_0$ cosmological model. Sharif and Zubair [20] presented Bianchi type-VI$_0$ models in the presence of an anisotropic DE and electromagnetic field with constant deceleration parameter. Further, Shrif and Zubair [21] obtained exact solutions of the field equations for a Bianchi type-I model by using the condition that the expansion scalar is proportional to the shear scalar. Sharif and Zubair [22] also discussed the dynamics of an anisotropic Bianchi type-VI$_0$ cosmological model with an anisotropic fluid in the presence of a magnetic field.

Recently some noticeable works have been carried out by several cosmologists in the context of anisotropic DE models within the framework of Lyra geometry. Adhav [23] investigated an LRS Bianchi type-I universe with anisotropic DE in Lyra geometry. Katore et al. [24] studied anisotropic cosmological models evolving with early deceleration to late-time acceleration in Lyra geometry. Pawar et al.[25] investigated magnetized anisotropic DE universe models of Bianchi type-VI$_0$ with time-dependent cosmological term in Lyra geometry by using special laws of variation of deceleration parameter proposed by Akarsu and Dereli [26], Abdussattar and Prajapati [27].

In this paper, we study the dynamics of the Bianchi type-VI$_0$ universe with anisotropic DE in the presence of a magnetic field within the framework of Lyra geometry. The paper is organized as follows: In Sect.2, we present the spatially homogeneous Bianchi type-VI$_0$ metric and the field equations for an anisotropic fluid together with an uniform magnetic field. In Sect.3, we obtain exact solutions of the field equations by utilizing the hybrid expansion law for the average scale factor of the model. In Sect.4, we discuss the physical and kinematical behaviors of the universe . Sect.5 contains some concluding remarks.

2 The metric and field equations

We consider the line-element of a Bianchi type-VI$_0$ model in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{2mx}dy^2 - C^2e^{-2mx}dz^2 \tag{2.1}$$

where the scale factors A, B, C are function of cosmic time t and m is a constant. For $m = 0$, the line-element (2.1) reduces to a spatially homogeneous and anisotropic Bianchi type-I model.

The Einstein's field equations based on Lyra manifold in normal gauge are given by

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \frac{3}{2}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_\alpha\phi^\alpha\right) = -T_{\mu\nu} \tag{2.2}$$

where $\phi_\mu = (0,0,0,\psi(t))$ is the time-dependent displacement vector, $\psi(t)$ being the gauge function (Sen[28). Here $T_{\mu\nu}$ is the energy-momentum tensor for the magnetized DE given by

$$T_\mu^\nu = \text{diag}\left[\rho + \rho_b, -p_x + \rho_b, -p_y - \rho_b, -p_z - \rho_b\right] \tag{2.3}$$

where ρ is the energy density of anisotropic DE and p_x, p_y, p_z are the pressures along x, y, z axes respectively and ρ_b denotes energy density of the magnetic field [13, 21].

The anisotropic DE is characterized by the equation of state $p = \omega\rho$, where ω is the EoS parameter, not necessarily a constant. From equation (2.3), we have

$$T_\mu^\nu = \text{diag}\left[\rho + \rho_b, -(\omega+\gamma)\rho + \rho_b, -(\omega+\eta)\rho - \rho_b, -(\omega+\delta)\rho - \rho_b\right] \tag{2.4}$$

where $\omega_x = \omega + \gamma$, $\omega_y = \omega + \eta$ and $\omega_z = \omega + \delta$ are the directional EoS parameters and γ, η and δ are deviations from ω on x, y and z axes respectively. We parameterize the directional EoS parameters in such a way that $\omega_x = \omega$, $\omega_y = \omega + \eta$, $\omega_z = \omega + \delta$. Then equation (2.4) can be written in the form

$$T_\mu^\nu = \text{diag}\left[\rho + \rho_b, -\omega\rho + \rho_b, -(\omega+\eta)\rho - \rho_b, -(\omega+\delta)\rho - \rho_b\right] \tag{2.5}$$

In comoving coordinate system, the field equations (2.2) for the metric (2.1) with the help of equation (2.5) yield the following system of equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{m^2}{A^2} + \frac{3}{4}\psi^2 = -\omega\rho + \rho_b, \tag{2.6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} + \frac{3}{4}\psi^2 = -(\omega+\eta)\rho - \rho_b, \tag{2.7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} + \frac{3}{4}\psi^2 = -(\omega+\delta)\rho - \rho_b, \tag{2.8}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} - \frac{3}{4}\psi^2 = \rho + \rho_b, \tag{2.9}$$

$$m\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0 \tag{2.10}$$

where an overdot denotes derivative with respect to t.

