Exploration

What Could Be the Role of Complexity Theory in TGD?

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Abstract

Chaotic (or actually extremely complex and only apparently chaotic) systems seem to be the diametrical opposite of completely integrable systems about which TGD is a possible example. There is however also something common: in completely integrable classical systems all orbits are cyclic and in chaotic systems they form a dense set in the space of orbits. Furthermore, in chaotic systems the approach to chaos occurs via steps as a control parameter is changed. Same would take place in adelic TGD fusing the descriptions of matter and cognition. In TGD Universe the hierarchy of extensions of rationals inducing finite-dimensional extension of p-adic number fields defines a hierarchy of adelic physics and provides a natural correlate for evolution. Galois groups and ramified primes appear as characterizers of the extensions. The sequences of Galois groups could characterize an evolution by phase transitions increasing the dimension of the extension associated with the coordinates of WCW in turn inducing the extension used at space-time and Hilbert space level. WCW decomposes to sectors characterized by Galois groups G_3 of extensions associated with the 3-surfaces at the ends of space-time surface at boundaries of causal diamond (CD) and G_4 characterizing the space-time surface itself. G_3 (G_4) acts on the discretization and induces a covering structure of the 3-surface (space-time surface). If the state function reduction to the opposite boundary of CD involves localization into a sector with fixed G_3, evolution is indeed mapped to a sequence of G_3s. Also the cognitive representation defined by the intersection of real and p-adic surfaces with coordinates of points in an extension of rationals evolve. The number of points in this representation becomes increasingly complex during evolution. Fermions at partonic 2-surfaces connected by fermionic strings define a tensor network, which also evolves since the number of fermions can change. The points of space-time surface invariant under non-trivial subgroup of Galois group define singularities of the covering, and the positions of fermions at partonic surfaces could correspond to these singularities - maybe even the maximal ones, in which case the singular points would be rational. There is a temptation to interpret the p-adic prime characterizing elementary particle as a ramified prime of extension having a decomposition similar to that of singularity so that category theoretic view suggests itself. One also ends up to ask how the number theoretic evolution could select preferred p-adic primes satisfying the p-adic length scale hypothesis as a survivors in number theoretic evolution, and ends up to a vision bringing strongly in mind the notion of conserved genes as analogy for conservation of ramified primes in extensions of extension. $h_{eff}/h = n$ has natural interpretation as the order of Galois group of extension. The generalization of $\hbar_{gr} = GMm/v_0 = \hbar_{eff}$ hypothesis to other interactions is discussed in terms of number theoretic evolution as increase of G_3, and one ends up to surprisingly concrete vision for what might happen in the transition from prokaryotes to eukaryotes.

1 Introduction

I have many times wondered what could be the role of chaos theory or better in TGD. In fact, I would prefer to talk about complexity theory since the chaos in the sense as it is used is only apparent and very different from thermodynamical chaos.

Wikipedia article (see `http://tinyurl.com/qexmowa`) gives a nice summary about the history of chaos theory and I repeat only some main points here. Chaos theory has roots already at the end of 18the century by the works of Poincare (non-periodic orbits in 3-body system) and Hadamard (free particle gliding frictionlessly on surface of constant negative curvature, "Hadamard billiard". In this case

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all trajectories are unstable diverging exponentially from each other: this is characterized by positive Lyapunov exponent.

Chaos theory got is start from ergodic theory (see `http://tinyurl.com/pfcrz4c`) studying dynamical systems with the original motivation coming from statistical physics. For instance, spin glasses are a representative example of non-ergodic system in which the trajectory of point does not go arbitrary near to every point. The study of non-linear differential equations George David Birkhoff, Andrey Nikolaevich Kolmogorov, Mary Lucy Cartwright and John Edensor Littlewood, and Stephen Smale provides was purely mathematical study of chaotic systems. Smale discovered strange attractor at which periodic orbits form a dense set. Chaos theory was formalized around 1950. At this time it was also discovered that finite-D linear systems do not allow chaos.

The emergence of computers meant breakthrough. Much of chaos theory involves repeated iteration of simple mathematical formulas. Edward Lorentz was a pioneer of chaos theory working with weather prediction and accidentally discovered initial value sensitivity. Benard Mandelbrot discovered fractality and Mitchell Feigenbaum the universality of chaos for iteration of functions of real variable.

Chaotic systems are as far from integrable systems as one could imagine: all orbits are cycles in integrable Hamiltonian dynamics. There are good reasons to suspect that TGD Universe is completely integrable classically. Chaos theory however describes also the emergence of complexity through phase transition like steps - period n-tupling and most importantly by period doubling for iteration of maps.

Chaotic (or actually extremely complex and only apparently chaotic) systems seem to be the diametrical opposite of completely integrable systems about which TGD is a possible example. There is however also something common: in completely integrable classical systems all orbits are cyclic and in chaotic systems they form a dense set in the space of orbits. Furthermore, in chaotic systems the approach to chaos occurs via steps as a control parameter is changed. Same would take place in adelic TGD fusing the descriptions of matter and cognition.

In TGD Universe the hierarchy of extensions of rationals inducing finite-dimensional extension of p-adic number fields defines a hierarchy of adelic physics and provides a natural correlate for evolution. Galois groups and ramified primes appear as characterizers of the extensions. The sequences of Galois groups could characterize an evolution by phase transitions increasing the dimension of the extension associated with the coordinates of "world of classical worlds" (WCW) in turn inducing the extension used at space-time and Hilbert space level. WCW decomposes to sectors characterized by Galois groups G_3 of extensions associated with the 3-surfaces at the ends of space-time surface at boundaries of causal diamond (CD) and G_4 characterizing the space-time surface itself. G_3 (G_4) acts on the discretization and induces a covering structure of the 3-surface (space-time surface). If the state function reduction to the opposite boundary of CD involves localization into a sector with fixed G_3, evolution is indeed mapped to a sequence of G_3s.

