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Simplified Form of the Semi-empirical Mass Formula

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Abstract

We have developed a simple semi-empirical relation for understanding nuclear binding energy. The key ideas are: (1) Considering the ratio of nucleons mass difference to electron mass, we show that $\exp\left[\left(m_n - m_p\right)/m_e\right] \cong 4\pi.$; (2) Close to the beta stability line, stable mass number $A_s \cong 2Z + (Z/4\pi)^2$.; (3) Considering $(1/4\pi)^2 \cong 0.006333 \cong k$ as a characteristic number, close to the beta stability line and for $Z \geq 26$, nuclear binding energy can be fitted with a single energy coefficient, 8.9 MeV assumed to be associated with strong and coulombic interactions.

Keywords: Semi-empirical mass formula, beta stability line, nuclear binding energy coefficient, strong coupling constant.

1. Introduction

According to the semi-empirical mass formula [1-3], (1) there exist 5 different energy terms and 5 different energy coefficients; (2) average binding energy per nucleon is approximately 8 MeV and maximum binding energy per nucleon is around 8.8 MeV.

In this paper, we will review our views [4-5] on nuclear binding energy with respect to beta stability line, nucleon and electron rest masses and SEMF in a unified approach. The basic idea is that, for $(Z \geq 26)$, close to the beta stability line, nuclear binding energy, $B_A \cong (A - X) \times 8.9$ MeV where $X \cong kN\left[1 + \left(kA^2/\sqrt{Z}\right)\right]$ and number $k = (1/4\pi)^2 \cong 0.006333$.

2. Liquid drop model & semi-empirical mass formula

According to liquid drop model:

1) Atomic nucleus can be considered as a drop of incompressible fluid; and

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2) Nuclear fluid is made of protons and neutrons, which are held together by the strong nuclear force.

Mathematical analysis of the theory delivers an equation which attempts to predict the binding energy of a nucleus in terms of the numbers of protons and neutrons it contains. This equation has five terms on its right hand side. These correspond to the cohesive binding of all the nucleons by the strong nuclear force, the electrostatic mutual repulsion of the protons, a surface energy term, an asymmetry term (derivable from the protons and neutrons occupying independent quantum momentum states) and a pairing term (partly derivable from the protons and neutrons occupying independent quantum spin states). The coefficients are calculated by fitting to experimentally measured masses of nuclei. Their values can vary depending on how they are fitted to the data. In the following formulae, let A be the total number of nucleons, Z the number of protons, and N the number of neutrons. The mass of an atomic nucleus is given by:

$$m = Zm_p + Nm_n - \left(B/c^2\right) \tag{1}$$

where m_p and m_n are the rest mass of a proton and a neutron respectively and B is the binding energy of the nucleus. The semi-empirical mass formula states that the binding energy will take the following form,

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \tag{2}$$

Here a_v = volume energy coefficient, a_s is the surface energy coefficient, a_c is the coulomb energy coefficient, a_a is the asymmetry energy coefficient and a_p is the pairing energy coefficient. If we consider the sum of the volume energy, surface energy, coulomb energy, asymmetry energy and pairing energy, then the picture of a nucleus as a drop of incompressible liquid roughly accounts for the observed variation of binding energy of the nucleus. See table-1 for the currently accepted semi empirical mass formula energy coefficients [3].

Table 1: Current SEMF binding energy coefficients

a_v MeV	a_s MeV	a_c MeV	a_a MeV	a_p MeV
15.78	18.34	0.71	23.21	12.0
15.258	16.26	0.689	22.20	10.08

3. Estimation of stable mass number with proton number

By maximizing $B(A,Z)$ with respect to Z, we find the number of protons Z of the stable nucleus of atomic weight A as,

$$\begin{cases} Z \approx \dfrac{A}{2+\left[\left(a_c/2a_a\right)A^{2/3}\right]}. \\ A-2Z \approx \dfrac{0.4A^2}{A+200} \end{cases} \tag{3}$$

