Article

On Bisector Surface in Minkowski Space

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Abstract

In this paper, we study bisector surfaces of some special curves in Minkowski 3-space. First, we give properties and the basic concepts of curves in $\mathbb{E}_1^3$. Then, we construct bisector surface generated by pedal and parallel curves of given timelike curve in $\mathbb{E}_1^3$. Moreover, we show how to generate the this surface. Finally, we give an example of these surfaces in $\mathbb{E}_1^3$.

Keywords: Minkowski space, bisector surface, parallel curve, pedal curve.

1 Introduction

The Bisctor surface is a special surface because this surface is defined by any two objects in 2-dimensional or 3-dimensional space. These objects can be point-curve, curve-curve or surface-surface. Moreover, the Bisector surface is the set of points which are equidistant from the two objects, [4]. This surface is often used in scientific research in the past. For example, Horvath proved that all bisectors are topological images of a plane of the embedding Euclidean 3-space if the shadow boundaries of the unit ball K are topological circles in [8], and Elber studyed a new computational model in $\mathbb{E}^3$ in [5]. The parallel curves are developed by Chrastinova in 2007. These curves are not easy to characterize in 3-dimensional space until this time. Additionally, he study parallel curves of a special curve as helices.

In this paper, we study bisector surfaces of some special curves in Minkowski 3-space. First, we give properties and the basic concepts of curves in $\mathbb{E}_1^3$. Then, we construct bisector surface generated by pedal and parallel curves of given timelike curve in $\mathbb{E}_1^3$. Moreover, we show how to generate the this surface. Finally, we give an example of these surfaces in $\mathbb{E}_1^3$.

2 Preliminaires

Given a spatial curve $\xi : s \to \xi(s)$, which is parameterized by arc-length parameter s. For each point of $\xi(s)$, the set $\{\mathbf{t}(s), \mathbf{n}(s), \mathbf{b}(s)\}$ is called the Frenet Frame along $\xi(s)$, where $\mathbf{t}(s)$, $\mathbf{n}(s)$, $\mathbf{b}(s)$ are the unit tangent, principal normal, and binormal vectors of the curve at

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the point $\xi(s)$, respectively. Derivative of the Frenet frame according to arc-length parameter is governed by the relations;

$$\begin{pmatrix} \mathbf{e}_1'(s) \\ \mathbf{e}_2'(s) \\ \mathbf{e}_3'(s) \end{pmatrix} = \begin{pmatrix} 0 & \kappa_1(s) & 0 \\ -\kappa_1(s) & 0 & \kappa_2(s) \\ 0 & -\kappa_2(s) & 0 \end{pmatrix} \begin{pmatrix} \mathbf{e}_1(s) \\ \mathbf{e}_2(s) \\ \mathbf{e}_3(s) \end{pmatrix},$$

where

$$\kappa_1(s) = \left\|\xi''(s)\right\|, \ \kappa_2(s) = \frac{(\xi'(s),\ \xi''(s),\ \xi'''(s))}{\left\|\xi''(s)\right\|^2}.$$

Assume that $\xi(s)$ is an arbitrary timelike curve in the space $\mathbb{E}_1^3$, then, the Frenet formulae of $\xi(s)$ are given by

$$\begin{pmatrix} \mathbf{e}_1'(s) \\ \mathbf{e}_2'(s) \\ \mathbf{e}_3'(s) \end{pmatrix} = \begin{pmatrix} 0 & \kappa_1(s) & 0 \\ \kappa_1(s) & 0 & \kappa_2(s) \\ 0 & -\kappa_2(s) & 0 \end{pmatrix} \begin{pmatrix} \mathbf{e}_1(s) \\ \mathbf{e}_2(s) \\ \mathbf{e}_3(s) \end{pmatrix},$$

where

$$\langle \mathbf{e}_1, \mathbf{e}_1 \rangle = -1, \ \langle \mathbf{e}_2, \mathbf{e}_2 \rangle = \langle \mathbf{e}_3, \mathbf{e}_3 \rangle = 1.$$

Assume that Minkowski 3-space is consider $\mathbb{E}_1^3 = \left[\mathbb{E}_1^3,\ (-,+,+)\right]$ and the Lorentzian inner product and vector product, respectively, are

$$\begin{aligned} \langle X, Y \rangle &= -x_1y_1 + x_2y_2 + x_3y_3, \\ X \times Y &= (x_2y_3 - x_3y_2, x_1y_3 - x_3y_1, x_2y_1 - x_1y_2). \end{aligned}$$

where $X = (x_1, x_2, x_3)$ and $Y = (y_1, y_2, y_3) \in \mathbb{E}_1^3$, [13].