Also the cognitive representation defined by the intersection of real and p-adic surfaces with coordinates of points in an extension of rationals evolve. The number of points in this representation becomes increasingly complex during evolution. Fermions at partonic 2-surfaces connected by fermionic strings define a tensor network, which also evolves since the number of fermions can change.

The points of space-time surface invariant under non-trivial subgroup of Galois group define singularities of the covering, and the positions of fermions at partonic surfaces could correspond to these singularities - maybe even the maximal ones, in which case the singular points would be rational. There is a temptation to interpret the p-adic prime characterizing elementary particle as a ramified prime of extension having a decomposition similar to that of singularity so that category theoretic view suggests itself.

One also ends up to ask how the number theoretic evolution could select preferred p-adic primes satisfying the p-adic length scale hypothesis as a survivors in number theoretic evolution, and ends up to a vision bringing strongly in mind the notion of conserved genes as analogy for conservation of ramified primes in extensions of extension. $h_{eff}/h = n$ has natural interpretation as the order of Galois group of extension. The generalization of $\hbar_{gr} = GMm/v_0 = \hbar_{eff}$ hypothesis to other interactions is discussed in terms of number theoretic evolution as increase of G_3, and one ends up to surprisingly concrete vision for what might happen in the transition from prokaryotes to eukaryotes.

2 Basic notions of chaos theory

It is good to begin by summarizing the basic concepts of chaos theory. Again Wikipedia article (see `http://tinyurl.com/qexmowa`) gives a more detailed representation and references. Citing Wikipedia freely: Within the apparent randomness of chaotic complex systems there are patterns, constant feedback loops, repetition, self-similarity, fractals, self-organization and there is sensitivity to initial conditions (butterfly effect) implying the loss of predictability although chaotic systems as such are deterministic.

2.1 Basic prerequisites for chaotic dynamics

Wikipedia article lists three basic conditions for chaotic dynamics. Dynamics must **a)** be sensitive to initial conditions, **b)** allow topological mixing, **c)** have dense set of periodic orbits.

1. Sensitivity to initial conditions.

   Mathematical formulation for the sensitivity to initial conditions can be formulated by perturbation theory for differential equations. The rate of separation of images of points initially near to each other increases exponentially as $exp(\lambda t)$ in initial value sensitive situation and the approximation fails soon. Lyapunov exponent λ characterizes the time evolution of the difference. In multi-dimensional case there are several Lyapunov exponents but the largest one is often enough to characterize the situation.

2. Topological mixing (transitivity).

   This notion corresponds to everyday intuition about mixing. For instance, the flow defined by a vector field mixes the marker completely with the fluid. Iteration of simple scaling operation is initial value sensitive but does not cause topological mixing. In 1-D case all points larger than one approach to infinity and smaller than 1 to zero so that the behavior is extremely simple.

3. Dense set of periodic orbits.

   Periodic orbits should form a dense set in the space of orbits: every point of space is approached arbitrarily closely by a periodic orbit. In completely integrable system all orbits would be periodic orbits so that the difference of these systems is very delicate and one can wonder whether the conditions a) and b) follow from this delicate difference. One can also ask whether there might be a deep connection between completely integrable and chaotic systems.

   Sharkovkii's theorem states that any 1-D system with dynamics determined by iteration of a continuous function of real argument exhibits a regular cycle of period 3 exhibits all other cycles. This theorem can be generalized further (see `http://tinyurl.com/l7q3rah`). Introduce Sharkovskii ordering of integers as union of sets consisting of odd integers multiplied by powers of 2. The generalization of the theorem states that if n is a peri89od and precedes k in Sharkovskii ordering then k is prime period (it is not a multiple of smaller period).

   The theorem holds true for reals but not for periodic functions at circle which are encountered for iterations defined by powers of cyclic group elements. The discrete subgroup of hyperbolic subgroups of Lie groups do not have not cycles at all.

2.2 Strange attractors and Julia sets

Logistic map $x \rightarrow kx(1-x)$ is chaotic everywhere but many systems are chaotic only in a subset of phase space. An interesting situation arises when the chaotic behavior takes place at attractor, since all initial positions in the basic of the attractor lead to the attractor and to a chaotic behavior. Lorentz attractor is a well-known example of strange attractor (see Wikipedia article for illustration). It contains dense sets of both periodic and aperiodic orbits.

Julia set (see http://tinyurl.com/l8jl5ne) is the boundary of the basin of attraction in chaotic systems defined by iteration of a rational function of complex argument mapping complex plane to itself. Both Julia sets and strange attractors have a fractal structure.

Strange attractors can appear only in spaces with dimension $D \geq 3$. Poincare-Bendixon theorem states that 2-D differential equations on Euclidian plane have very regular behavior. In non-Euclidian geometry situation changes and the hyperbolic character of the geometry implying initial value sensitivity of geodesic motion is the reason for this. Also infinite-D linear systems can exhibit chaotic behavior.

3 How to assign chaos/complexity theory with TGD?

Completely integrable systems can be seen as a diametric opposite of chaotic systems. If classical TGD indeed represents a completely integrable system meaning that space-time surfaces as preferred extremals can be constructed explicitly, one might think that chaos theory need not have much to do with classical TGD. Chaos is however the end product of transitions making the system more complex, and it might well be that the understanding about the emergence of complexity in chaotic systems could help to develop the vision about emergence of complexity in TGD. Note also that periodic orbit are dense in chaotic systems so that diametrical opposites might actually meet.

