Article

Generalized Ghost Pilgrim Dark Energy with Sign-Changeable Interaction

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Abstract

In this paper, I have considered a Bianchi type-III space-time filled with interacting generalized ghost pilgrim dark energy and dark matter. I have considered here a sign-changeable interaction. To obtain the exact solutions of the Einstein's field equations, I have taken the relation between the expansion scalar and the shear scalar, I have also taken the relation between the Hubble parameter and the average scale factor. I have discussed the physical as well as the dynamical properties of the model along with the coincidence parameter. The results are found to be consistent with the present day observations.

Keywords: Bianchi type-III spacetime, generalized ghost pilgrim dark energy, sign-changeable interaction.

1. Introduction

The accelerated expansion of the universe is believed to be the result of some exotic dark energy, whose nature and composition is still unknown (Permutter et al. 1999; Reiss et al. 1998; Spergel et al. 2003, 2007; Copeland et al. 2006). As we know that the simplest candidate of dark energy is the cosmological constant. But it has the cosmological coincidence and fine-tuning problem. Thus, several alternative dark energy models have been proposed which can be characterized by the equation of state parameter ω .

The cosmological coincidence problem has a significant role in the dark energy cosmology. The cosmological coincidence problem is "why we are living in an epoch in which the densities of the dark energy and matter are comparable?"(Hao 2011). With the expansion of the universe, the densities of dark energy and dark matter scale differently, thus there should be ome fine-tunings. As the nature of dark energy and dark matter is still unknown, it is not possible to derive the precise form of the interaction from fundamental theory. Thus literally it is assumed that dark energy and dark matter interact with a coupling term Q. Earlier researchers (Wei et al. 2009; Sheykhi 2009; Amendola 1999, 2000; Zimdahl et al. 2001; Pavon et al.; Chimento et al. 2003; Guo et al. 2005, 2007 etc) have taken the most familiar form of interaction as $Q = 3bH\rho$, where b is a coupling constant and if b is positive means dark energy decays into dark matter,

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again negative b gives the converse situation that is the dark matter decays into dark energy. ρ is the density of dark matter, dark energy or the sum of them. These interactions are either positive or negative and cannot change their signs.

In 2010, Cai and Su have shown that the interaction term changes its sign in the approximate redshift range of $0.45 \le z \le 0.9$. Thus a sign-changeable interaction was introduced $Q = q(\alpha\dot{\rho} + 3\beta H\rho)$, where α and β are dimensionless constants, q is the deceleration parameter and H is the Hubble's parameter. In this work we have considered a composed fluid component with interaction term as $Q = q^{y}\left(3ba^{\chi}H\rho + \gamma a^{\varepsilon}\dot{\rho}\right)$, where q is the deceleration parameter, H the Hubble parameter, a the average scale factor and ρ is the energy density of the universe. y, b and γ are constants and can be determined by recent day observations. Also, χ and ε are another set of constants, which could be either positive or negative.

As mentioned earlier that different types of dark energy models are there, the holographic dark energy has been considered as an interesting candidate of dark energy. This dark energy model is proposed in the context of fundamental principle of quantum gravity (Susskind 1995). In 2012, Wei has proposed a new model of dark energy known as pilgrim dark energy based on the fact that black hole formation can be avoided through the strong repulsive force of the type of dark energy.

It is known that everything will be completely torn up before our universe ends in the big rip caused by the phantom-like dark energy. Thus the phantom-like dark energy has that kind of repulsive force which is strong enough to destroy the black hole. In the later years Babichev et al. (2004, 2005a, 2005b) has shown that accretion of phantom-like dark energy is accompanied with the gradual decrease of the black hole mass. Wei (2012) has developed cosmological parameters for pilgrim dark energywith Hubble horizon and provided different possibilities for avoiding the black hole formation. Fernandez (2012), Malekjani (2013), Zubair and Abbas (2015) have worked on different aspects of ghost and generalized ghost dark energy models. The generalized ghost dark energy density in terms of pilgrim dark energy is known as generalized ghost pilgrim dark energy.

Bianchi type cosmological models play a significant role in the description of large scale behavior of the universe.

