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A Bianchi Type-V Cosmological Model in the $f(R,T)$ Theory of Gravity

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Abstract

In this paper we have studied Bianchi type V cosmological models in the $f(R,T)$ modified theory of gravity. To solve Einstein's field equations, we assume that the anisotropy (σ/θ) is inversely proportional to m^{th} power of scale factor a i.e. $\frac{\sigma}{\theta} \propto \frac{1}{a^m}$. Physical properties of the model have also been discussed.

Keywords: Bianchi type-V, $f(R,T)$, theory of gravity, deceleration parameter, cosmological parameter.

1. Introduction

The research on Bianchi Type-V cosmological models is more attractive than that on isotropic special cases and it permits arbitrary small anisotropy models at some momentary cosmic time. The natural generalizations of open FRW models are among different models of Bianchi type-V universes that finally become isotropic and are key players in understanding phenomena like formation of galaxies in the early universe. The quadrature form of metric functions for Bianchi Type-V cosmological models with perfect fluid and viscous fluid has been obtained by following the work of Saha [1] and Singh and Chaubey [2,3]. The solutions of EFEs for homogeneous but anisotropic models have been obtained by many authors [4-11] by using different generation techniques. Bianchi Type-V cosmological models with a constant deceleration parameter in general relativity has been presented by Singh and Baghel [12]. Some Bianchi-V models of an accelerating universe with dark energy have been studied by Kumar and Yadav [13]. Tiwari and Singh [14] have investigated Bianchi type-V cosmological models with time-evolving cosmological and gravitational constants in the presence of perfect fluid distribution.

A variety of theoretical models have been introduced to explain the nature of dark energy and accelerated expansion such as quintessence, phantom energy, k-essence, tachyon, f-essence and chaplygin gas etc. Dark energy has an important role in explaining recent cosmic observations. In view of the late time acceleration of the universe and the existence of dark energy and dark matter, many useful modified theories of gravity have been developed and studied. The primary goal of modern cosmology is to determine the large-scale structure of the Universe. The astronomical observation of type-Ia supernova experiments [15-17] suggests that the observable

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Universe is expanding; observations such as cosmic microwave background radiation [18] and large-scale structure [19] provide indirect evidence for late time accelerated expansion of the Universe.

Dark Energy can be explored by modifying the geometric part of the Einstein-Hilbert action [20]. This is considered the most efficient way among several to explore dark energy. According to the modifications, several alternative theories of gravity came into existence. $f(R)$, $f(T)$, $f(G)$ and $f(R,T)$ gravity are some of the modified theories of gravity. These models are proposed to explore dark energy and other cosmological problems. Sharif and Azeem [21] discussed the cosmological evolution for Dark Energy Models in $f(T)$ Gravity.

Out of the various models of Dark Energy, the modified gravity theories that have been able to explain the late-time accelerated expansion of the Universe are $f(R)$ gravity [22-26] and Gauss-Bonnet gravity [27-31]. Shamir [32] investigated the $f(R)$ gravity model that explains the unification of early time inflation and late time acceleration. Also, Harko *et al* [33] developed the $f(R,T)$ theory of gravity by using an arbitrary function of a Ricci scalar R and a Trace T of an energy-momentum tensor. Ahmed and Pradhan [34] have investigated the cosmological models with a cosmological constant in $f(R,T)$ gravity for different Bianchi types of spacetime.

In the past few years, the Bianchi universe has been playing a vital role in observational cosmology, since it required an addition in the WMAP data [35-37], the standard cosmological model with positive cosmological constant. According to this, the universe should approach a slightly anisotropic special geometry in spite of the inflation against generic inflationary models [38-44] in order to explain the flatness and homogeneity of the expanding universe. It is usually understood that this expansion has occurred as an exponential expansion [40-42]. In most cases, an accelerating universe is described within the framework of Friedman-Robertson-Walker (FRW) cosmology. Various authors have studied certain cases of anisotropic models and have concluded that the scenario by the FRW model, and therefore the anisotropy Bianchi models, are of particular academic interest.

Inspired by the above analysis, we propose to study cosmological models in $f(R,T)$ gravity in Bianchi type-V spacetime by considering $f(R,T) = f_1(R) + f_2(T)$. The framework of this paper follows the gravitational field equation in $f(R,T)$ modified gravity theory established in Section 2. In section 3, we present explicit field equations in $f(R,T)$ gravity for a general class of Bianchi cosmological models. In Section 4, we consider a relation $\frac{\sigma}{\theta} = \frac{k_1}{a^m}$.The solutions of the field equations and physical features of the models are discussed in detail. Section 5 depicts the summarised conclusion of the proposed work.