The most relevant TGD based ingredients used in the sequel are following: the world of classical worlds (WCW) [14]; strong form of holography (SH) [7], quantum classical correspondence (QCC), zero energy ontology (ZEO) [8], dark matter as hierarchy of phases with effective Planck constant $h_{eff}/h = n$ [2, 11, 13], p-Adic physics as physics of cognition [4, 3, 9] [21], adelic physics [21] fusing the physics of matter and cognition by integrating reals and extensions of various p-adic number fields induced by an extension of rationals to a larger structure, and the notions of adelic manifold and associated cognitive representation [19] , Negentropy Maximization Principle (NMP) [3] satisfied automatically in statistical sense in adelic physics [21].

3.1 Complexity in TGD

Complexity is often taken to mean computational complexity for classical computations. Complexity as it is understood in the sequel relates very closely cognition. Too complex looks chaotic since our cognitive abilities do not allow to discern too complex patterns. Hence complexity theory should characterize cognitive representations whatever they are.

Number theoretic vision about TGD serves as the guideline here.

1. In adelic TGD [16] cognitive representations correspond to the intersections of real space-time surfaces and their p-adic variants obeying same field equations and representing correlates for cognition. In these intersections the coordinates of points belong to an extension of rationals defining adele [19].

   One ends up with a generalization of the notion of manifold to adelic manifold. Intersection defines a common discrete spine consisting of points with coordinates in the extension of rationals defining the adele. These poins are shared by the real and p-adic variants of the adelic manifold. I have called this manifold also monadic manifold since there is strong resemblance with the ideas of Leibniz. In real sector this manifold differs from ordinary manifold in that the open sets are labelled by a discrete set of points in the intersection.

   In TGD framework it is essential that the spine of the space-time surface consists of points of imbedding space for which it is convenient to use preferred coordinates.

2. Complexity corresponds roughly to the dimension of extension of rationals defining the adeles. p-Adic differential equations are non-deterministic due to the existence of p-adic pseudo constants

depending on finite number of p-adic digits of the p-adic number. This non-determinism is identified as a correlate for imagination. p-Adic variants of space-time surfaces are not uniquely determined this means finite cognitive resolution.

By SH [10] the data associated with string world sheets, partonic 2-surfaces, and discretization allow to construct space-time surfaces as preferred extremals of the action principle defining classical TGD and to find the Kähler function for WCW geometry. It is quite well possible that the data allowing to construct p-adic space-time surfaces does not allow continuation to a preferred extremal: all imaginations are not realizable!

The algebraic dimension of the extension could be relevant for the ability of mathematical cognition to imagine spaces with dimension higher than that for the real 3-space. Besides the extensions of p-adics induced by algebraic extensions of rationals also those induced by some root of e are algebraically finite-dimensional. One can imagine also other extensions involving transcendentals in real sense but it is not clear whether there are finite dimensional extensions among them. The finiteness of cognition suggests that only these extensions can be allowed. All imaginations are not realizable!

3. Extension is characterized partially by Galois group (see `http://tinyurl.com/mrvqhz2`) acting as automorphisms meaning that Galois group permutes the roots of the n:th order polynomials defining extensions of rationals via their non-rational roots. So called ramified primes (see `http://tinyurl.com/m32nvcz` and `http://tinyurl.com/oh7tgsw`) provide additional characteristics.

   Iteration cycles appearing in complexity theory for iteration of functions and repeated action of an element Galois group defining a finite Abelian group are mathematically similar notions. Now only cycles are present whereas chaotic systems have aperiodic orbits. The cyclic subgroups of Galois group do not seem to have an natural realization as iterative dynamics except in quantum sense meaning that cyclic orbits are replaced with wave functions labelled by number theoretic integer valued "momenta" for the action of the analog of Cartan subgroup as maximal commutative subgroup for the Galois group. The maximal Abelian Galois group is analog of Cartan subgroup for Galois group of algebraic numbers and states are in its irreducible representations.

   Remark: What is interesting that for polynomials with order larger than 4, one cannot write closed analytic expressions for the roots of the polynomials. This obviously means a fundamental limitation on symbolic cognitive representations provided by explicit formulas. The realization of was a huge step in the evolution of mathematics. Could also the emergence of Galois groups with order larger at space-time level than 5 have meant cognitive revolution - probably at much lower level in the hierarchy? Could this relate also to the fact that space-time dimension is $D = 4$ and thus imaginable using 4-D algebraic extension of rationals?

   A possible measure for the cognitive complexity is the dimension of the Galois group of the extension. One can speak also about the complexity of the Galois group itself - the non-Abelianity of Galois group brings in additional complexity. The number of generators and number of relations between them serve as a measure for complexity of Galois group.

   Extension of rationals is also characterized by so called ramified primes and should have a profound physical meaning. p-Adic length scale hypothesis states that physically preferred primes are near powers of 2 and perhaps also other small primes. Could they correspond to ramified primes. Why just these ramified primes would be survivors in the number theoretic evolution, is the fascinating question to be addressed later.

4. The increase of the dimension of extension or complexity of its Galois group corresponds naturally to evolution interpreted as emergence of algebraic complexity and evolutionary paths could be seen as sequences of inclusions for Galois groups. Chaos would correspond to the limit when the extension of rationals approaches to infinite sub-field of algebraic numbers - say maximal Abelian extension of rationals - so that the number of points in the cognitive representation becomes infinite.

The Galois group of algebraic numbers - the magic Absolute Group - would characterize this limit as a kind of never achievable mathematical enlightenment. A more practical definition would be that external system is experienced as complex, when its number theoretical complexity exceeds that of the conscious observer so that it is impossible to form a faithful cognitive representation about the system. Note that these cognitive representations could be formulated as homomorphisms between Galois groups. This would suggest a rather nice category theoretical picture about cognitive representations in the self hierarchy.