The above discussion motivates us to study the generalized ghost pilgrim dark energy model with sign-changeable interaction between generalized ghost pilgrim dark energy and dark matter in Bianchi type-III universe. After the introduction in Sect. 2 we have discussed the metric and the field equations. In Sect. 3 the solutions of the field equations are obtained. Sect. 4 is the discussion of the physical properties of the model. Lastly the conclusions are given in Sect.5.

2. Metric and Field Equations

Let us consider here a spatially homogeneous and anisotropic Bianchi type – III metric of the form

$$ds^2 = dt^2 - A^2\, dx^2 - B^2\, e^{-2x}\, dy^2 - C^2\, dz^2 . \qquad (1)$$

where A, B and C are directional scale factors and are functions of cosmic time t.

Einstein's field equations are given by

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon}\, R = -\left(T_{\mu\upsilon} + \tau_{\mu\upsilon}\right). \qquad (2)$$

where $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar and $T_{\mu\nu}, \tau_{\mu\nu}$ are the energy momentum tensors of dark matter and generalized ghost pilgrim dark energy respectively.
The energy momentum tensor for matter and the generalized ghost pilgrim dark energy are

$$T_{\mu\nu} = \rho_m u_\mu u_\nu , \qquad \tau_{\mu\nu} = \left(\rho_{GPD} + p_{GPD}\right) u_\mu u_\nu + g_{\mu\nu}\, p_{GPD} . \qquad (3)$$

where ρ_m, ρ_{GPD} are the energy densities of matter and the generalized ghost pilgrim dark energy respectively and p_{GPD} is the pressure of the generalized ghost pilgrim dark energy.

The Einstein's field equations (2), for our model (1) together with (3) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -p_{GPD} , \qquad (4)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -p_{GPD} , \qquad (5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -p_{GPD} , \qquad (6)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = \rho_m + \rho_{GPD} , \qquad (7)$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 . \qquad (8)$$

Here we have taken the cosmological constant $\Lambda = 0$ and $c = 8\pi G = 1$.

The energy conservation for a composed fluid is given by

$$\dot{\rho}_m + \dot{\rho}_{GPD} + 3H\left(\rho_m + \rho_{GPD} + p_{GPD}\right) = 0 . \qquad (9)$$

In this present day situation, an interaction between the fluid components is very much important. So, mathematically we can write

$$\dot{\rho}_m + 3H\rho_m = Q , \qquad (10)$$

and

$$\dot{\rho}_{GPD} + 3H\left(\rho_{GPD} + p_{GPD}\right) = -Q . \qquad (11)$$

where H is the Hubble parameter and Q denotes the phenomenological interaction. The usual forms for the interaction term Q are

$$Q = 3Hb\rho_d \ , \quad Q = 3Hb(\rho_d + \rho_m) \ , \quad Q = 3Hb\rho_m , \tag{12}$$

where ρ_m, ρ_d are the energy densities of matter and dark energy respectively. From the above expressions we can say that the interactions are always positive or negative and can not change their sign in the evolution of the universe. In 2010, Cai and Su (2010) have introduced a sign-changeable interaction in the approximate redshift range of $0.45 \le z \le 0.9$. Later in 2012, Wi (2012) has proposed a new type of interaction. Motivated by Wi (2011) we have investigated the interacting modified Holographic Ricci dark energy model. (Das and Sultana, 2015)

Another phenomenological modification of the interaction term was done by Chen in 2014(Chen et al. 2014). The interaction term Q considered in their work were particular cases of more general form

$$Q = q^y \left(3ba^\chi \ H\rho + \gamma a^\varepsilon \dot{\rho}\right). \tag{13}$$

where q is the deceleration parameter, H the Hubble parameter, a the average scale factor and ρ is the energy density of the universe. y, b and γ are constants and can be determined by recent day observations. Also, χ and ε are another set of constants, which could be either positive or negative.

The generalized ghost dark energy density in terms of pilgrim dark energy is known as generalized ghost pilgrim dark energy and it has the form

$$\rho_{GPD} = \left(\alpha H + \beta H^2\right)^u . \tag{14}$$

where u is the dimensionless constant.

For the space-time (1), the physical and kinematical parameters are as follows

The scale factor $$a = (ABC)^{1/3} , \tag{15}$$

The spatial volume V is

$$V = a^3 = ABC , \tag{16}$$

The average Hubble parameter is

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{17}$$

The expressions for scalar expansion θ and shear scalar σ^2 are

$$\theta = 3H = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{18}$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{1}{6}\theta^2 , \tag{19}$$

The expression for anisotropic parameter is

$$\overline{A}_{GPD} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 . \tag{20}$$

where $H_i \ (i = x, y, z)$ are the directional Hubble parameters in the directions of x, y and z.