This is roughly $A/2$ for light nuclei, but for heavy nuclei there is an even better agreement with nature. By substituting the above value of Z back into B one obtains the binding energy as a function of the atomic weight, $B(A)$. Maximizing $B(A)/A$ with respect to A gives the nucleus which is most strongly bound or most stable. In this context, we would like to suggest that, independent of SEMF concepts, nuclear beta stability line can be understood with neutron, proton and electron rest masses. It is also possible to show that [6],

$$\begin{aligned} &\exp\left(\frac{\left(m_n-m_p\right)c^2}{m_e c^2}\right) \cong 12.5659102 \cong 4\pi \\ &\rightarrow \left(\frac{\left(m_n-m_p\right)c^2}{m_e c^2}\right) \cong \ln\left(4\pi\right) \cong 2.53102425 \end{aligned} \tag{4}$$

where, $m_n c^2 \cong 939.565413$ MeV, $m_p c^2 \cong 938.272081$ MeV and $m_e c^2 \cong 0.5109989461$ MeV. Based on this observation and without considering the binding energy coefficients, beta stability line can be understood with the following empirical relations.

$$\left.\begin{aligned} &\text{Let, } k \cong \left(\frac{1}{4\pi}\right)^2 \cong 0.006333 \\ &A_s \cong 2Z + \left(\frac{Z}{4\pi}\right)^2 \cong 2Z + 0.00633Z^2 \cong 2Z + kZ^2 \\ &N_s \cong Z + \left(\frac{Z}{4\pi}\right)^2 \cong Z + 0.00633Z^2 \cong Z + kZ^2 \\ &\Rightarrow A_s - 2Z \cong \left(\frac{Z}{4\pi}\right)^2 \cong 0.00633Z^2 \cong kZ^2 \end{aligned}\right\} \tag{5}$$

These relations can be compared with the computational relation pertaining to isotonic shift and drip lines proposed in reference [3], $N_s = 0.968051Z + 0.006558803Z^2$. With even-odd corrections much better correlations can be observed. For light and medium atomic nuclides, there is some mismatch. It can be attributed to shell structure and needs for further study. See table-2 for fitting the stable nucleon number with its corresponding proton number. Interesting point to be noted is that, considering k as a characteristic number, nuclear binding energy can be fitted.

Table 2 : To fit the stable mass numbers

Proton number	Estimated stable mass number	Observed stable mass number(s)		Proton number	Estimated stable mass number	Observed stable mass number(s)
11	22.77	23		51	118.47	121
21	44.79	45		53	123.78	127
23	49.34	50,51		55	129.16	133
25	53.96	55		57	134.57	138,139
27	58.62	59		59	140.0	141
29	63.33	63,65		63	151.13	151
31	68.08	69,71		65	156.75	159
33	72.90	75		67	162.43	165
35	77.76	79		69	168.15	169
37	82.67	85,87		71	173.92	175,176
39	87.63	89		73	179.75	180,181
41	92.64	93		75	185.6	185,187
43	97.71	--		79	197.52	197
45	102.82	103		81	203.55	203,205
47	107.99	107,109		83	209.62	209
49	113.20	113/115		92	237.60	234,235,238

In this table-1, one can see the remarkable fitting of estimated and actual stable isotopes. It is also true that, in between $Z = 51 \text{ and } 57$, there is some continuous deviation in the estimated data and needs fine tuning. Super heavy stable elements can also be predicted with this relation. Results obtained with the above relation can be compared with SEMF stability relation (3).

4. Characteristic binding energy coefficient

To fit the nuclear binding energy, we consider an ad hoc energy unit of approximately $B_0 \cong 8.9 \text{ MeV}.$ Its physical significance can be understood in the following ways.

$$\text{Significance-1: } \frac{B_0}{a_c} \cong 4\pi \cong \sqrt{\frac{1}{k}} \tag{6}$$

$$\text{Significance-2: } \frac{B_0^2}{\left(m_p c^2\right) a_c} \cong \alpha_s \cong 0.1186 \tag{7}$$

where $\alpha_s =$ Strong coupling constant [7].

$$\text{Significance-3: } \frac{B_0}{m_p c^2} \cong \sqrt{k}\alpha_s \cong \frac{\alpha_s}{4\pi} \tag{8}$$

Based on these relations, $\frac{a_c}{m_p c^2} \cong k\alpha_s$. With further study, other hidden relations can also be developed.