5. Galois group acts on the cognitive representation associated with the space-time sheet and in general gives n-fold covering of the space-time sheet: n is naturally the order of Galois group since Galois group acts on the discretization and implies n-sheeted structure for it and therefore also for the space-time surface.

   The value of the effective Planck constant assigned with dark matter as phases of ordinary matter $h_{eff}/h = n$ was identified from very beginning as number of sheets for some kind of covering space of imbedding space. n would correspond to the order of Galois group for discretized imbedding space consisting of points with coordinates in extension of rational. The increase of h_{eff} corresponds to the emergence of also cognitive complexity. Physically it is accompanied by the emergence of quantum coherence and non-locality in increasingly long scales.

3.2 General vision about evolution as emergence of complexity

Evolution would mean emergence of number theoretical complexity. Evolutionary paths would naturally correspond to sequences of inclusions (note that recent view allows also temporary "de-evolutions" but in statistical sense evolution occurs). There are infinitely many evolutionary pathways of this kind.

There is a strong resemblance with the inclusion sequences of hyper-finite factors of type II_1 (HHFs) for von Neumann algebras [6] also playing a central role in TGD and assignable to a fractal hierarchy of isomorphic sub-algebras of super-symplectic algebra associated with the isometries of WCW and related Kac-Moody algebras. It is difficult to believe that this could be an accident.

Evolution must mean a discrete time evolution of some kind - most naturally by non-deterministic quantum version of discrete dynamics, which can be deterministic only in statistical sense. By QCC this evolution should have classical correlates at space-time level. ZEO and TGD inspired theory of consciousness, which can be regarded as a generalization of quantum measurement theory in ZEO, is essential in attempts to concretize this intuition.

1. Galois group codes for the complexity and evolution means the emergence of increasingly complex Galois groups assignable to spacetime surface in a sector of WCW for which WCW coordinates are in corresponding extension of rationals. One can say that evolution defines a path in the space of sectors of WCW characterized by Galois groups. Although the space-time dynamics is expected to be integrable, the notion of complexity still has meaning, and ultimate chaos would emerge at the limit of algebraic numbers as extension of rationals.

2. One can assign Galois group G_5 to space-time surface. Suppose that one an assign Galois groups $G_3 \subset G_4$ with the 3-surfaces at the ends of space-time surfaces at boundaries of CD. This point will be discused below in more detail.

3. At quantum level conscious entities - selves - correspond to sequences off steps consisting of unitary evolution followed by a localization in the moduli space of CD. State function reduction to the opposite boundary of CD means death of self and re-incarnation of self with opposite arrow of time: also this means localization to a definite sector of WCW [21, 20]. The sequence of pairs of selves and their time reversals associated with the opposite boundaries of CD (, which itself increases in size) defines a candidate for the non-deterministic quantum analog of iteration in complexity theory.

4. There is a temptation to assume that for the passive boundary of CD all 3-surfaces in quantum superposition have same G_3 - the G_3 that emerged in the first state function reduction to the passive boundary when this self was born. G_3 so would be automatically measured observable and sequence of reductions would define a sequence of G_3s analogous to iteration sequence and also to evolution.

   But can one assume that G_3 is measured automatically in the re-incarnation of self as its time-reversal [1, 16]? Could only some charateristics of G_3 - say order $n = h_{eff}/h$ - be measured? Also ramified primes characteristize extensions and their measurement is also possible and proposed to characterize elementary particles: they do not fix G_3. These uncertainties are not relevant for the general vision.

5. For the active boundary one would have a superposition of 3-surfaces with different Galois groups and the sequence of the steps consisting of unitary evolution followed by a localization in the moduli space of CDs including also a localization in clock time determined by distance between the tips of CD. Also this would give to quantal discrete dynamics. Also now one can wonder whether Galois group is measured or not. If not, one would have a dispersion like process in the space of Galois groups labelling sectors of WCW.

6. Also the evolution of the tensor net defined by fermionic strings connecting the positions of fermions at partonic 2-surfaces would define a discrete dynamics in the space of these networks both at classical and quantum level [18]. The dynamics of many-fermion states would determine this evolution.

In the sequel this picture is discussed in more detail.

3.2.1 How can one assign an extension of rationals to WCW, imbedding space, and a region of space-time surface?

What fixes the extension used at both WCW level, imbedding space level, and space-time level? The natural assumption is that the extension used for WCW coordinates induces the extension used at imbedding space level and space-time level. At the level of space-time surfaces WCW coordinates appear as moduli (parameters) characterizing preferred extremals and would have values in an extension of rationals characterizing the adele by inducing the extensions of p-adic sectors.

1. The simplest option is that the extension is dictated by WCW. Preferred WCW coordinates - made possible by maximal isometries and fixed apart from the isometries of WCW - are in the extension: this makes the space of allowed 3-surfaces discrete. This in turn induces a constraint on space-time surfaces: WCW coordinates define parameters characterizing the space-time surface as a preferred extremal. One could use also other coordinates of WCW but these would not be optimal as cognitive representations.

   This applies also at the level of imbedding space. Contrary to what I first thought, it is not actually absolutely necessary to use preferred space-time coordinates (subset of imbedding space coordinates) since cognitive representation depends on coordinates in finite measurement resolution: consider only spherical and Cartesian coordinates with given resolution defining different discretizations. The preferred coordinates would be preferred because they are cognitively optimal.