A surface in $\mathbb{E}_1^3$ is called a timelike surface if the normal vector on the surface is spacelike vector. A surface is called spacelike surface if the normal vector on surface is the timelike vector.

3 The Bisector Surface Obtained a Point and a Curve

In this paper, our goal is construct bisector surface generated by pedal and parallel curves of a given timelike curve in $\mathbb{E}_1^3$.

3.1 Bisector Surface Generated by a Point and a Parallel Curve of a Given Timelike Curve in $\mathbb{E}_1^3$

Let $\alpha : I \to \mathbb{E}_1^3$ be a timelike curve in $\mathbb{E}_1^3$ and $\mathcal{P}$ be its a parallel curve. Then, Parallel curve of a given timelike curve obtained by;

$$\mathcal{P} = \alpha - \frac{1}{\kappa_\alpha}\mathbf{n}_\alpha + \sqrt{t^2 - \kappa_\alpha^{-2}}\mathbf{b}_\alpha. \tag{3.1.1}$$

Theorem 3.1 Assume that $Q = (q_1, q_2, q_3)$ is a fixed point and $\mathcal{P}$ is a regular parallel curve of timelike curve. Then, the rational ruled bisector surface $\mathfrak{B}(s,t)$ is

$$\mathfrak{B}(s,t) = \mathfrak{B}(s) + t\mathcal{N}(s)\,; \text{ for } s,t \in IR,$$

where

$$\begin{aligned}\mathcal{N}(s) &= \mathbf{t_p} \times (\mathcal{P} - Q) \\ &= (\mathbf{t_p}^3(p_2 - q_2) - \mathbf{t_p}^2(p_3 - q_3), \\ &\quad \mathbf{t_p}^3(p_1 - q_1) - \mathbf{t_p}^1(p_3 - q_3), \\ &\quad \mathbf{t_p}^1(p_2 - q_2) - \mathbf{t_p}^2(p_1 - q_1)),\end{aligned}$$

and

$$\begin{aligned} b_1 &= \frac{1}{\mathfrak{J}}\begin{vmatrix} r_1 & \mathbf{t}_p^2 & \mathbf{t}_p^3 \\ r_2 & n_2 & n_3 \\ r_3 & (p_2 - q_2) & (p_3 - q_3)\end{vmatrix}, \\ b_2 &= \frac{1}{\mathfrak{J}}\begin{vmatrix} -\mathbf{t}_p^1 & r_1 & \mathbf{t}_p^3 \\ -n_1 & r_2 & n_3 \\ -(p_1 - q_1) & r_3 & (p_3 - q_3)\end{vmatrix}, \\ b_3 &= \frac{1}{\mathfrak{J}}\begin{vmatrix} -\mathbf{t}_p^1 & \mathbf{t}_p^2 & r_1 \\ -n_1 & n_2 & r_2 \\ -(p_1 - q_1) & (p_2 - q_2) & r_3\end{vmatrix}, \\ \mathfrak{J} &= \begin{vmatrix} -\mathbf{t}_p^1 & \mathbf{t}_p^2 & \mathbf{t}_p^3 \\ -n_1 & n_2 & n_3 \\ -(p_1 - q_1) & (p_2 - q_2) & (p_3 - q_3)\end{vmatrix}.\end{aligned}$$

Proof. Taking the derivative of the eq (3.1.1), we can computed by

$$\dot{\mathcal{P}} = \mathbf{t_p} = \left(\frac{1}{\kappa_\alpha^2} - \sqrt{t^2 - \kappa_\alpha^{-2}}\tau_\alpha\right)\mathbf{n}_\alpha + \left(\frac{1}{\kappa_\alpha^3\sqrt{t^2 - \kappa_\alpha^{-2}}} - \frac{\tau_\alpha}{\kappa_\alpha}\right)\mathbf{b}_\alpha.$$

On the other hand, let $\mathfrak{B}$ be a bisector point of $\mathcal{P}(s)$ and Q with its foot points at $\mathcal{P}(s)$ and Q, respectively. Then, it is clear that the point $\mathfrak{B}$ is contained both the normal plane $\mathcal{L}(s_0)$ and the bisector plane $\mathcal{L}_b(s_0)$. In that case $\mathcal{L}(s_0)$ and $\mathcal{L}_b(s_0)$ intersect in a line $l(s_0)$.

