

Report

## On Anomalous Effects in 2D Nanocrystals

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### Abstract

In this report, we point out that in the case of two-dimensional nanocrystals or quasi-crystals like graphene, there is an interesting anomalous behaviour of Fermions. This could potentially be useful. The point is that some electromagnetic properties depend on the non-commutative geometry of the crystal structure rather than on the chemistry of the crystal material. Issues related to magnetism are also commented upon.

**Keywords:** Nanocrystal, 2D, anomalous effect, graphene, Fermion, magnetism.

Recently Andre Geim, who got the joint Nobel Prize for the discovery of Graphene and his coworkers observed that Graphene is porous only to protons and opaque otherwise. He hinted that this property could be used for harvesting hydrogen for use in fuel cells [1]. On the other hand, the author discussed the magical and anomalous properties of two dimensional crystals beginning 1995 – long before the discovery of Graphene [2, 3, 4, 5, 6]. He has also pointed out that these properties of minimum conductivity and the like are a characteristic not just of Graphene but of any two dimensional crystal or quasi crystal, arguing on the following grounds: All this is a consequence of the noncommutative geometry caused by the lattice like structure which is not peculiar to Graphene alone [7].

The author had gone on to point out that any two dimensional crystal would exhibit such apparently anomalous properties. Indeed this was confirmed in the case of Stanene which is a two dimensional crystal with tin in place of carbon.

Returning to these two dimensional crystals, the protons would be described by the well known Weyl like two component equation [8]

$$\nu_F \vec{\sigma} \cdot \vec{\nabla} \psi(r) = E \psi(r) \quad (0.1)$$

Essentially (0.1) arises as only two of the three momenta ( $p_x p_y p_z$ ) would survive owing to the two dimensionality.

In the above  $\sigma$ s are the Pauli matrices and  $\nu_F$  is the Fermi velocity  $\sim 10^6 m/s$  This is about 300 times less compared to the velocity of light. However, there is a correspondence between graphene and the Minkowski world as detailed in an isomorphism [9] which enables us to go from the one to the other as far as mathematical formalism is considered. In this case a beam behaves like a mono energetic beam of Fermions all with energy  $\sim mc^2$  or  $m\nu_F^2$  discussed earlier in detail [10].

What would happen is that there would be, as discussed earlier, a Bose Einstein type of condensation of, curiously enough the protons. To see this in greater detail, we start with the well known formula for the occupation number of a Fermion gas [11]

$$\bar{n}_p = \frac{1}{z^{-1} e^{bE_p} + 1} \quad (0.2)$$

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where,  $z' \equiv \frac{\lambda^3}{v} \equiv \mu z \approx z$  because, here, as can be easily shown  $\mu \approx 1$  (Cf.ref [10, 12])

$$v = \frac{V}{N}, \lambda = \sqrt{\frac{2\pi\hbar^2}{m/b}}$$

$$b \equiv \left(\frac{1}{kT}\right), \quad \text{and} \quad \sum \bar{n}_p = N \quad (0.3)$$

where the symbols have their usual meaning.

Let us consider in particular a collection of Fermions which is somehow made nearly mono-energetic, as when they stream through a sheet of graphene or Stanene, that is, given by the distribution,

$$n'_p = \delta(p - p_0)\bar{n}_p \quad (0.4)$$

where  $\bar{n}_p$  is given by (0.2). (0.4) would also bring us back to the 2D case (Cf. also Appendix).

This is not possible in general. By the usual formulation we have,

$$N = \frac{V}{\hbar^3} \int d\vec{p} n'_p = \frac{V}{\hbar^3} \int \delta(p - p_0) 4\pi p^2 \bar{n}_p dp = \frac{4\pi V}{\hbar^3} p_0^2 \frac{1}{z^{-1}e^\theta + 1} \quad (0.5)$$

where  $\theta \equiv bE_{p_0}$ .

It must be noted that in (0.5) there is a loss of dimension in momentum space, due to the  $\delta$  function in (0.4). This is also the case in 2D crystals like single layer graphene.