Integration of equation (2.10) yields $B = kC$, where k is a constant of integration. For no loss of generality, we choose $k = 1$. Thus, we have

$$B = C. \tag{2.11}$$

Using equation (2.11) in equations (2.7) and (2.8), we see that $\gamma = \delta$. Therefore the system of equations (2.6)-(2.9) reduces to

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{m^2}{A^2} + \frac{3}{4}\psi^2 = -\omega\rho + \rho_b, \tag{2.12}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{m^2}{A^2}+\frac{3}{4}\psi^2=-(\omega+\delta)\rho-\rho_b, \tag{2.13}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{m^2}{A^2}-\frac{3}{4}\psi^2=\rho+\rho_b. \tag{2.14}$$

From the energy conservation equation $T^{\nu}_{\mu;\nu}=0$, we obtain

$$\dot{\rho}+(1+\omega+\delta)\rho\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)=0, \tag{2.15}$$

$$\dot{\rho}_b+4\rho_b(\frac{\dot{B}}{B})=0, \tag{2.16}$$

$$\psi\dot{\psi}+\psi^2\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)=0. \tag{2.17}$$

Solving equations (2.16) and (2.17), we obtain

$$\rho_b=\frac{c}{B^4}, \tag{2.18}$$

$$\psi=\frac{l}{AB^2} \tag{2.19}$$

where c and l are constants of integration.

Now, we define certain parameters for the metric (2.1) which are important in cosmological observations. The average scale factor a and the volume are defined as

$$a^3=AB^2=V. \tag{2.20}$$

The scalar expansion θ shear scalar σ, mean Hubble parameter H and the anisotropy parameter A_m are given by

$$\theta=3H=\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right), \tag{2.21}$$

$$\sigma^2=\frac{1}{3}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)^2, \tag{2.22}$$

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \tag{2.23}$$

where H_1, H_2, H_3 are the directional Hubble parameters in the directions of x, y and z axes respectively, and are given by

$$H_1=\frac{\dot{A}}{A}, H_2=H_3=\frac{\dot{B}}{B}. \tag{2.24}$$

An important observational quantity in cosmology is the deceleration parameter q defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{2.25}$$

The sign of q indicates whether the model inflates or not. The positive sign of q corresponds to standard decelerating models whereas the negative sign indicates inflation.

3 Solution of the field equation

We now obtain exact solution of the fields equations by applying the hybrid expansion law (HEL) for the average scale factor defined by

$$a(t) = kt^{\alpha}e^{\beta t} \tag{3.1}$$

where $k > 0$, $\alpha \geq 0$ and $\beta \geq 0$ are constants (Akarsu et al. [29]). We observe that HEL leads to power-law cosmology for $\beta = 0$ and to the exponential law cosmology for $\alpha = 0$. Thus, the case $\alpha > 0$ and $\beta > 0$ leads to a new cosmology arising from HEL. Kumar [30] studied the dynamics of universe within the framework of a Bianchi type-V space-time in the presence of a perfect fluid composed of non-interacting matter and dynamical DE. Shri Ram and Chandel [31] discussed the dynamics of a magnetized string cosmological model of Bianchi type-V in f(R, T) gravity theory by using HEL for the average scale factor. Further, Shri Ram et al. [32] investigated hypersurface homogeneous cosmological models with dynamical EoS in Lyra geometry.