Assume that $\mathcal{N}(s)$ is the direction vector of $l(t)$, then it is clear that $\mathcal{N}(s)$ is contained in both $\mathcal{L}(s)$ and $\mathcal{L}_b(s)$ and it is orthogonal to the normal vectors of $\mathcal{L}(s)$ and $\mathcal{L}_b(s)$. Therefore, the following equation can be written easily

$$\begin{aligned}\mathcal{N}(s) &= \mathbf{t_p}(s) \times (\mathcal{P}(s) - Q) \\ &= (\mathbf{t_p}^3(p_2 - q_2) - \mathbf{t_p}^2(p_3 - q_3), \\ &\quad \mathbf{t_p}^3(p_1 - q_1) - \mathbf{t_p}^1(p_3 - q_3), \\ &\quad \mathbf{t_p}^1(p_2 - q_2) - \mathbf{t_p}^2(p_1 - q_1)),\end{aligned}$$

which is a rational vector field.

An auxiliary plane ($\mathcal{AP}$) $\mathcal{L}_n(s)$ is orthogonal to the intersection line $l(s)$ and passes through the fixed point Q. So $\mathfrak{B}(s)$ is the closest point of $l(s)$ to Q, $\mathcal{AP}$ can be written as:

$$\mathcal{L}_n(s): \quad \langle \mathfrak{B}-Q, \mathcal{N}(s)\rangle = 0.$$

If the above equations are considered together, we obtain the following equations for intersection point $\mathfrak{B}$:

$$\begin{aligned} \langle \mathfrak{B}, \mathbf{t_p}(s)\rangle &= \langle \mathcal{P}(s), \mathbf{t_p}(s)\rangle, \\ \langle \mathfrak{B}, \mathcal{N}(s)\rangle &= \langle Q, \mathcal{N}(s)\rangle, \\ \langle \mathfrak{B}, \mathcal{P}(s) - Q\rangle &= \frac{1}{2}(\|\mathcal{P}(s)\|^2 - \|Q\|^2). \end{aligned}$$

Then, we have the following matrix equation,

$$\begin{bmatrix} -\mathbf{t}_p^1 & \mathbf{t}_p^2 & \mathbf{t}_p^3 \\ -n_1 & n_2 & n_3 \\ -(p_1-q_1) & (p_2-q_2) & (p_3-q_3) \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}, \tag{3.1.2}$$

where

$$\begin{aligned} r_1 &= -p_1\mathbf{t}_p^1 + p_2\mathbf{t}_p^2 + p_3\mathbf{t}_p^3, \\ r_2 &= -q_1 n_1 + q_2 n_2 + q_3 n_3, \\ r_3 &= \frac{1}{2}(\|\mathcal{P}(s)\|^2 - \|Q\|^2). \end{aligned}$$

If eq. (3.1.2) can be solved, then following equation is obtained by

$$b_1 = \frac{1}{\mathfrak{J}} \begin{vmatrix} r_1 & \mathbf{t}_p^2 & \mathbf{t}_p^3 \\ r_2 & n_2 & n_3 \\ r_3 & (p_2-q_2) & (p_3-q_3) \end{vmatrix},$$

$$b_2 = \frac{1}{\mathfrak{J}} \begin{vmatrix} -\mathbf{t}_p^1 & r_1 & \mathbf{t}_p^3 \\ -n_1 & r_2 & n_3 \\ -(p_1-q_1) & r_3 & (p_3-q_3) \end{vmatrix},$$

$$b_3 = \frac{1}{\mathfrak{J}} \begin{vmatrix} -\mathbf{t}_p^1 & \mathbf{t}_p^2 & r_1 \\ -n_1 & n_2 & r_2 \\ -(p_1-q_1) & (p_2-q_2) & r_3 \end{vmatrix},$$

where $\mathfrak{J}$ is

$$\begin{vmatrix} -\mathbf{t}_p^1 & \mathbf{t}_p^2 & \mathbf{t}_p^3 \\ -n_1 & n_2 & n_3 \\ -(p_1-q_1) & (p_2-q_2) & (p_3-q_3) \end{vmatrix}.$$