2. *f*(R,T) Modified Gravity Theory

As per modified gravity action can be mentioned below:

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\,d^4x + \int L_m\sqrt{-g}\,d^4x, \tag{1}$$

where $f(R,T)$ is an arbitrary function dependent on Ricci scalar R & the trace T of the stress-energy tensor of the matter while $T_{\mu\nu}$. L_m is the matter Lagrangian density. Also, the stress-energy tensor of matter can be defined as:

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta\sqrt{-g}L_m}{\delta g^{\mu\nu}}, \tag{2}$$

where trace $T = g^{\mu\nu} T_{\mu\nu}$. Assuming that the Lagrangian density L_m of matter is dependent only on the components of metric tensor $g_{\mu\nu}$ and not on its derivatives leads to

$$T_{\mu\nu} = g_{\mu\nu} L_m - 2\frac{\partial L_m}{\partial g^{\mu\nu}} \tag{3}$$

where the unit of $G = c = 1$. Previous theories suggest that Harko and Lobo [45] have explored a theory where Lagrangian density is described by an arbitrary function of R and the Lagrangian density of matter as $f(R, L_m)$. Along similar lines, Poplawski [46] has given a theory where the cosmological constant is written by the function of the trace of the stress-energy tensor $\Lambda(T)$.

Changing the action S with respect to the metric tensor components $g^{\mu\nu}$, the gravitational field equations of $f(R,T)$ gravity are obtained as

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2} f(R,T)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu) f_R(R,T)$$

$$= 8\pi T_{\mu\nu} - f_T(R,T)T_{\mu\nu} - f_T(R,T)\Theta_{\mu\nu} \tag{4}$$

with $\Theta_{\mu\nu} \equiv g^{ij}\left(\frac{\delta T_{ij}}{\delta g^{\mu\nu}}\right)$, which follows from the relation $\delta\left(\frac{g^{ij}T_{ij}}{\delta g^{\mu\nu}}\right) = T_{\mu\nu} + \Theta_{\mu\nu}$ and $\Box = \nabla^i\nabla_i$, $f_R(R,T) \equiv \frac{\partial f(R,T)}{\partial R}$, $f_T(R,T) \equiv \frac{\partial f(R,T)}{\partial T}$ and ∇_i denotes the covariant derivative. The contraction of Eq. (4) yields $f_R(R,T)R + 3\Box f_R(R,T) - 2f(R,T) = (8\pi - f_T(R,T))T - f_T(R,T)\Theta$ with $\Theta \equiv g^{\mu\nu}\Theta_{\mu\nu}$. Combining Eq. (4) and the contracted equation and eliminating the $\Box f_R(R,T)$ term from these equations, we obtain

$$f_R(R,T)\left(R_{\mu\nu} - \frac{1}{3}Rg_{\mu\nu}\right) + \frac{1}{6} f(R,T)g_{\mu\nu}$$

$$= (8\pi - f_T(R,T))\left(T_{\mu\nu} - \frac{1}{3}Tg_{\mu\nu}\right) - f_T(R,T)\left(\Theta_{\mu\nu} - \frac{1}{3}\Theta g_{\mu\nu}\right) + \nabla_\mu\nabla_\nu f_R(R,T) \tag{5}$$

On the other hand, through the covariant divergence of Eq. (1) as well as the energy-momentum conservation law $\nabla^{\mu}\left[f_R(R,T) - \frac{1}{2} f(R,T) g_{\mu\nu} + \left(g_{\mu\nu} \Box - \nabla_{\mu}\nabla_{\mu} \right) f_R(R,T) \right] \equiv 0,$

that corresponds to the divergence of the left hand side of Eq. (1), we acquire the divergence of $T_{\mu\nu}$ as

$$\nabla^{\mu} T_{\mu\nu} = \frac{f_T(R,T)}{8\pi - \frac{1}{2} f_T(R,T)} \left[\left(T_{\mu\nu} + \Theta_{\mu\nu} \right) \nabla^{\mu} \ln f_T(R,T) + \nabla^{\mu}\Theta_{\mu\nu} \right] \tag{6}$$