5. Proposed new semi-empirical relations of nuclear binding energy for $(Z \geq 26)$

Based on the above points, in a trial - error approach, we developed the following semi empirical relations. We are working on understanding their back ground physics.

Step 1: Close to the beta stability line,

$$\begin{aligned} &B_{A_s} \cong (A_s - X_s) \times 8.9 \text{ MeV} \\ &\text{where } X_s \cong \left[kN_s \left(1 + \frac{kA_s^2}{\sqrt{Z}} \right) \right] \text{ and } N_s = (A_s - Z) \end{aligned} \tag{9}$$

Based on this relation, for $A \approx (A_s \text{ m}4)$, it is possible estimate the nuclear binding energy by the following general relation:

$$\begin{aligned} &B_A \approx \left\{ A - \left[kN \left(1 + \frac{kA^2}{\sqrt{Z}} \right) \right] \right\} \times 8.9 \text{ MeV} \\ &\text{where } \quad N = (A - Z) \end{aligned} \tag{10}$$

Step 2: Nuclear binding energy for $(N < N_s)$

$$(B_A)_{(N<N_s)} \approx \left(\frac{N}{N_s} \right)^{\frac{2}{3}} B_{A_s} \tag{11}$$

Step 3: Nuclear binding energy for $(N > N_s)$

$$(B_A)_{(N>N_s)} \approx \left(\frac{N}{N_s} \right)^{\frac{1}{2}} B_{A_s} \tag{12}$$

6. Fitting the nuclear binding energy

Case A: See table-3 for estimating the nuclear binding energy close to beta stability line

Case B: See table-4 for estimating the nuclear binding energy of isotopes of Z=50

Table 3. Fitting the actual binding energy of stable isotopes of Z=26 to 100

Proton number	Estimated Stable mass number	Estimated stable mass number with even-odd corrections	Estimated stable neutron number with even-odd corrections	Estimated binding energy in MeV From elations (5) and (9)	Actual or reference binding energy in MeV [8] and *[2]
26	56	56	30	490.13	492.258
27	59	59	32	515.65	517.313
28	61	60	32	524.43	526.846
29	63	63	34	549.85	551.385
30	66	66	36	575.16	578.136
31	68	67	36	583.92	*582.1
32	70	70	38	609.12	610.521
33	73	73	40	634.21	*634.34
34	75	74	40	642.95	642.891
35	78	77	42	667.92	*667.66
36	80	80	44	692.78	695.434
37	83	83	46	717.53	*720.46
38	85	84	46	726.23	728.906
39	88	87	48	750.85	*755.05
40	90	90	50	775.35	783.893
41	93	93	52	799.72	805.765
42	95	94	52	808.39	814.256
43	98	97	54	832.63	*836.26
44	100	100	56	856.74	861.928
45	103	103	58	880.72	884.163
46	105	104	58	889.35	892.82
47	108	107	60	913.19	915.263
48	111	110	62	936.89	940.646
49	113	113	64	960.46	963.094
50	116	116	66	983.89	988.684
51	118	117	66	992.47	*992.78
52	121	120	68	1015.74	1017.282
53	124	123	70	1038.88	*1038.77
54	126	126	72	1061.87	1063.909
55	129	129	74	1084.72	*1085.08
56	132	132	76	1107.41	1110.038
57	135	135	78	1129.96	*1131
58	137	136	78	1138.45	1138.792
59	140	139	80	1160.83	*1160.56
60	143	142	82	1183.06	1185.142
61	146	145	84	1205.13	*1203.86
62	148	148	86	1227.04	1225.392
63	151	151	88	1248.79	1244.141
64	154	154	90	1270.38	1266.627
65	157	157	92	1291.81	*1287.38