Now, subtracting equation (2.12) from equation (2.13), we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{B}}{B}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = \frac{2m^2}{A^2} - \delta\rho - 2\rho_b. \tag{3.2}$$

This equation can be written in thee form

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = \frac{2m^2}{A^2} - \delta\rho - 2\rho_b. \tag{3.3}$$

Integrating equation (3.3), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{X}{V}\exp\left[\int\left(\frac{2m^2}{A^2} - \delta\rho - 2\rho_b\right)V\,dt\right] \tag{3.4}$$

where X is an integration constant. The integral term in equation (3.4) vanishes when

$$\frac{2m^2}{A} - \delta\rho - 2\rho_b = 0. \tag{3.5}$$

Then equation (3.4) reduces to

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{X}{V}. \tag{3.6}$$

Differentiating equation (2.20), we get

$$\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}=\frac{\dot{V}}{V}. \tag{3.7}$$

From equations (3.6) and (3.7), we obtain

$$\frac{\dot{A}}{A}=\frac{2}{3}\frac{X}{V}+\frac{1}{3}\frac{\dot{V}}{V}, \tag{3.8}$$

$$\frac{\dot{B}}{B}=-\frac{1}{3}\frac{X}{V}+\frac{1}{3}\frac{\dot{V}}{V}. \tag{3.9}$$

From equations (3.8) and (3.9), we can obtain the scale factors A and B if V is a known function of cosmic time t. From equations (2.20) and (3.1), we get

$$V=t^{3\alpha}e^{3\beta t}, \tag{3.10}$$

taking $k=1$ without loss of generality. Substituting for V in equations (3.8) and (3.9), we obtain

$$\frac{\dot{A}}{A}=\frac{\alpha}{t}+\beta+\frac{2X}{3t^{3\alpha}e^{3\beta t}}, \tag{3.11}$$

$$\frac{\dot{B}}{B}=\frac{\alpha}{t}+\beta-\frac{X}{3t^{3\alpha}e^{3\beta t}}. \tag{3.12}$$

Integration of equations (3.11) and (3.12) provides the expressions of the scale factors as A and B as

$$A(t)=c_1t^{\alpha}e^{\beta t}\exp\left[-\frac{2X}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha,3\beta t)\right], \tag{3.13}$$

$$B(t)=c_2t^{\alpha}e^{\beta t}\exp\left[\frac{X}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha,3\beta t)\right] \tag{3.14}$$

where $\Gamma(s,x)$ is the lower incomplete gamma function. The constants of integration c_1 and c_2 can be taken unity.

Hence, the metric (1) with scale factors $A(t)$ and $B(t)$, given by equations (3.13) and (3.14) respectively, represents a Bianchi type VI$_0$ cosmological model with anisotropic DE and a uniform magnetic field in Lyra geometry. From equations (2.18) and (2.19) the energy density of the magnetic field ρ_b and the gauge function $\psi(t)$ are obtained as

$$\rho_b=\frac{c}{t^{4\alpha}e^{4\beta t}}\exp\left[-\frac{4X}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha,3\beta t)\right], \tag{3.15}$$

$$\psi(t)=\frac{l}{t^{3\alpha}e^{3\beta t}}. \tag{3.16}$$

4 Some physical and kinematical features of the model

Now we discuss some physical and kinematical features of the derived model in Sect.3. From equations (2.12)-(2.14) and equations (3.15)-(3.16), the energy density ρ of DE and EoS parameter ω turn out to be

$$\rho = 3\left(\frac{\alpha}{t}+\beta\right)^2 - \frac{X^2}{3t^{3\alpha}e^{3\beta t}} - \frac{3l^2}{4t^{6\alpha}e^{6\beta t}} - \left(\frac{m^2}{t^{2\alpha}e^{2\beta t}} + \frac{c}{t^{4\alpha}e^{4\beta t}}\right)\exp\left[-\frac{4X}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha,3\beta t)\right], \tag{4.1}$$

$$\omega = -\frac{1}{\rho}\left[-\frac{2\alpha}{t^2} + 3\left(\frac{\alpha}{t}+\beta\right)^2 + \frac{X^2}{3t^{6\alpha}e^{6\beta t}} + \frac{3l^2}{4t^{6\alpha}e^{6\beta t}} - \left(\frac{m^2}{t^{2\alpha}e^{2\beta t}} - \frac{c}{t^{4\alpha}e^{4\beta t}}\right)\right.$$
$$\left.\exp\left\{-\frac{4X}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha,3\beta t)\right\}\right]. \tag{4.2}$$

Using, equations (3.13) (3.15) and (4.1) in equation (3.5), we obtain the skewness parameter δ as

$$\delta = \frac{1}{\rho}\left[\frac{2m^2}{t^{2\alpha}e^{2\beta t}} - \frac{2c}{t^{4\alpha}e^{4\beta t}}\right]\exp\left\{-\frac{4X}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha,3\beta t)\right\}. \tag{4.3}$$