In addition, from $T_{\mu\nu} = g_{\mu\nu} L_m - 2\left(\frac{\partial L_m}{\partial g^{\mu\nu}} \right)$ we have

$$\frac{\delta T_{ij}}{\delta g^{\mu\nu}} = \left(\frac{\delta g_{ij}}{\delta g^{\mu\nu}} + \frac{1}{2} g_{ij} g_{\mu\nu} \right) L_m - \frac{1}{2} g_{ij} T_{\mu\nu} - 2 \frac{\partial^2 L_m}{\partial g^{\mu\nu} \partial g^{ij}} \tag{7}$$

Using the relation $\frac{\delta g_{ij}}{\delta g^{\mu\nu}} = - g_{i\gamma} g_{j\sigma} \delta_{\mu\nu}^{\gamma\sigma}$ with $\delta_{\mu\nu}^{\gamma\sigma} = \frac{\delta g^{\gamma\sigma}}{\delta g^{\mu\nu}}$,which follows from $g_{i\gamma} g^{\gamma j} = \delta_i^j$, we obtain $\Theta_{\mu\nu}$ as given by

$$\Theta_{\mu\nu} = -2T_{\mu\nu} + g_{\mu\nu} L_m - 2g^{ij} \frac{\partial^2 L_m}{\partial g^{\mu\nu} \partial g^{ij}} . \tag{8}$$

Provided that matter is regarded as a perfect fluid, the stress-energy tensor of the matter Lagrangian is given by

$$T_{\mu\nu} = (\rho + p) u_{\mu} u_{\nu} - p g_{\mu\nu} \tag{9}$$

where $u^{\mu} = (0, 0, 0, 1)$ is the four velocity in the moving coordinates that satisfies the conditions $u^{\mu} u_{\nu} = 1$ and $u^{\mu} \nabla_{\nu} u_{\mu} = 0$. ρ and p are the energy density and pressure of the fluid, respectively. With the use of equation (8), we obtain

$$\Theta_{\mu\nu} = -2T_{\mu\nu} - p g_{\mu\nu} \tag{10}$$

Since the field equations in $f(R,T)$ gravity also depend on the physical nature of the matter field (through the tensor $\Theta_{\mu\nu}$), several theoretical models can be obtained for each choice of f. Three explicit specifications of the functional form of f have been considered in Harko et al. [33]

$$f(R,T)=\begin{cases} R+2f(T) \\ f_1(R)+f_2(T) \\ f_1(R)+f_2(R)f_3(T) \end{cases}$$

The cosmological consequences for the class $f(R,T) = R + 2f(T)$ have recently been discussed in detail by many authors [47-52,53]. Shamir et al. [54] and Chaubey & Shukla [47] have discussed the Bianchi type-I & V and a general class of Bianchi models respectively in $f(R,T)$ gravity by considering $f(R,T) = R + 2f(T)$. In this paper, we are considering the cosmological consequences of the class for which $f(R,T) = f_1(R) + f_2(T)$. Our derived cosmological model is totally different and new from those of other authors mentioned here. So far, the physically important cosmological term Λ, which is a candidate for dark energy, remains less attended. So, our derived model may lead to a better understanding of the characteristics of Bianchi type-V models.

The gravitational field equation (4) becomes

$$f_1^{'}(R)R_{\mu\nu}-\frac{1}{2}f_1(R)g_{\mu\nu}+\left(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu\right)f_1^{'}(R)=$$

$$8\pi T_{\mu\nu}+f_2^{'}(T)T_{\mu\nu}+\left(f_2^{'}(T)p+\frac{1}{2}f_2(T)\right)g_{\mu\nu}, \qquad (11)$$

where the prime denotes differentiation with respect to the argument. The field equations of the standard $f(R)$ gravity can be recovered for $p = 0$ (the dust case) and $f_2(T) = 0$. We consider a particular form of the functions $f_1(R) = \lambda_1 R$ and $f_2(T) = \lambda_2 T$ where λ_1 and λ_2 are arbitrary parameters. In this article, we take $\lambda_1 = \lambda_2 = \lambda$ so that $f(R,T) = \lambda(R + T)$.