2. Real imbedding space is replaced with a discrete set of points of H with preferred coordinates in an extension of rationals. The direct identification of the points of extension as real numbers with p-adic numbers is extremely discontinuous although it would respect algebraic symmetries. The situation is saved by the lower dimensionality of space-time surfaces for which the set of points with coordinates in extension is discrete and even finite in the generic case. The surface $x^n + y^n = z^n$ has only one rational point for $n > 2$! $D = 4 < 8$ for space-time surfaces automatically brings in finite measurement resolution and cognitive resolution induced directly from the restriction on WCW parameters.

SH has as data the intersection plus string world sheets (SH). String world sheets are in the intersection of reality and p-adicities defined by rational functions with coefficients of polynomials in extension, and makes sense both in real and p-adic sense. To these initial data one can assign as a preferred extremal of Kähler action a smooth p-adic space-time surface such that each point is contained in an open set consisting of points with p-adic coordinates having norm smaller than some power of p. This extremal is not unique in the p-adic sectors. In real sector it might not exist at all as already discussed.

3. 3-surface is seen as pair of 3-surfaces assigned to the ends of the space-time surface at boundaries of CD. WCW coordinates parameterize this pair and correspond to extension in 4-D sense. These parameters are expected to decompose to sets of parameters characterizing the 3-D members of pair and parameters characterizing the connecting space-time surface unless it is unique. If so, one can assign to the initial and final 3-surfaces subsets of WCW coordinates.

   The extensions associated with the ends of CD would be extensions in 3-D sense and sub-extensions of the extension in 4-D sense. Hence one can say that classical space-time evolution connecting initial and final 3-surfaces can modify the extension, its Galois group, and therefore also $h_{eff}/h = n$. This would be the classical view a about number theoretic evolution and also about quantum critical fluctuation changing the value of $h_{eff}/h = n$.

4. The extension of rationals for WCW coordinates induces the cognitive representation posing constraints of p-adic space-time surfaces. Adelic sub-WCW consisting of preferred extremals inside given CD decomposes to sectors characterized by an extension of rationals and evolution should correspond number theoretically to a path in the space of WCW sectors.

   This is a restriction on p-adic space-time sheets and thus cognition: the larger the number of points in the intersection, the more precise the cognitive representation is. The increase of the dimension of extension implies that the number of points of cognitive representation increases and it becomes more precise. The cognitive abilities of the system evolve. p-Adic pseudo constants allow imagination but also make the representation imprecise in scales below that defined by the cognitive representation. The continuation to smooth p-adic surface would however explain the highly non-trivial fact that we automatically tend to associate continuous structures with discrete data.

5. The fermions at partonic 2-surfaces are at positions for which preferred space-time coordinates are in extension and can be said to actualize the cognitive representation. It turns out that these positions could naturally correspond to the singularities of the space-time surfaces as n-fold covering in the sense that the dimension of the orbit of Galois group would be reduced at these points.

3.2.2 Can one assign the analog of discrete dynamics to TGD at fundamental level?

Could one assign a discrete symbolic dynamics to classical and quantum TGD?

At classical level the dynamics would correspond to space-time surface connecting the boundaries of CD and 3-surfaces at them. As already explained, the WCW coordinates characterizing space-time surface as a preferred extremal correspond to what might be called Galois group in 4-D sense. These coordinates decompose to coordinates characterizing the coordinates at the 3-surfaces at the ends of of space-time at boundaries of CD in extensions characterized by Galois groups in 3-D sense - the initial and final Galois group. The classical evolutionary step would be a step leading from the initial fo final Galois group serving as classical correlate for quantum evolution.

What about quantum level?

1. One expects that zero energy state in general is a superposition of space-time surfaces with different Galois groups in 4-D sense, G_4. The Galois groups in 3-D sense - G_3 - assignable to the ends of space-time surface would be sub-groups of G_4. If the first state function reduction to the opposite

boundary of CD involves a localization to a sector of WCW having same G_3 at passive boundary for all 3-surfaces in the superposition.

Subsequent reductions at opposite boundaries would define evolutionary pathway in the space of Galois groups G_3 leading in statistical sense to the increase of complexity.

2. The original vision was that Negentropy Maximization Principle (NMP) [3] is needed as a separate principle to guarantee evolution but adelic physics implies it in statistical sense automatically [21]. There is infinite number of extensions more complex than given one and only finite number of them less complex.

3. At quantum level the basic notion is self. It corresponds to a discrete sequence steps consisting of unitary evolution followed by a localization in the moduli space of CDs. This would correspond to a dispersion in WCW to sectors characterized by different Galois groups G_4 and G_3 associated with the 3-surface at active boundary. As explained, the state function reduction to the opposite boundary of CD analogous to a halting of quantum computation would correspond to a localization to a sector with definite Galois group G_3.

4. These time discrete time evolutions are non-deterministic unlike the dynamical evolutions studied in chaos theory defined by differential equations or iteration of function. The sequence of unitary time evolutions involving localization in the moduli of CD would however give rise to a quantum analog of iteration and one can ask whether the quantum counterparts for the notions of cycle, super-stable cycle etc... could make sense for the quantum superpositions of 4-surfaces involved. One expects dispersion in the space of Galois groups so that this idea does not look promising. One can also wonder if the sequence of unitary transformations could lead to some kind of asymptotic self-organization pattern before the first state function reduction to the opposite boundary of CD.

It is natural to consider also the evolution of the cognitive representation itself both at the space-time level and forced by the change of the many-fermion state and at quantum level.

1. For a given preferred extremal cognitive representation defines a discrete set of points in an extension of rationals and the number of points in the extension increases as it grows. The positions of fermions at partonic 2-surfaces define the nodes of a graph with strings connecting fermions at different partonic 2-surfaces serving as edges. Evolution of fermionic state changes the topology of this network by adding vertices and changing the connection.