The rational ruled bisector surface $\mathfrak{B}(s,t)$ can be constructed as follows:

$$\mathfrak{B}(s,t) = \mathfrak{B}(s) + t\mathcal{N}(s)\,; \text{ for } s,t \in IR.$$

3.2 Bisector Surface Generated by a Point and a Pedal Curve of a Given Timelike Curve in $\mathbb{E}_1^3$

Let $\alpha : I \to \mathbb{E}_1^3$ be a timelike curve in $\mathbb{E}_1^3$ and $\mathfrak{D}$ be a developable ruled surface given in $\mathbb{E}_1^3$. Thus, for the pedal of $\mathfrak{D}$, we can write

$$\mathfrak{T}(s) = \alpha(s) + Q(s)\,\mathbf{t}_\alpha(s)\,,\ \ \|\mathbf{t}_\alpha(s)\| = \|\dot{\alpha}(s)\| = 1, \tag{3.2.1}$$

where Q is the distance between the points $\alpha(s)$ and $\mathfrak{T}(s)$, [10].

Theorem 3.2 Assume that $\mathfrak{M} = (m_1, m_2, m_3)$ is a fixed point and $\mathfrak{T}$ is a regular pedal curve of timelike curve. Then the rational ruled bisector surface $\mathfrak{B}_2(s,t)$ is

$$\mathfrak{B}_2(s,t) = \mathfrak{B}_2(s) + t\mathfrak{N}(s)\,;\ \text{for } s,t \in IR,$$

where

$$\begin{aligned}
\mathfrak{N}(s) &= \mathbf{t}_{\mathfrak{T}}(s) \times (\mathfrak{T}(s) - \mathfrak{M}) \\
&= (\mathbf{t}_{\mathfrak{T}}^3(t^2 - m^2) - \mathbf{t}_{\mathbf{p}}^2\left(t^3 - m^3\right), \\
&\quad \mathbf{t}_{\mathfrak{T}}^3\left(t^1 - m^1\right) - \mathbf{t}_{\mathbf{p}}^1(t^3 - m^3), \\
&\quad \mathbf{t}_{\mathfrak{T}}^1(t^2 - m^2) - \mathbf{t}_{\mathbf{p}}^2\left(t^1 - m^1\right)),
\end{aligned}$$

and

$$\begin{aligned}
b_1 &= \frac{1}{\mathfrak{U}}\begin{vmatrix} r^1 & \mathbf{t}_{\mathfrak{T}}^2 & \mathbf{t}_{\mathfrak{T}}^3 \\ r^2 & n^2 & n^3 \\ r^3 & \left(t^2 - m^2\right) & \left(t^3 - m^3\right) \end{vmatrix}, \\
b_2 &= \frac{1}{\mathfrak{U}}\begin{vmatrix} -\mathbf{t}_{\mathfrak{T}}^1 & r^1 & \mathbf{t}_{\mathfrak{T}}^3 \\ -n^1 & r^2 & n^3 \\ -\left(t^1 - m^1\right) & r^3 & \left(t^3 - m^3\right) \end{vmatrix}, \\
b_3 &= \frac{1}{\mathfrak{U}}\begin{vmatrix} -\mathbf{t}_{\mathfrak{T}}^1 & \mathbf{t}_{\mathfrak{T}}^2 & r^1 \\ -n^1 & n^2 & r^2 \\ -\left(t^1 - m^1\right) & \left(t^2 - m^2\right) & r^3 \end{vmatrix}, \\
\mathfrak{U} &= \begin{vmatrix} -\mathbf{t}_{\mathfrak{T}}^1 & \mathbf{t}_{\mathfrak{T}}^2 & \mathbf{t}_{\mathfrak{T}}^3 \\ -n^1 & n^2 & n^3 \\ -\left(t^1 - m^1\right) & \left(t^2 - m^2\right) & \left(t^3 - m^3\right) \end{vmatrix}.
\end{aligned}$$