66	160	160	94	1313.07	1309.455
67	162	161	94	1321.45	*1314.32
68	165	164	96	1342.53	1336.447
69	168	167	98	1363.44	*1356.45
70	171	170	100	1384.18	1378.13
71	174	173	102	1404.75	*1397.78
72	177	176	104	1425.15	1418.801
73	180	179	106	1445.37	*1438.48
74	183	182	108	1465.41	1459.335
75	186	185	110	1485.28	1478.341
76	189	186	110	1493.53	1484.807
77	192	191	114	1524.46	1518.088
78	195	194	116	1543.78	1539.577
79	198	197	118	1562.91	1559.386
80	201	200	120	1581.86	1581.181
81	204	203	122	1600.62	1600.87
82	207	206	124	1619.19	1622.325
83	210	209	126	1637.56	1640.23
84	213	212	128	1655.75	*1655.76
85	216	215	130	1673.74	*1669.2
86	219	218	132	1691.53	*1685.75
87	222	221	134	1709.13	*1701.68
88	225	224	136	1726.53	*1719.13
89	228	227	138	1743.72	*1735.53
90	231	230	140	1760.72	*1753.97
91	234	233	142	1777.51	*1770.78
92	238	238	146	1802.50	1801.69
93	241	241	148	1818.69	*1817.31
94	244	244	150	1834.67	*1835.45
95	247	247	152	1850.45	*1851.73
96	250	250	154	1866.01	*1868.97
97	254	253	156	1881.35	*1884.38
98	257	256	158	1896.49	*1901.62
99	260	259	160	1911.41	*1917.39
100	263	262	162	1926.11	*1935.12

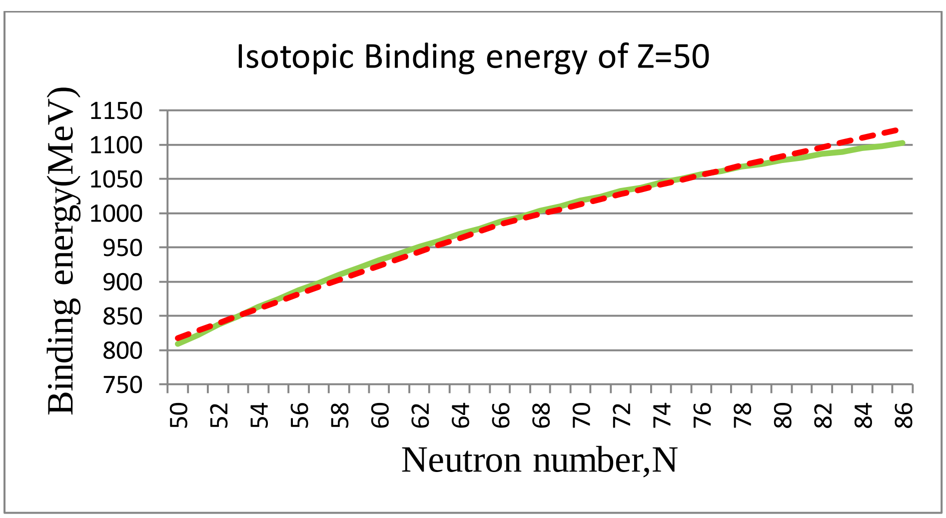
Table 4. Fitting and estimating the binding energy of isotopes of Z=50

Proton number	Estimated Stable mass number	Neutron number	Mass number	SEMF Binding energy in MeV	Estimated binding energy in MeV From relations (5), (9), (11) and (12)
50	116	50	100	809.3	817.6
50	116	51	101	822.3	828.5
50	116	52	102	837.2	839.3
50	116	53	103	849.2	850.0
50	116	54	104	863.2	860.7
50	116	55	105	874.5	871.3
50	116	56	106	887.6	881.8
50	116	57	107	898.1	892.3