   One can assign a complexity theory to these graphs. A connection with tensor nets [18] emerging in the description of quantum complexity is highly suggestive. The nodes of the tensor net would correspond to fermions at partonic 2-surfaces. As the number of fermions increases, the complexity of this network increases and also the space-time surface itself becomes more complex. The maximum number of fermions increases with the dimension of extension.

   An interesting proposal is that fermion lines are accompanied by magnetic flux tubes taking the role of wormholes in ER-EPR correspondence (see `http://tinyurl.com/hzqlo6r`), which emerged more than half decade after its TGD analog. The discrete evolution of many-fermion state in state function reductions in the fermionic sector induces the evolution of this network.

2. In the case of graphs one can speak about various kinds of cycles, in particular Hamiltonian cycles going through all points of graph and having no self-intersections. Interestingly, Hamiltonian cycles for icosahedron (here the isometry group of icosahedron is involved as an additional structure) lead to a vision about genetic code and music harmonies [17].

3. An interesting question concerns the extensions of rationals having as Galois group the isometry groups of Platonic solids: they probably exist. One can also consider the counterparts of Galois groups as discrete subgroups of the Galois group $SO(3)$ of quaternions. They emerge naturally

for algebraic discretizations of M^4 regarded as a subspace of complexified quaternions with time axis identified as the real axis for quaternions (for $M^8 - H$ correspondence [5, 15] see `http://tinyurl.com/mdvazmr`). Platonic solids correspond to *finite* discretizations with finite isometry groups belonging to a hierarchy of finite discrete subgroups of $SO(3)$ labelling the hierarchy of inclusions of HFFs: a connection between HFFs and quaternions is suggestive. For HFFs Platonic solids are in unique role in the sense that only for them the action of $SO(3)$ is genuinely 3-D. In Mac Kay correspondence they correspond to exceptional groups.

For this generalization evolution would correspond to evolution in the space of Galois groups for finite-dimensional extensions of rational valued quaternions. p-Adic quaternions do not however form a field since p-adic quaternion can have vanishing norm squared.

4. The wave functions in the Galois group G reduce to wave functions in its coset space G/H if they are invariant under subgroup H. One can also perform the analog of second quantization for fermions in Galois group labelling the space-time sheets (or those of 3-space). In the model of harmony based on Hamilton's cycles the notes of 12-note scale would correspond to vertices of icosahedron obtained as coset space of I/Z_5, where I is icosahedral group with 60 elements. 3-chords of the harmony for a given Hamiltonian cycle would correspond to faces, which are triangles. Single particle fermion states localized at vertices (points of coset space) would correspond to notes of the scale and 3-fermion states localized at vertices of triangle to allowed 3-chords. The observation that one can understand the degeneracies of vertebrate genetic code by introducing besides icosahedron also tetrahedron suggests that both music and genetic code could relate directly to cognition described number theoretically.

5. It is also known that graphs can be identified as representations for Boolean statements (see `http://tinyurl.com/myrkhny`). Many-fermion states represent in TGD framework quantum Boolean statements with fermion number taking the role of bit. Could it be that this graphs indeed represent entanglement many-fermion states having interpretation as quantum Boolean statements?

Can one imagine a quantum counterpart of iteration cycle? The space-time sheets can be seen as covering spaces with the number of sheets equal to the order $n = h_{eff}/h$ of Galois group. This gives additional discrete degrees of freedom and one could have wave functions in Galois group and also in its cyclic subgroup. These might serve as quantum counterparts for iteration cycles. An open question is whether n is always accompanied by $1/n$ fractionization of quantum numbers so that dark elementary particles would have same quantum numbers as ordinary ones but could be said to decompose to n pieces corresponding to sheets of covering.

One can also imagine that the cycles appear in the statistical description. At this limit one obtains deterministic kinetic equations and by their non-linearity one expects that they exhibit chaotic behavior in the usual sense.

3.3 Why would primes near powers of two (or small primes) be important?

p-Adic length scale hypothesis states that physically preferred p-adic primes come as primes near prime powers of two and possibly also other small primes. Does this have some analog to complexity theory, period doubling, and with the super-stability associated with period doublings?

Also ramified primes characterize the extension of rationals and would define naturally preferred primes for a given extension.

1. Any rational prime p can be decomposes to a product of powers P_i^k of primes P_i of extension given by $p = \prod_i P_i^{k_i}$, $\sum k_i = n$. If one has $k_i \neq 1$ for some i, one has ramified prime. Prime p is Galois invariant but ramified prime decomposes to lower-dimensional orbits of Galois group formed by a subset of $P_i^{k_i}$ with the same index k_i . One might say that ramified primes are more structured and informative than un-ramified ones. This could mean also representative capacity.

2. Ramification has as its analog criticality leading to the degenerate roots of a polynomial or the lowering of the rank of the matrix defined by the second derivatives of potential function depending on parameters. The graph of potential function in the space defined by its arguments and parameters if n-sheeted singular covering of this space since the potential has several extrema for given parameters. At boundaries of the n-sheeted structure some sheets degenerate and the dimension is reduced locally . Cusp catastrophe with 3-sheets in catastrophe region is standard example about this.

   Ramification also brings in mind super-stability of n-cycle for the iteration of functions meaning that the derivative of n:th iterate $f(f(...)(x) \equiv f^{n)}(x)$ vanishes. Superstability occurs for the iteration of function $f = ax + bx^2$ for $a = 0$.

3. I have considered the possibility that that the n-sheeted coverings of the space-time surface are singular in that the sheet co-incide at the ends of space-time surface or at some partonic 2-surfaces. One can also consider the possibility that only some sheets or partonic 2-surfaces co-incide.