Proof. Taking the derivative of the eq (3.2.1), we can computed by

$$\dot{\mathfrak{T}} = \mathbf{t}_{\mathfrak{T}} = \left(1 + \dot{Q}\right)\mathbf{t}_\alpha + Q\kappa_\alpha \mathbf{n}_\alpha.$$

On the other hand, let $\mathfrak{B}_2$ be a bisector point of $\mathfrak{T}(s)$ and $\mathfrak{M}$ with its foot points at $\mathfrak{T}(s)$ and $\mathfrak{M}$, respectively. Then, it is clear that the point $\mathfrak{B}_2$ is contained both the normal plane $\mathcal{L}_2(s_0)$ and the bisector plane $\mathcal{L}_{b_2}(s_0)$. In that case $\mathcal{L}_2(s_0)$ and $\mathcal{L}_{b_2}(s_0)$ intersect in a line $l_2(s_0)$.

Assume that $\mathfrak{N}(s)$ is the direction vector of $l_2(t)$, then it is clear that $\mathfrak{N}(s)$ is contained in both $\mathcal{L}_2(s)$ and $\mathcal{L}_{b_2}(s)$ and it is orthogonal to the normal vectors of $\mathcal{L}_2(s)$ and $\mathcal{L}_{b_2}(s)$. Therefore, the following equation can be written easily

$$\begin{aligned} \mathfrak{N}(s) &= \mathbf{t}_{\mathfrak{T}}(s) \times (\mathfrak{T}(s) - \mathfrak{M}) \\ &= (\mathbf{t}_{\mathfrak{T}}^3(t^2 - m^2) - \mathbf{t}_{\mathbf{p}}^2\left(t^3 - m^3\right), \\ &\quad \mathbf{t}_{\mathfrak{T}}^3\left(t^1 - m^1\right) - \mathbf{t}_{\mathbf{p}}^1(t^3 - m^3), \\ &\quad \mathbf{t}_{\mathfrak{T}}^1(t^2 - m^2) - \mathbf{t}_{\mathbf{p}}^2\left(t^1 - m^1\right)), \end{aligned}$$

which is a rational vector field.

An auxiliary plane $(\mathcal{AP})$ $\mathcal{L}_{n_2}(s)$ is orthogonal to the intersection line $l_2(s)$ and passes through the fixed point $\mathfrak{M}$. So $\mathfrak{B}_2(s)$ is the closest point of $l_2(s)$ to Q, $\mathcal{AP}$ can be written as:

$$\mathfrak{L}_{n_2}(s): \quad \langle \mathfrak{B}_2 - \mathfrak{M}, \mathfrak{N}(s) \rangle = 0.$$

If the above equations are considered together, we obtain the following equations for intersection point $\mathfrak{B}_2$:

$$\begin{aligned} \left\langle \mathfrak{B}_2, \dot{\mathfrak{T}}(s) \right\rangle &= \left\langle \mathfrak{T}(s), \dot{\mathfrak{T}}(s) \right\rangle, \\ \langle \mathfrak{B}_2, \mathfrak{N}(s) \rangle &= \langle \mathfrak{M}, \mathfrak{N}(s) \rangle, \\ \langle \mathfrak{B}_2, \mathfrak{T}(s) - \mathfrak{M} \rangle &= \frac{1}{2}(\|\mathfrak{T}(s)\|^2 - \|\mathfrak{M}\|^2). \end{aligned}$$

Then, we have the following matrix equation,

$$\begin{bmatrix} -\mathbf{t}_{\mathfrak{T}}^1 & \mathbf{t}_{\mathfrak{T}}^2 & \mathbf{t}_{\mathfrak{T}}^3 \\ -n^1 & n^2 & n^3 \\ -\left(t^1 - m^1\right) & \left(t^2 - m^2\right) & \left(t^3 - m^3\right) \end{bmatrix} \begin{bmatrix} b^1 \\ b^2 \\ b^3 \end{bmatrix} = \begin{bmatrix} r^1 \\ r^2 \\ r^3 \end{bmatrix}, \tag{3.2.2}$$

where

$$\begin{aligned} r^1 &= -t^1\mathbf{t}_{\mathfrak{T}}^1 + t^2\mathbf{t}_{\mathfrak{T}}^2 + t^3\mathbf{t}_{\mathfrak{T}}^3, \\ r^2 &= -m^1n^1 + m^2n^2 + m^3n^3, \\ r^3 &= \frac{1}{2}(\|\mathfrak{T}(s)\|^2 - \|\mathfrak{M}\|^2). \end{aligned}$$