Equation (11) can now be rewritten as

$$\lambda R_{\mu\nu}-\frac{1}{2}\lambda\left(R+T\right)g_{\mu\nu}+\left(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu\right)\lambda$$

$$=8\pi T_{\mu\nu}-\lambda T_{\mu\nu}+\lambda\left(2T_{\mu\nu}+pg_{\mu\nu}\right). \qquad (12)$$

Setting $\left(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu\right)\lambda=0,$ we get

$$\lambda G_{\mu\nu}=8\pi T_{\mu\nu}+\lambda T_{\mu\nu}+\left(\lambda p+\frac{1}{2}\lambda T\right)g_{\mu\nu}, \qquad (13)$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ is the Einstein tensor. This could be rearranged as

$$G_{\mu\nu} - \left(p + \frac{1}{2}T\right) g_{\mu\nu} = \frac{8\pi + \lambda}{\lambda} T_{\mu\nu}. \qquad (14)$$

Recalling Einstein equations with cosmological constant

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = -8\pi T_{\mu\nu}. \qquad (15)$$

We choose a negative small value for the arbitrary λ so that we have the same sign of the RHS of (13) and (14); we keep this choice of λ throughout. The term $\left(p + \frac{1}{2}T\right)$ can now be regarded as a cosmological constant. Hence, we write

$$\Lambda \equiv \Lambda(T) = p + \frac{1}{2}T. \qquad (16)$$

In the past, the dependence on cosmological constant Λ on the trace of the energy momentum tensor T has been highlighted by Poplawski [46] such that the gravitational Lagrangian was a function of the trace of energy momentum tensor. Hence as per Poplawski [46], the model was denoted "$\Lambda(T)$ gravity". Furthermore, it has also been suggested (Magnano [55], Poplawski [56,57]) that cosmological data favours a variable cosmological constant that is consistent with $\Lambda(T)$ gravity. Considering the perfect fluid case, the trace $T = -3p + \rho$ for our model, Eq. (16) reduces to

$$\Lambda = \frac{1}{2}(\rho - p) \qquad (17)$$

3. Metric and Field Equations

As per [35-37], the universe cannot be considered completely symmetric with regards to modern observations. Thus to describe the universe which has symmetry than the standard FRW models. Bianchi models (which represent spatially homogeneous and anisotropic spaces) would be more appropriate. Hence the general class of Bianchi cosmological models would be as mentioned below:

$$ds^2 = dt^2 - A^2(t)\,dx^2 - e^{2\beta x}\left[B^2(t)\,dy^2 + C^2(t)\,dz^2\right] \qquad (18)$$

where β is a constant and the functions $A(t)$, $B(t)$ and $C(t)$ are the three anisotropic directions of expansion in normal three-dimensional space. The average scale factor a , the spatial volume V and the average Hubble's parameter H are defined as

$$V = a^3 = ABC\ , \tag{19}$$

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right), \tag{20}$$

Here $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble parameters in the directions of x, y and z respectively. In the following equation and at all places here onwards, the dot indicates differentiation with respect to cosmic time t. The physical quantities of observational interest in cosmology are the expansion scalar θ, declaration parameter q and shear scalar σ which are defined as

$$\theta = 3H \tag{21}$$

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \tag{22}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\,\sigma^{ij} \tag{23}$$

The anisotropy parameter (A_m) is defined as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{24}$$

Now the cosmological equations for the energy momentum tensor (9) and the metric (18) are

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\beta^2}{A^2} = \left(\frac{8\pi + \lambda}{\lambda}\right)p - \Lambda \tag{25}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\beta^2}{A^2} = \left(\frac{8\pi + \lambda}{\lambda}\right)p - \Lambda \tag{26}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\beta^2}{A^2} = \left(\frac{8\pi + \lambda}{\lambda}\right)p - \Lambda \tag{27}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{3\beta^2}{A^2}=-\left(\frac{8\pi+\lambda}{\lambda}\right)\rho-\Lambda \tag{28}$$

$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0. \tag{29}$$

In the next proceeding of the paper we take constant (β) as unity without any loss of generality. Integrating Eq. (29) and absorbing the integration constant into *B* or *C*, we obtain

$$A^2=BC, \tag{30}$$

From equations (25)-(28), we obtain the following three relations respectively:

$$\frac{A}{B}=\alpha_1\exp\left(\beta_1\int\frac{dt}{a^3}\right), \tag{31}$$

$$\frac{A}{C}=\alpha_2\exp\left(\beta_2\int\frac{dt}{a^3}\right), \tag{32}$$

$$\frac{B}{C}=\alpha_3\exp\left(\beta_3\int\frac{dt}{a^3}\right), \tag{33}$$

α_1, α_2, α_3, β_1, β_2 and β_3 are constants of integration. Finally, using $a=\left(ABC\right)^{\frac{1}{3}}$ we write the metric functions from (31−33) in explicit form as