   The extreme option is that the singularities occur only at the points representing fermions at partonic 2-surfaces. Fermions could in this case correspond to different ramified primes. The graph of $w = z^{1/2}$ defining 2-fold covering of complex plane with singularity at origin gives an idea about what would be involved. This option looks the most attractive one and conforms with the idea that singularities of the coverings in general correspond to isolated points. It also conforms with the hypothesis that fermions are labelled by p-adic primes and the connection between ramifications and Galois singularities could justify this hypothesis.

4. Category theorists love structural similarities and might ask whether there might be a morphism mapping these singularities of the space-time surfaces as Galois coverings to the ramified primes so that sheets would correspond to primes of extension appearing in the decomposition of prime to primes of extension.

   Could the singularities of the covering correspond to the ramification of primes of extension? Could this degeneracy for given extension be coded by a ramified prime? Could quantum criticality of TGD favour ramified primes and singularities at the locations of fermions at partonic 2-surfaces?

   Could the fundamental fermions at the partonic surfaces be quite generally localize at the singularities of the covering space serving as markings for them? This also conforms with the assumption that fermions with standard value of Planck constants corresponds to 2-sheeted coverings.

5. What could the ramification for a point of cognitive representation mean algebraically? The covering orbit of point is obtained as orbit of Galois group. For maximal singularity the Galois orbit reduces to single point so that the point is rational. Maximally ramified fermions would be located at rational points of extension. For non-maximal ramifications the number of sheets would be reduced but there would be several of them and one can ask whether only maximally ramified primes are realized. Could this relate at the deeper level to the fact that only rational numbers can be represented in computers exactly.

6. Can one imagine a physical correlate for the singular points of the space-time sheets at the ends of the space-time surface? Quantum criticality as analogy of criticality associated with super-stable cycles in chaos theory could be in question. Could the fusion of the space-time sheets correspond to a phenomenon analogous to Bose-Einstein condensation? Most naturally the condensate would correspond to a fractionization of fermion number allowing to put n fermions to point with same M^4 projection? The largest condensate would correspond to a maximal ramification $p = P_i^n$.

Why ramified primes would tend to be primes near powers of two or of small prime? The attempt to answer this question forces to ask what it means to be a survivor in number theoretical evolution. One can imagine two kinds of explanations.

1. Some extensions are winners in the number theoretic evolution, and also the ramified primes assignable to them. These extensions would be especially stable against further evolution representing analogs of evolutionary fossils. As proposed earlier, they could also allow exceptionally large cognitive representations that is large number of points of real space-time surface in extension.

2. Certain primes as ramified primes are winners in the sense the further extensions conserve the property of being ramified.

   (a) The first possibility is that further evolution could preserve these ramified primes and only add new ramified primes. The preferred primes would be like genes, which are conserved during biological evolution. What kind of extensions of existing extension preserve the already existing ramified primes. One could naively think that extension of an extension replaces P_i in the extension for $P_i = Q_{ik}^{k_i}$ so that the ramified primes would remain ramified primes.

   (b) Surviving ramified primes could be associated with a exceptionally large number of extensions and thus with their Galois groups. In other words, some primes would have strong tendency to ramify. They would be at criticality with respect to ramification. They would be critical in the sense that multiple roots appear.

   Can one find any support for this purely TGD inspired conjecture from literature? I am not a number theorist so that I can only go to web and search and try to understand what I found. Web search led to a thesis (see `http://tinyurl.com/mkhrssy`) studying Galois group with prescribed ramified primes.

   The thesis contained the statement that not every finite group can appear as Galois group with prescribed ramification. The second statement was that as the number and size of ramified primes increases more Galois groups are possible for given pre-determined ramified primes. This would conform with the conjecture. The number and size of ramified primes would be a measure for complexity of the system, and both would increase with the size of the system.

   (c) Of course, both mechanisms could be involved.

Why ramified primes near powers of 2 would be winners? Do they correspond to ramified primes associated with especially many extension and are they conserved in evolution by subsequent extensions of Galois group. But why? This brings in mind the fact that $n = 2^k$-cycles becomes super-stable and thus critical at certain critical value of the control parameter. Note also that ramified primes are analogous to prime cycles in iteration. Analogy with the evolution of genome is also strongly suggestive.

3.4 $h_{eff}/h = n$ hypothesis and Galois groups

The natural hypothesis is that $h_{eff}/h = n$ equals to the order of Galois group in the case that it gives the number of sheets of the covering assignable to the space-time surfaces. The stronger hypothesis is that $h_{eff}/h = n$ is associated with flux tubes and is proportional to the quantum numbers associated with the ends.

1. The basic idea is that Mother Nature is theoretician friendly. As perturbation theory breaks down, the interaction strength expressible as a product of appropriate charges divided by Planck constant, is reduced in the phase transition $\hbar \rightarrow \hbar_{eff}$.

2. In the case of gravitation $GMm \rightarrow= GMm(h/h_{eff})$. Equivalence Principle is satisfied if one has $\hbar_{eff} = \hbar_{gr} = GMm/v_0$, where v_0 is parameter with dimensions of velocity and of the order of some rotation velocity associated with the system. If the masses move with relativistic velocities the interaction strength is proportional to the inner product of four-momenta and therefore to Lorentz boost factors for energies in the rest system of the entire system. In this case one must assume quantization of energies to satisfy the constraint or a compensating reduction of v_0. Interactions

strength becomes equal to $\beta_0 = v_0/c$ having no dependence on the masses: this brings in mind the universality associated with quantum criticality.