3. Solutions of field equations

From equation (8), we have

$$A = c_0 \, B . \quad (21)$$

where c_0 is a constant of integration.

Thus using (21), equations (4)-(8) takes the form

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -p_{GPD} , \quad (22)$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = -p_{GPD} , \quad (23)$$

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{B}\dot{C}}{BC} = \rho_m + \rho_{GPD} . \quad (24)$$

which are a system of three independent and highly non-linear differential equations in five unknowns B, C, ρ_m, ρ_{GPD} and p_{GPD}. To solve the system completely we use the conditions which are physically significant.

(i) the expansion scalar θ is proportional to the shear scalar σ . This gives

$$B = C^m . \quad (25)$$

where m is a positive constant.

(ii) Following Berman (1983) , we assume the relation between the average Hubble parameter H and average scale factor a as

$$H = na^{-1/n} . \quad (26)$$

where n is a positive constant.

Using equation (26), we get

$$a = V^{1/3} = (t + c_1)^n . \quad (27)$$

where c_1 is a constant of integration.

Again we have

$$V = ABC . \quad (28)$$

Using equations (21), (25) and (27), we have the expressions for metric coefficients as

$$A = (t + c_1)^{\frac{3mn}{1+2m}} , \quad (29)$$

$$B = (t + c_1)^{\frac{3mn}{1+2m}}, \qquad \text{(taking } c_0 = 1\text{)} \tag{30}$$

$$C = (t + c_1)^{\frac{3n}{1+2m}}. \tag{31}$$

Using equations (29), (30) and (31), the metric (1) can be written as

$$ds^2 = dt^2 - (t + c_1)^{\frac{6mn}{1+2m}} dx^2 - e^{-2x} (t + c_1)^{\frac{6mn}{1+2m}} dy^2 - (t + c_1)^{\frac{6n}{1+2m}} dz^2. \tag{32}$$

4. Discussion of the physical model

The following physical and kinematical parameters are very important for the physical discussion of the model.
The expression for the deceleration parameter q is

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{1}{H^2}\frac{\ddot{a}}{a} = \frac{1-n}{n}. \tag{33}$$

The above expression shows that for $0 < n < 1$ the model is in decelerating phase but for $n > 1$ the model accelerates.

The average Hubble's parameter is

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = n(t + c_1)^{-1}, \tag{34}$$

The scalar expansion is given by

$$\theta = 3H = 3n(t + c_1)^{-1}, \tag{35}$$

The shear scalar is

$$\sigma^2 = \frac{1}{2}\frac{9n^2}{(1+2m)^2}(2m^2 + 1)(t + c_1)^{-2} - \frac{3n^2}{2}(t + c_1)^{-2}. \tag{36}$$

From equations (34), (35) and (36), we can conclude that the average Hubble's parameter, scalar expansion and shear scalar are decreasing functions of cosmic time t.

The average anisotropy parameter is

$$\bar{A}_{GPD} = 2\frac{(m-1)^2}{(1+2m)^2}. \tag{37}$$

Using equation (34) in equation (14), the generalized ghost pilgrim dark energy density comes out to be

$$\rho_{GPD} = \left[\alpha n(t + c_1)^{-1} + \beta n^2 (t + c_1)^{-2}\right]^u. \tag{38}$$

From equation (24), using equations (30), (31) and (38), the matter energy density is found to be as

$$\rho_m = \frac{9\, m n^2 (m+1)}{(1+2m)^2}(t+c_1)^{-2} - \left[\alpha\, n (t+c_1)^{-1} + \beta n^2 (t+c_1)^{-2}\right]^u . \qquad (39)$$