If eq. (3.2.2) can be solved, then following equation is obtained by

$$b_1 = \frac{1}{\mathfrak{U}} \begin{vmatrix} r^1 & \mathbf{t}_{\mathfrak{T}}^2 & \mathbf{t}_{\mathfrak{T}}^3 \\ r^2 & n^2 & n^3 \\ r^3 & \left(t^2 - m^2\right) & \left(t^3 - m^3\right) \end{vmatrix},$$

$$b_2 = \frac{1}{\mathfrak{U}} \begin{vmatrix} -\mathbf{t}_{\mathfrak{T}}^1 & r^1 & \mathbf{t}_{\mathfrak{T}}^3 \\ -n^1 & r^2 & n^3 \\ -\left(t^1 - m^1\right) & r^3 & \left(t^3 - m^3\right) \end{vmatrix},$$

$$b_3 = \frac{1}{\mathfrak{U}} \begin{vmatrix} -\mathbf{t}^1_{\mathfrak{T}} & \mathbf{t}^2_{\mathfrak{T}} & r^1 \\ -n^1 & n^2 & r^2 \\ -\left(t^1 - m^1\right) & \left(t^2 - m^2\right) & r^3 \end{vmatrix},$$

where $\mathfrak{U}$ is

$$\begin{vmatrix} -\mathbf{t}^1_{\mathfrak{T}} & \mathbf{t}^2_{\mathfrak{T}} & \mathbf{t}^3_{\mathfrak{T}} \\ -n^1 & n^2 & n^3 \\ -\left(t^1 - m^1\right) & \left(t^2 - m^2\right) & \left(t^3 - m^3\right) \end{vmatrix}.$$

The rational ruled bisector surface $\mathfrak{B}_2(s,t)$ can be constructed as follows:

$$\mathfrak{B}_2(s,t) = \mathfrak{B}_2(s) + t\mathfrak{N}(s)\,; \text{ for } s,t \in IR$$

3.3 Application

Let us consider a unit speed timelike curve in $\mathbb{E}^3_1$ by

$$\alpha = \alpha(s) = (\sqrt{2}s, \cos s, \sin s). \tag{3.3.1}$$

One can calculate its Frenet-Serret apparatus as the following, [9],

$$\begin{aligned} \mathbf{t}(s) &= (\sqrt{2}, -\sin s, \cos s), \\ \mathbf{n}(s) &= (0, -\cos s, -\sin s), \\ \mathbf{b}(s) &= (-1, \sqrt{2}\sin s, -\sqrt{2}\cos s). \end{aligned}$$

Then, the curvatures of α is given by

$$\begin{aligned} \kappa(s) &= 1, \\ \tau(s) &= \sqrt{2}. \end{aligned}$$

On the other hand, $\mathcal{P}(s)$ parallel curve of a timelike $\alpha(s)$ curve with parametrized by arc-length in $\mathbb{E}^3_1$ obtained as follows

$$\mathcal{P}(s) = (\sqrt{2}s - 2, 2\cos s + 2\sqrt{2}\sin s, 2\sin s - 2\sqrt{2}\cos s), \tag{3.3.2}$$

where we choose $t = \sqrt{5}$. Taking the derivative of the eq. (3.3.2), we can computed by

$$\mathcal{P}(s) = \mathbf{t_P} = (\sqrt{2}, -2\sin s + 2\sqrt{2}\cos s, 2\cos s + 2\sqrt{2}\sin s).$$

Now, if we choose $Q = (-2, 2, 2)$ be a fixed point in $\mathbb{E}^3_1$, the direction vector $\mathcal{N}(s)$ of the intersection line $l(t)$ between two planes $\mathcal{L}(s)$ and $\mathcal{L}_b(s)$ obtained by

$$\begin{aligned} \mathcal{N}(s) = \; & (12 - (4 - 4\sqrt{2})\cos s - (4 + 4\sqrt{2})\sin s, \\ & 2\sqrt{2} + (4 + 2\sqrt{2}s)\cos s - (2\sqrt{2} - 4s)\sin s, \\ & -2\sqrt{2} + (2\sqrt{2} - 4s)\cos s + (4 + 2\sqrt{2}s)\sin s). \end{aligned}$$