3. The hypothesis applies to all interactions. For electromagnetism one would have the replacements $Z_1Z_2\alpha \to Z_1Z_2\alpha(h/h_{em})$ and $\hbar_{em} = Z_1Z_2\alpha/\beta_0$ giving Universal interaction strength. In the case of color interactions the phase transition would lead to the emergence of hadron and it could be that inside hadrons the valence quark have $h_{eff}/h = n > 1$. In this case one could consider a generalization in which the product of masses is replaced with the inner product of four-momenta. In this case quantization of energy at either or both ends is required. For astrophysical energies one would have effective energy continuum.

This hypothesis suggests the interpretation of $h_{eff}/h = n$ as either the dimension of the extension or the order of its Galois group. If the extensions have dimensions n_1 and n_2, then the composite system would be n_2-dimensional extension of n_1-dimensional extension and have dimension $n_1 \times n_2$. This could be also true for the orders of Galois groups. This would be the case if Galois group of the entire system is free group generated by the G_1 and G_2. One just takes all products of elements of G_1 and G_2 and assumes that they commute to get $G_1 \times G_2$.

Consider gravitation as example.

1. The order of Galois group should coincide with $\hbar_{eff}/\hbar = n = \hbar_{gr}/\hbar = GMm/v_0\hbar$. The transition occurs only if the value of $\hbar_{gr}/\hbar$ is larger than one. One can say that the order of Galois group is proportional the product of masses using as unit Planck mass. Rather large extensions are involved and the number of sheets in the Galois covering is huge.

   Note that it is difficult to say how larger Planck constants are actually involved since by gravitational binding the classical gravitational forces are additive and by Equivalence principle same potential is obtained as sum of potentials for splitting of masses into pieces. Also the gravitational Compton length $\lambda_{gr} = GM/v_0$ for m does not depend on m at all so that all particles have same $\lambda_{gr} = GM/v_0$ irrespective of mass (note that v_0 is expressed using units with $c = 1$).

   The maximally incoherent situation would correspond to ordinary Planck constant and the usual view about gravitational interaction between particles. The extreme quantum coherence would mean that both M and m behave as single quantum unit. In many-sheeted space-time this could be understood in terms of a picture based on flux tubes. The interpretation for the degree of coherence is discussed in terms of flux tube connections mediating gravitational flux is discussed in [11].

2. h_{gr}/h would be order of Galois group, and there is a temptation to associated with the product of masses the product $n = n_1n_2$ of the orders n_i of Galois groups associated masses M and m. The order of Galois group for both masses would have as unit $m_P/\sqrt{\beta_0}$, $\beta_0 = v_0/c$, rather than Planck mass m_P. For instance, the reduction of the Galois group of entire system to a product of Galois groups of parts would occur if Galois groups for M and m are cyclic groups with orders with have no common prime factors but not generally.

   The problem is that the order of the Galois group associated with m would be smaller than 1 for masses $m < m_P/\sqrt{\beta_0}$. Planck mass is about 1.3×10^{19} proton masses and corresponds to a blob of water with size scale 10^{-4} meters - size scale of a large neuron so that only above these scale gravitational quantum coherence would be possible. For $v_0 < 1$ it would seem that even in the case of large neurons one must have more than one neurons. Maybe pyramidal neurons could satisfy the mass constraint and would represent higher level of conscious as compared to other neurons and cells. The giant neurons discovered by the group led by Christof Koch in the brain of of mouse having axonal connections distributed over the entire brain might fulfil the constraint (see `http://tinyurl.com/gvwggsc`).

3. It is difficult to avoid the idea that macroscopic quantum gravitational coherence for multicellular objects with mass at least that for the largest neurons could be involved with biology. Multicellular

systems can have mass above this threshold for some critical cell number. This might explain the dramatic evolutionary step distinguishing between prokaryotes (mono-cellulars consisting of Archaea and bacteria including also cellular organelles and cells with sub-critical size) and eukaryotes (multi-cellulars).

4. I have proposed an explanation of the fountain effect appearing in super-fluidity and apparently defying the law of gravity. In this case m was assumed to be the mass of ^4He atom in case of super-fluidity to explain fountain effect [11]. The above arguments however allow to ask whether anything changes if one allows the blobs of superfluid to have masses coming as a multiple of $m_P/\sqrt{\beta_0}$. One could check whether fountain effect is possible for super-fluid volumes with mass below $m_P/\sqrt{\beta_0}$.

What about h_{em}? In the case of super-conductivity the interpretation of h_{em}/h as product of orders of Galois groups would allow to estimate the number $N = Q/2e$ of Cooper pairs of a minimal blob of super-conducting matter from the condition that the order of its Galois group is larger than integer. The number $N = Q/2e$ is such that one has $2N\sqrt{\alpha/\beta_0} = n$. The condition is satisfied if one has $\alpha/\beta_0 = q^2$, with $q = k/2l$ such that N is divisible by l. The number of Cooper pairs would be quantized as multiples of l. What is clear that em interaction would correspond to a lower level of cognitive consciousness and that the step to gravitation dominated cognition would be huge if the dark gravitational interaction with size of astrophysical systems is involved [13]. Many-sheeted space-time allows this in principle.

These arguments support the view that quantum information theory indeed closely relates not only to gravitation but also other interactions. Speculations revolving around blackhole, entropy, and holography, and emergence of space would be replaced with the number theoretic vision about cognition providing information theoretic interpretation of basic interactions in terms of entangled tensor networks [18]. Negentropic entanglement would have magnetic flux tubes (and fermionic strings at them) as topological correlates. The increase of the complexity of quantum states could occur by the "fusion" of Galois groups associated with various nodes of this network as macroscopic quantum states are formed. Galois groups and their representations would define the basic information theoretic concepts. The emergence of gravitational quantum coherence identified as the emergence of multi-cellulars would mean a major step in biological evolution.

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