In an earlier communication [10] we showed that in the one dimensional case, corresponding to nanotubes we would have

$$kT = \frac{3}{5} kT_F \quad (0.6)$$

where  $T_F$  is the Fermi temperature. We can see that for the two dimensional case too  $kT$  would be very small (Cf.ref.[1]). This is because using the well known formula for two dimensions we have

$$kT = \frac{e\hbar\pi}{m\nu_F} \quad (0.7)$$

$$(kT)^3 = \frac{6e\hbar\nu_F}{\pi} \quad (0.8)$$

Whence we have

$$(kT)^2 = 6 \cdot \nu_F^2 \pi^2 m \quad (0.9)$$

Remembering that  $\nu_F \sim 10^8$ , even for a particle whose mass is that of an electron or a proton,  $kT$  in (0.9) is very small. By way of a comparison for the Fermi temperature we get,

$$kT_F = \frac{\hbar}{2}(z6\pi)^{1/3} \cdot \nu_F$$

We would now have,  $kT = \langle E_p \rangle \approx E_p$  because of the mono energetic feature so that,  $\theta \approx 1$ . But we can proceed without giving  $\theta$  any specific value.

Using the expressions for  $v$  and  $z$  given in (0.3) in (0.4), we get

$$(z^{-1}e^\theta + 1) = (4\pi)^{5/2} \frac{z'^{-1}}{p_0}; \text{whence}$$

$$z'^{-1}A \equiv z'^{-1} \left( \frac{(4\pi)^{5/2}}{p_0} - e^\theta \right) = 1, \quad (0.10)$$

where we use the fact that in (0.3),  $\mu \approx 1$  as can be easily deduced.

From (0.10) if,

$$\begin{aligned} A &\approx 1, \text{ i.e.,} \\ p_0 &\approx \frac{(4\pi)^{5/2}}{1+e} \end{aligned} \quad (0.11)$$

where  $A$  is given in (0.10), then  $z' \approx 1$ . Remembering that in (0.3),  $\lambda$  is of the order of the de Broglie wave length and  $v$  is the average volume occupied per particle, this means that the gas gets very densely packed for momenta given by (0.11). Infact for a Bose gas, as is well known, this is the condition for Bose-Einstein condensation at the level  $p = 0$  (cf.ref.[11]).

In any case there is an anomalous behaviour of the Fermions accompanied by a "fusion" type effect for protons.

On the other hand, if,

$$A \approx 0 \text{ (that is } \frac{(4\pi)^{5/2}}{e} \approx p_0)$$

then  $z' \approx 0$ . That is, the gas becomes dilute, or  $V$  increases.

More generally, equation (0.10) also puts a restriction on the energy (or momentum), because  $z' > 0$ , viz.,

$$\begin{aligned} A &> 0 \text{ (i.e. } p_0 < \frac{(4\pi)^{5/2}}{e}) \\ \text{But if } A &< 0, \text{ (i.e. } p_0 > \frac{(4\pi)^{5/2}}{e}) \end{aligned}$$

then there is an apparent contradiction.

The contradiction disappears if we realize that  $A \approx 0$ , or

$$p_0 = \frac{(4\pi)^{5/2}}{e} \quad (0.12)$$

(corresponding to a temperature given by  $KT = \frac{p_0^2}{2m}$ ) is a threshold momentum (phase transition). For momenta greater than the threshold given by (0.12), the collection of Fermions behaves like Bosons. This is the bosonization effect [13]. In this case, the occupation number is given by

$$\bar{n}_p = \frac{1}{z^{-1}e^{bE_p} - 1},$$

instead of (0.2), and the right side equation of (0.10) would be given by  $' - 1'$  instead of  $+1$ , so that there would be no contradiction. Thus in this case there is an anomalous behaviour of the Fermions.

The following comment is relevant: It is commonly believed that the spin features of bosons and fermions are intrinsic properties. That is true in the usual 3D space, where there is some form of entanglement with the environment as described e.g. in ref.[14]. Once we deal with the 2D case, this disappears and we can have "transmutation" of Fermions to Bosons or vice versa.

## APPENDIX(Cf.ref.[15])

To illustrate some of the above statements let us consider the case of 2D crystals in the context of the fractional Quantum Hall Effect. We have reiterated that the "graphene" effects are valid for all 2D crystals.

The Quantum Hall Effect was discovered experimentally in the 1980s [16]. In this case, as is by now well known, for a two dimensional system of electrons, the Hall conductivity is found to be of the form

$$G = \lambda \cdot \frac{e^2}{h} \quad (0.13)$$

where  $\lambda$  takes on values

$$\lambda = m/n \quad (0.14)$$

$m$  and  $n$  being integers [17].

There have been attempts to explain this strange phenomenon from theory. Particularly by invoking gauge invariance [18]. However there have been some persisting puzzles.