The directional Hubble parameters and the mean Hubble parameter are obtained as

$$H_1 = \frac{\alpha}{t} + \beta + \frac{2X}{3t^{3\alpha}e^{3\beta t}}, \tag{4.4}$$

$$H_2 = H_3 = \frac{\alpha}{t} + \beta - \frac{X}{3t^{3\alpha}e^{3\beta t}}, \tag{4.5}$$

$$H = \frac{\alpha}{t} + \beta. \tag{4.6}$$

The expansion scalar θ, shear scalar σ and the anisotropy parameter are found to be

$$\theta = 3\left(\frac{\alpha}{t}+\beta\right), \tag{4.7}$$

$$\sigma = \frac{X}{\sqrt{3}t^{3\alpha}e^{3\beta t}}, \tag{4.8}$$

$$A_m = \frac{2X^2}{9t^{6\alpha}e^{6\beta t}}\left(\frac{\alpha}{t}+\beta\right)^{-2}. \tag{4.9}$$

From equations (3.13) and (3.14), we observe that the spatial volume is zero at $t = 0$ and increases as t increases and ultimately become infinite for large time . The expansion scalar and shear scalar are infinite at $t = 0$ and decrease with the increase of cosmic time. At $t = 0$ the energy density is infinite and is a decreasing function of time . Therefore, the cosmological model has a point type singularity at $t = 0$. Thus the universe starts evolving with zero volume at the initial time $t = 0$ with infinite rate of expansion and the expansion rate slows down with time and ultimately assumes the constant value 3β.

The component of the magnetic field reduces the energy density of the anisotropic fluid. The energy density decreases with time and converges to $3\beta^2$ as $t \to \infty$. We also see that as $t \to \infty$, the EoS parameter $\omega = -1$ and skewness parameter $\delta = 0$. Therefore, the present model is accelerating at late-time due to the dominance of DE. The anisotropic parameter of expansion tends to zero as $t \to \infty$, which means that the universe approaches isotropy at large time.

The deceleration parameter q is calculated as

$$q = -1 + \frac{\alpha}{(\alpha + \beta t)^2}. \tag{4.10}$$

We see that the deceleration parameter is positive for $t < \frac{\sqrt{\alpha}-\alpha}{\beta}$ and is negative for $t > \frac{\sqrt{\alpha}-\alpha}{\beta}$, which indicate that the universe evolves with variable deceleration parameter and the transition from deceleration to acceleration takes place at $t = \frac{\sqrt{\alpha}-\alpha}{\beta}$. This restricts α in the range $0 < \alpha < 1$. The gauge function ψ contributes significantly to the expressions of the physical parameters ρ, ω and δ.

Thus, for sufficiently large tomes, we find that

$$H \sim \beta, \qquad q \sim -1, \qquad \rho \sim 3\beta^2, \qquad \omega \sim -1 \tag{4.11}$$

which lead to the conclusion that the present universe asymptotically achieves the de Sitter phase and hence expands forever with the dominance of DE with constant energy density $3\beta^2$.

5 Conclusion

In this paper we have investigated a spatially homogeneous Bianchi type-VI$_0$ cosmological model with magnetized anisotropic DE with dynamical EoS in the framework of Lyra geometry. We have presented exact solutions of the field equations by using the hybrid expansion law for the average scale factor of the model. The component of the magnetic field reduces the energy density. The universe is anisotropic for finite time and isotropizes for large time in consistent with the recent observations on present-day universe. The expansion in the universe is found to be infinite at he initial time which decreases with the increase in time and ultimately becomes 3β at late time. At late time, $\omega = -1$, $\delta = 0$ and $q = -1$ which indicate the inflationary behaviour of the universe with constant energy density $3\beta^2$. Therefore the universe is accelerating with constant rate of expansion for large time due to the dominance of DE. The gauge function $\psi(t)$ is a decreasing function of time and tends to zero at late time. The content of this paper may be useful for better understanding of the present-day accelerating universe with and without the magnetic field. This study will throw some light on the structure formation of the universe, which has astrophysical significance.

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