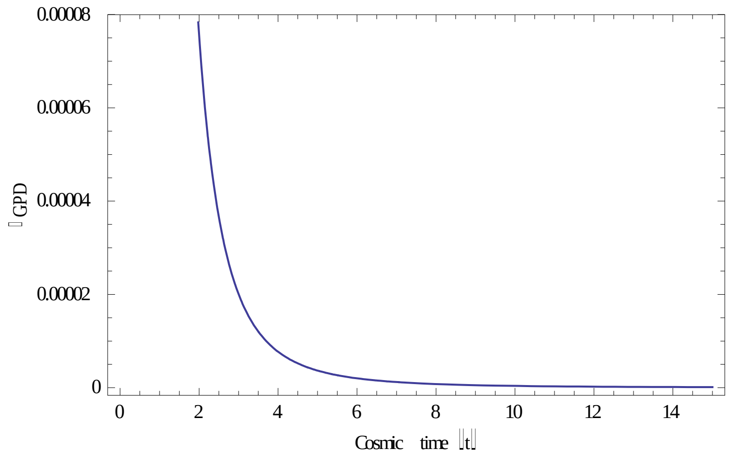

Fig.1 The plot of energy density of generalized ghost pilgrim dark energy ρ_{GPD} vs. cosmic time t with $c_1 = 0.35,\ \alpha = 0.00015,\ \beta = 0.00019,\ u = 2,\ n = 15,\ m = 75$.

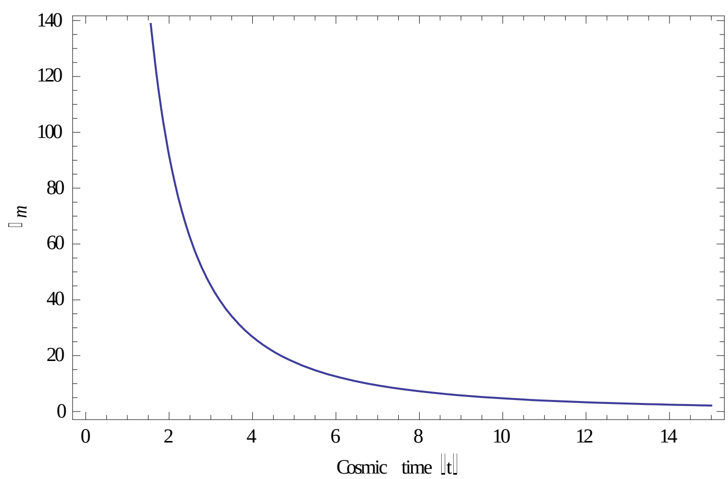

Fig.2 The plot of matter energy density ρ_m vs. cosmic time t with $c_1 = 0.35,\ \alpha = 0.00015,\ \beta = 0.00019,\ u = 2,\ n = 15,\ m = 75$.

It is observed from fig.1 and fig.2 that both the matter energy density and generalized ghost pilgrim dark energy density decrease as the universe expands and tends to zero.

From equation (11), using equations (34) and (38), the expression for the EoS parameter ω_{GPD} is found to be as

$$\omega_{GPD} = -1 - \frac{\dot{\rho}_{GPD}}{3H\,\rho_{GPD}} - \frac{Q}{3H\,\rho_{GPD}}\,. \qquad (40)$$

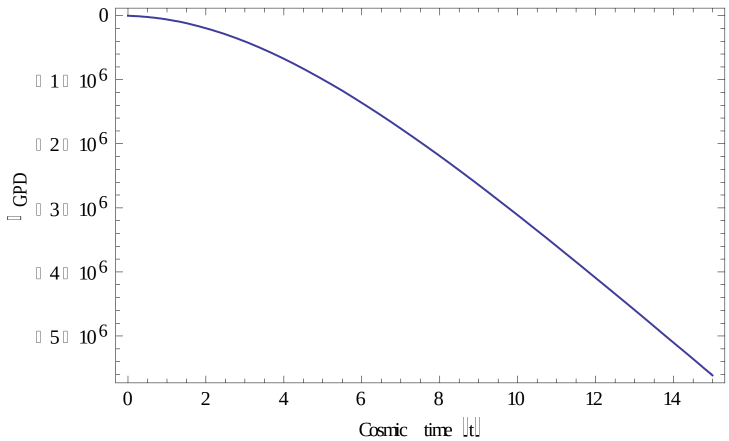

Fig.3 The plot of EoS ω_{GPD} parameter vs. cosmic time t with $c_1 = 0.35,\ \alpha = 0.00015,\ \beta = 0.00019,\ u = 2,\ n = 15,\ m = 75,$ $y = 0.5,\ \chi = 0.02,\ \varepsilon = 0.03,\ \gamma = 0.02,\ \eta = 0.13,\ b = 0.5$

Fig.3 shows that the EoS parameter decreases with time and always less than -1. Thus it behaves like phantom dark energy.