$$A(t)=m_1a\exp\left(n_1\int a^{-3}dt\right), \tag{34}$$

$$B(t)=m_2a\exp\left(n_2\int a^{-3}dt\right), \tag{35}$$

$$C(t)=m_3a\exp\left(n_3\int a^{-3}dt\right), \tag{36}$$

where

$$m_1=\sqrt[3]{\alpha_1\alpha_2},\quad m_2=\sqrt[3]{\alpha_1^{-1}\alpha_3},\quad m_3=\sqrt[3]{\left(\alpha_2\alpha_3\right)^{-1}} \tag{37}$$

and

$$n_1 = \frac{\beta_1 + \beta_2}{3}, n_2 = \frac{\beta_3 - \beta_1}{3}, n_3 = \frac{-(\beta_2 + \beta_3)}{3} \tag{38}$$

The constants n_1, n_2, n_3 and m_1, m_2, m_3 satisfy the fallowing two relations:

$$n_1 + n_2 + n_3 = 0, \quad m_1 m_2 m_3 = 1. \tag{39}$$

Substituting Eq. (30) in Eq. (34− 36), we obtain

$$m_1 = 1,\ m_2 = m_3^{-1} = c_1,\ n_1 = 0,\ n_2 = -n_3 = c_2, \tag{40}$$

where c_1 and c_2 are constants. Again, substituting Eq. (40) in Eqs. (34−36), the quadrature form of the metric functions in terms of average scale factor a can be written as

$$A(t) = a, \tag{41}$$

$$B(t) = c_1 a \exp\left(c_2 \int \frac{dt}{a^3}\right), \tag{42}$$

$$C(t) = \frac{a}{c_1} \exp\left(-c_2 \int \frac{dt}{a^3}\right), \tag{43}$$

4. Solution of the field equations

Recent observations of CMBR (Cosmic Microwave Background Radiation) provide that the universe is expanding quite isotropically since the epoch in which it became definitively transparent to radiation. We have analyzed the anisotropy parameter dependent on scale factor, therefore study of cosmological models in which anisotropy of space existing at the beinging of universe are damped out in the course of evolution leading to isotropic phase. The anisotropy of space is given by $A_m = 6\frac{\sigma^2}{\theta^2}$.

The modified gravity field equations can be solved if we assume that anisotropy (σ/θ) is inversely proportional to the m^{th} power of scale factor a [58]. This gives

$$\frac{\sigma}{\theta} = \frac{k_1}{a^m} \tag{44}$$

where k_1 and m are constants.

From equation (44), we obtain average scale factor a as,

$$a = \left[\frac{(3-m)k}{3k_1}t + k_2\right]^{\frac{1}{3-m}} \qquad , m \neq 3 \qquad (45)$$

$$= k_3 e^{\frac{k}{3k_1}t} \qquad , m = 3 \qquad (46)$$

where k, k_1, k_2 and k_3 are constants of integration.

4.1 Properties of the Model for $m \neq 3$

For $m \neq 3$, using equations (41)-(43), we obtain

$$A(t) = \left[\frac{(3-m)k}{3k_1}t + k_2\right]^{\frac{1}{3-m}} \qquad (47)$$

$$B(t) = c_1\left[\frac{(3-m)k}{3k_1}t + k_2\right]^{\frac{1}{3-m}} \exp\left[\frac{-3k_1c_2}{mk}\left(\frac{(3-m)k}{3k_1}t + k_2\right)^{\frac{m}{m-3}}\right] \qquad (48)$$

$$C(t) = \frac{1}{c_1}\left[\frac{(3-m)k}{3k_1}t + k_2\right]^{\frac{1}{3-m}} \exp\left[\frac{3k_1c_2}{mk}\left(\frac{(3-m)k}{3k_1}t + k_2\right)^{\frac{m}{m-3}}\right] \qquad (49)$$

For this solution, the metric (18) assumes the following form after a suitable transformation of coordinates,

$$ds^2 = -dt^2 + \tau^{\frac{2}{3-m}}dx^2 + \tau^{\frac{2}{3-m}}\left[\exp\left(2\beta x - \frac{6k_1c_2}{mk}\tau^{\frac{m}{m-3}}\right)dy^2 + \exp\left(2\beta x + \frac{6k_1c_2}{mk}\tau^{\frac{m}{m-3}}\right)dz^2\right]$$

where $\tau = \frac{(3-m)k}{3k_1}t + k_2$ (50)