We will now look at the Fractional Quantum Hall Effect (FQHE) from a completely novel perspective. Let us consider graphene. As is well known this is a single layer graphite with almost magical properties. Some of these have been predicted by the author starting 1995 [19, 20, 21, 10]. Graphene has a honeycomb like lattice structure so that the space of graphene resembles a chessboard with "holes" in space itself as pointed out by Mecklenburg and Regan [7]. This means that there is a fundamental minimum length underpinning the system.

This fundamental length  $L$  leads to a non-commutative geometry as pointed out by Snyder a long time ago [22]. This was in the context of Quantum Electrodynamics [23]. This means that if  $(x, y)$  are the coordinates,  $xy \neq yx$ . A verification for this is the theoretical deduction of the entire suite of Fractional Quantum Hall Effect i.e. equation (0.20), as we will see below.

Indeed, as pointed out, graphene therefore provides a test bed for these principles of physics which play a role in Quantum Gravity approaches. In such a situation it has been shown by the author and independently Saito [24, 25, 26] that there is a strong magnetic field. Further, the author showed that this field is given by

$$BL^2 = hc/e \quad (0.15)$$

$L^2$  defines a Quantum of area exactly as in Quantum Gravity approaches [27, 28]. This in our case is the area of individual lattices.

To elaborate, the author had argued that (0.15) holds in the case of a noncommutative geometry. This happens when there is a fundamental length  $L$  which acts as a minimum length of the system. In this latter case Snyder had shown that commutation relations like

$$[x, y] = (\iota L^2/\hbar) L_x \text{ etc.} \quad (0.16)$$

hold good.

In these considerations for graphene as is very well known, the Fermi velocity  $\nu_F$  replaces the velocity of light. So we have for the electron mobility and conductivity

$$\mu = \nu_F/|E| \quad (0.17)$$

$$\sigma = (n/A)e \cdot \frac{\nu_F}{|E|}, A \sim L^2 \quad (0.18)$$

where  $A$ , as in the usual theory is the area and  $n$  is the number of electrons. In our case as noted above  $A$ , the area is made up of a number of honeycomb lattice areas, each with area  $\sim L^2$ , that is

$$A = mL^2$$

where  $m$  is an integer.

Using these inputs we get (Cf.ref.[4] for details)

$$\sigma = \frac{n}{m} \cdot \frac{e\nu_F}{|B|L^2} \quad (0.19)$$

If we now use (0.15) in (0.19) (with  $\nu_F$  replacing  $c$ ) we get for the conductance

$$\sigma = \frac{n}{m} \cdot \frac{e^2}{h} \quad (0.20)$$

which defines the fractional Quantum Hall Effect.

The author had also shown that it is this non-commutative space feature in two dimensional structures that explains also Landau levels [29] or the minimum conductivity that exists in 2D crystals even when there are practically no electrons at the Dirac points [22]. In other words several supposedly diverse phenomena arise from the non-commutative space of these two dimensional structures.

It must be mentioned that the idea of trying to consider graphene from the perspective of its non-commutative space has been studied by several authors [30, 31, 32, 33]. Some of the approaches were motivated by the Quantum Electrodynamics in the spirit of Wilson's Lattice Gauge Theory. We would like to point out that unlike in the other approaches we have added two new inputs not used earlier which have lead to the rather comprehensive and neat deduction of (0.20) like (0.15) and (0.16). These are firstly (0.15) which was deduced several years ago in the context of high energy physics and quantum gravity and secondly (??), which as shown applies to two dimensional systems.

It may be mentioned that over the years the Integral Hall Effect and Fractional Quantum Hall Effects have not only been observed experimentally but several excellent simulations exist [34]. Furthermore while it has been known that both the integral and the fractional effects may be qualitatively related, exact theories to explain this have not been fully developed. It must be mentioned that there is another approach that of composite Fermions which could potentially provide a unified description, for example that of the approach of J. Jain of Pennsylvania State University. To put it briefly a composite Fermion describes an electron together with an even number of vortices.

Finally it may be mentioned that the work of the author and Saito briefly described above and which shows the production of a magnetic field due to noncommutative space, provides an explanation for the Quantum Anomalous Hall Effect which takes place in the absence of an external magnetic field. This effect was observed recently [35]. We can expect that in the process of the fermion boson transmutation from equation (0.11)ff., there would be a surge of magnetism or its sudden disappearance owing to fermion spin alignments.

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