The matter energy density parameter Ω_m and the dark energy density parameter Ω_{GPD} are defined as

$$\Omega_m = \frac{\rho_m}{3H^2} \text{ and } \Omega_{GPD} = \frac{\rho_{GPD}}{3H^2}\,. \qquad (41)$$

And the total energy density parameter is given by

$$\Omega = \Omega_m + \Omega_{GPD} = \frac{3m}{(1+2m)^2}(1+m). \qquad (42)$$

From equation (42), we can conclude that the total energy density parameter $\Omega \approx 1$ as $m > 0$, which tally with the recent observational data.

The coincidence parameter, the ratio between the dark energy density and the matter energy density is given by

$$\mu = \frac{\rho_{GPD}}{\rho_m} = \frac{\left[\alpha n (t+c_1)^{-1} + \beta n^2 (t+c_1)^{-2}\right]^u}{\frac{9\,mn^2(m+1)}{(1+2m)^2}(t+c_1)^{-2} - \left[\alpha n (t+c_1)^{-1} + \beta n^2 (t+c_1)^{-2}\right]^u}\,. \qquad (43)$$

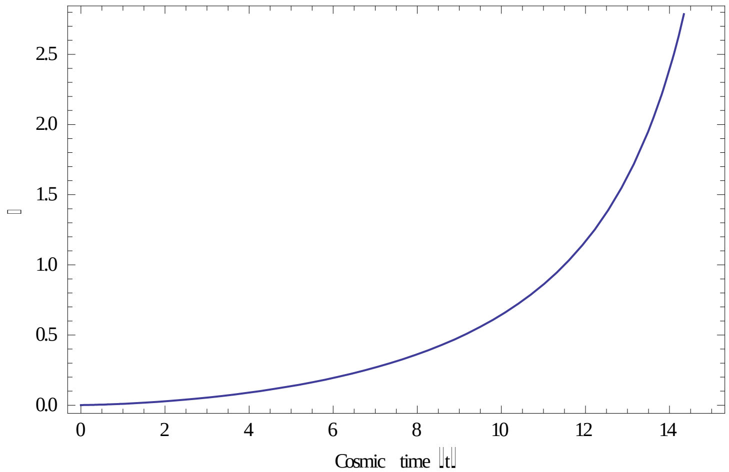

Fig 4. the coincidence parameter μ vs. cosmic time t with
$c_1 = 0.35,\ \alpha = 15,\ \beta = 0.0019,\ u = 0.2,\ n = 15,\ m = 5$

Fig 4. shows the behavior of the coincidence parameter which is an increasing function of cosmic time. The figure gives the value of the coincidence parameter $\mu \approx 2.33$at the present cosmic time $13.789(Gyr)$. So our result is consistent with the present day observations (Ade et al. 2013).

5. Conclusions

In this article we considered a generalized ghost pilgrim dark energy model with sign-changeable interaction with dark matter. To obtain the exact solutions of Einstein's field equations we assume firstly that shear scalar is proportional to the expansion scalar and secondly, we take a special form of Hubble parameter. We have also investigated some of the basic geometrical and kinematical properties of the model. The deceleration parameter q is positive for $0 < n < 1$ and hence the universe is in early decelerating phase and q is negative for $n > 1$ and so the universe is at accelerating phase· It is interesting that in this model both decelerating and accelerating phases of the universe can be explained by changing the value of n. Also it is found that $\frac{\sigma^2}{\theta^2} \neq 0$ and the mean anisotropy parameter $\overline{A}_{GPD} \neq 0$ except at $m = 1$. So our universe is anisotropic at all the time except at $m = 1$. It is clear from Fig. 1 and 2 that both the energy densities ρ_m and ρ_{GPD} tend to zero at late times.

In the accelerating phase of the universe i.e. when $n > 1$, the total energy density $\Omega \to 1$, thus at present our universe becomes flat. We have investigated the coincidence parameter which is found to be an increasing function of time (Fig.4). Recent observational data (Ade et al. 2013) shows that the value of the coincidence parameter is 2.33while the age of the universe is $13.798(Gyr.)$. Hence our results are consistent with the recent day observations.

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