For the model (50) spatial volume (V), Hubble parameter (H), expansion scalar (θ), deceleration parameter (q) and shear scalar (σ) are respectively given by

$$V = a^3 = \tau^{\frac{3}{3-m}} \qquad (51)$$

$$H = \frac{k}{3k_1\tau} \qquad (52)$$

$$\theta = \frac{k}{k_1\tau} \qquad (53)$$

$$q = 2 - m \qquad (54)$$

$$\sigma = \frac{c_2}{\tau^{3/3-m}} \qquad (55)$$

Anisotropy parameter (A_m) for the model can be expressed as

$$A_m = \frac{6c_2^2 k_2^2}{k^2}\frac{1}{\tau^{2m/3-m}} \qquad (56)$$

using equations (47-49) in (25-28) and solving with (17), we obtain the expression for pressure (p), energy density (ρ) and cosmological constant (Λ) for the model (50) as

$$p(t) = c_3\frac{1}{\tau^2} - c_4\frac{1}{\tau^{2/3-m}} + c_5\frac{1}{\tau^{6/3-m}} - c_6\frac{1}{\tau^{6-m/3-m}} \qquad (57)$$

$$\rho(t) = c_7\frac{1}{\tau^{2/3-m}} + c_8\frac{1}{\tau^2} + c_5\frac{1}{\tau^{6/3-m}} - c_6\frac{1}{\tau^{6-m/3-m}} \qquad (58)$$

$$\Lambda(t) = \left(\frac{\lambda}{8\pi + 2\lambda}\right)\left\{2\beta^2\frac{1}{\tau^{2/3-m}} - \frac{mk^2}{9k_1^2}\frac{1}{\tau^2}\right\} \qquad (59)$$

From the model we observe that the spatial volume (V) is zero at $\tau = 0$ and the expansion scalar θ is infinite, which show that the universe starts evolving with zero volume at $\tau = 0$ which is big bang scenario. Hubble parameter H and shear scalar σ diverge at $\tau = 0$. From eqs. (47) - (49), we observe that the scale factors A, B and C are vanish at initial epoch $\tau = 0$. The model has singularity at τ =0 known as point type singularity. The physical quantities isotropic pressure (p) , proper energy density (ρ) and cosmological constant (Λ) diverge at τ =0. Also at τ =0, anisotropy parameter (A_m) are infinitely large indicating that the universe in the model was anisotropic at the initial time. In the limit of large times i.e. as $\tau \to\infty$, Hubble parameter H, expansion scalar θ, shear scalar σ, pressure p, density ρ and cosmological constant Λ become negligible whereas the scale factors A ,B and C become infinite. As $\tau \to\infty$, spatial volume becomes infinite and anisotropy parameter (A_m) tend to zero. This shows that the universe in the model turns isotropic at late time.

It can also be seen that $\frac{\sigma^2}{\theta^2} \to 0$ as $\tau \to\infty$ showing that the model approaches isotropy, which agrees Collins and Hawking [59]. From equation (54), we find that q is constant. For $m < 3$, q is positive and the universe will be in decelerating phase of expansion whereas for $m > 3$, q is negative which shows that the universe in the model is in accelerating phase.

4.2 Properties of the Model for $m = 3$

For $m=3$, Eqs. (41) - (43), give

$$A(t)=k_3 \exp\left(\frac{k}{3k_1}t\right) \tag{60}$$

$$B(t)=c_1 k_3 \exp\left(\frac{k}{3k_1}t\right)\exp\left(-\frac{k_1 c_2}{kk_3^3}e^{\frac{-k}{k_1}t}\right) \tag{61}$$

$$C(t)=\frac{k_3}{c_1}\exp\left(\frac{k}{3k_1}t\right)\exp\left(\frac{k_1 c_2}{kk_3^3}e^{\frac{-k}{k_1}t}\right) \tag{62}$$

For this solution the metric (18) assumes the following form after a suitable transformation of coordinates

$$ds^2 = -dt^2 + k_3^2 e^{2Nt}dx^2 + e^{2\beta x}\left[c_1^2 k_3^2 \exp\left(2Nt - \frac{4c_2}{3k_3^3}\frac{1}{N}e^{\frac{-3Nt}{2}}\right)dy^2 + \frac{1}{c_1^2}k_3^2 \exp\left(2Nt + \frac{4c_2}{3k_3^3}\frac{1}{N}e^{\frac{-3Nt}{2}}\right)dz^2\right]$$

where $N=\frac{k}{3k_1}$ (63)