50	116	58	108	910.4	902.7
50	116	59	109	920.1	913.0
50	116	60	110	931.8	923.3
50	116	61	111	940.7	933.5
50	116	62	112	951.6	943.7
50	116	63	113	960.0	953.8
50	116	64	114	970.2	963.9
50	116	65	115	977.9	973.9
50	116	66	116	987.5	983.9
50	116	67	117	994.6	991.3
50	116	68	118	1003.6	998.7
50	116	69	119	1010.1	1006.0
50	116	70	120	1018.5	1013.3
50	116	71	121	1024.4	1020.5
50	116	72	122	1032.3	1027.6
50	116	73	123	1037.8	1034.8
50	116	74	124	1045.1	1041.8
50	116	75	125	1050.1	1048.8
50	116	76	126	1056.9	1055.8
50	116	77	127	1061.4	1062.7
50	116	78	128	1067.8	1069.6
50	116	79	129	1071.8	1076.4
50	116	80	130	1077.7	1083.2
50	116	81	131	1081.4	1090.0
50	116	82	132	1086.9	1096.7
50	116	83	133	1090.1	1103.4
50	116	84	134	1095.2	1110.0
50	116	85	135	1098.0	1116.6
50	116	86	136	1102.7	1123.1

See Figure 1 pertaining to isotopic binding energy of Z=50. Solid green curve indicates the binding energy estimated from SEMF and dashed red curve indicates our estimation.

Figure 1. Binding energy of isotopes of Z=50



7. Discussion

From the above proposed relations and tables it is possible to say that:

1) Strong coupling constant seems to play an interesting role in nuclear binding energy scheme.
2) 8.9 MeV can be considered as an effective nuclear binding energy potential pertaining to strong interaction and coulombic repulsion.
3) Interesting point to be noted is that, close to the line of beta stability, $\frac{k^2 A_s^2 N_s}{\sqrt{Z}} \approx \frac{(N_s - Z)^2}{Z}$ and $\left(k^2 A_s N_s \sqrt{Z}\right) \approx \frac{(N_s - Z)^2}{A_s}$. Based on these observations, we are trying to understand the physical back ground of $X_s \cong \left[kN_s \left(1 + \frac{kA_s^2}{\sqrt{Z}} \right) \right]$.
4) $Z = 53$ is estimated to be stable $A_s = 123$ and its estimated binding energy is 1038.9 MeV. Actually, it is stable at $A_s = 127$. From the proposed relations, estimated binding energy of ${}_{53}I^{127}$ is $\left(\frac{127-53}{123-53}\right)^{\frac{1}{2}} 1038.9 \cong 1068.17 \text{ MeV}$ and its actual binding energy is 1072.57 MeV.
5) Binding energy of $Z < 26$ can be approximated with: $B_A \approx \left(A - Z^{\frac{1}{3}} \right) \times 8.9 \text{ MeV}.$ It needs further study. We are working on fine tuning this relation.
6) Similar to the new model proposed by N. Ghahramany et al [8], our method is also very simple to understand.

8. Conclusion

Even though some % error is persisting in the proposed binding energy relations, qualitatively they are very simple to follow. Interesting point to be noted is that, number of energy coefficients can be minimized to one and its existence can be interrelated with strong interaction. We are confident to say that, with further research, back ground physics of the proposed relations can be understood and thereby a clear and simple model can be developed.

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References

[1] Weizsäcker, Carl Friedrich von, On the theory of nuclear masses; Journal of Physics 96 (1935) pages 431- 458.
[2] W. D. Myers et al. Table of Nuclear Masses according to the 1994 Thomas-Fermi Model.(from nsdssd.lbl.gov)
[3] P. Roy Chowdhury et al. Modified Bethe-Weizsacker mass formula with isotonic shift and new driplines. (http://arxiv.org/abs/nucl-th/0405080).
[4] Seshavatharam U. V. S, Lakshminarayana, S., On the role of strong interaction in understanding nuclear beta stability line and nuclear binding energy. Proceedings of the DAE-BRNS Symp. on Nucl. Phys. 60, 118-119 (2015)
[5] Seshavatharam U. V. S, Lakshminarayana, S., On the Ratio of Nuclear Binding Energy & Protons Kinetic Energy. Prespacetime Journal, Volume 6, Issue 3, pp. 247-255 (2015)
[6] Seshavatharam U. V. S, Lakshminarayana, S., Consideration on Nuclear Binding Energy Formula. Prespacetime Journal, Volume 6, Issue 1, pp.58-75 (2015)
[7] C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update
[8] N. Ghahramany et al. A new approach to nuclear binding energy in integrated nuclear model. T.8, No 2 (165). 169-181. (2011)