In case of $m=3$, spatial volume (V), Hubble parameter (H), expansion scalar (θ), deceleration parameter (q) and shear scalar (σ) take the form

$$V = a^3 = k_3^3 e^{3Nt} \tag{64}$$

$$H = N \tag{65}$$

$$\theta = 3N \tag{66}$$

$$q = -1 \tag{67}$$

$$\sigma = \frac{c_2}{k_3^{\ 3}} e^{-3Nt} \tag{68}$$

The anisotropy parameter (A_m) appear as

$$A_m = \frac{2c_2^2}{3k_3^6 N^2} \mathrm{e}^{-6Nt} \tag{69}$$

The using equation (60-62) in (25-28) and solving with (17) we obtain the expressions for pressure (p), energy density (ρ) and cosmological constant (Λ) for the model (63) as

$$p(t) = k_4 + k_5 \exp\left(\frac{-2k}{k_1}\right)t + k_6 \exp\left(\frac{-2k}{3k_1}\right)t \tag{70}$$

$$\rho(t) = k_7 + k_5 \exp\left(\frac{-2k}{k_1}\right)t + k_8 \exp\left(\frac{-2k}{3k_1}\right)t \tag{71}$$

$$\Lambda(t) = \left(\frac{\lambda}{8\pi + 2\lambda}\right)\left[\frac{2\beta^2}{k_3^2} \exp\left(\frac{-2k}{3k_1}\right)t - \frac{k^2}{3k_1^2}\right] \tag{72}$$

Then using the exponentially form of the average scale factor we have obtained the solutions of the scale factors. The scale factor admit constant values at early times of the universe ($t \to 0$).The spatial volume V increases exponentially with cosmic time and becomes infinitely large at late times. The expansion scalar for these scale factors exhibits the constant value which is $\theta = 3N$ .This shows uniform exponential expansion for all time i.e., the universe expands homogeneously. Since $H = N$,the mean Hubble parameter is constant, whereas directional Hubble parameters are dynamical. The deceleration parameter $q = -1$ implies accelerating expansion of the universe as one can expect for exponential volumetric expansion. We also observe that σ, A_m , p, ρ and Λ decrease exponentially with cosmic time t .

Since $\frac{\sigma}{\theta} \neq 0$ at $t = 0$ and $\frac{\sigma}{\theta} = 0$ at $t = \infty$, which shows that the universe in the model attains isotropy at late time.

5. Conclusion

Evolution of Bianchi type-V cosmological model is studied in presence of perfect fluid and variable cosmological constant in $f\,(R,T)$ theory of gravity [Harko et al.33] . For each choice of the function $f(R,T)$, we get different theoretical models. In this paper, the gravitational field equation has been established by taking case (ii) into consideration. We have considered a special variation law in which the anisotropy (σ/θ) is a function of scale factor a .The model gives two types of cosmologies viz. for $m \neq 3$ and $m = 3$ respectively. Cosmology for $m \neq 3$ indicates the power law expansion of the universe whereas $m = 3$ represents the exponential expansion of the universe.

(i) The power law expansion shows the singular model where the spatial scale factors and volume scalar vanish at τ =0.The pressure, energy density and Hubble parameter are infinite at the initial epoch. As $\tau \to \infty$, the pressure (p), ρ and H tend to zero. A_m and σ are very large at initial time but decrease with cosmic time and vanish as $\tau \to \infty$ .The model shows isotropic state in the late time of its evolution. It is also observed that in the power law expansion, the value of deceleration parameter q is $2 - m$ .For $m < 3$, we obtain a decelerating phase of the universe whereas $m > 3$ gives an accelerating phase of the universe.

(ii) The exponential solution represents singularity free model of the universe. In this case the model represents uniform expansion and the volume grows exponentially with time .All the other physical parameters viz. p, ρ, σ, Λ and A_m being constant at t=0 decrease exponentially with cosmic time t and become negligible at late times except H, θ which remain constant throughout the evolution. The deceleration parameter $q = -1$ which implies that the model gives the greatest value of Hubble parameter and the fastest rate of expansion of the actual universe.

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