The intersection point $\mathfrak{B} = (b_1, b_2, b_3)$ of three planes: $\mathcal{L}(s)$, $\mathcal{L}_n(s)$, and $\mathcal{L}_b(s)$ can be computed by solving the following simultaneous linear equations in $\mathfrak{B}$:

$$\begin{aligned} \langle \mathfrak{B}, \mathbf{t_p}(s) \rangle &= r_1, \\ \langle \mathfrak{B}, \mathcal{N}(s) \rangle &= r_2, \\ \langle \mathfrak{B}, \mathcal{P}(s) - Q \rangle &= r_3, \end{aligned}$$

where

$$\begin{aligned} r_1 &= \langle \mathcal{P}(s), \mathbf{t_p}(s) \rangle \\ r_2 &= \langle Q, \mathcal{N}(s) \rangle, \\ r_3 &= \frac{1}{2}(\|\mathbf{P}(s)\|^2 - \|Q\|^2). \end{aligned}$$

Then, by Cramer's rule, eq. (3.3.3) can be solved as follows:

$$\begin{aligned} b_1 = \ & \frac{1}{\mathfrak{J}}[r_2(12 + (-4 + 4\sqrt{2})\cos s + (-4 + 4\sqrt{2})\sin s) \\ & + n_2((-10 + 4\sqrt{2}s - 4\sqrt{2} + 2s)\cos s + (2\sqrt{2} \\ & + 2\sqrt{2}s - 4s - 8)\sin s + 4s - 4\sqrt{2}) + n_3((4s + 8 \\ & - 2\sqrt{2} - 2\sqrt{2}s)\cos s + (4\sqrt{2}s - 10 - 4\sqrt{2} + 2s)\sin s \\ & - 4s + 4\sqrt{2})], \end{aligned}$$

$$\begin{aligned} b_2 = \ & \frac{1}{\mathfrak{J}}[n_1((-10 - 4\sqrt{2} + 2s + 4\sqrt{2}s)\cos s - (8 - 2\sqrt{2} \\ & + 4s - 2\sqrt{2}s)\sin s - 4\sqrt{2} + 4s) + r_2((4 + 2\sqrt{2}s)\cos s \\ & - (2\sqrt{2} - 4s)\sin s + 2\sqrt{2}) + n_3(4 + \sqrt{2} - 4s - \sqrt{2}s \\ & + 2\sqrt{2}s^2)], \end{aligned}$$

$$\begin{aligned} b_3 = \ & \frac{1}{\mathfrak{J}}[n_1((8 - 2\sqrt{2} + 4s - 2\sqrt{2}s)\cos s - (10 + 4\sqrt{2} - 2s \\ & - 4\sqrt{2}s)\sin s - 4s + 4\sqrt{2}) + n_2(-4 - \sqrt{2} + 4s + \sqrt{2}s \\ & - 2\sqrt{2}s^2) + r_2((2\sqrt{2} - 4s)\cos s + (4 + 2\sqrt{2}s)\sin s - 2\sqrt{2})], \end{aligned}$$

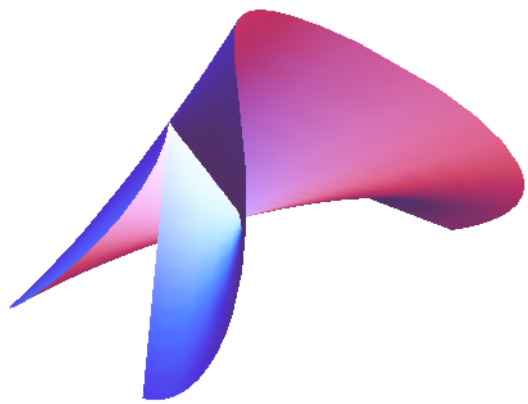
where

$$\begin{aligned} \mathfrak{J} = -[- \quad & 12 + (4 - 4\sqrt{2})\cos s + (4 + 4\sqrt{2})\sin s]^2 \\ & + [2\sqrt{2} + (4 + 2\sqrt{2}s)\cos s - (2\sqrt{2} - 4s)\sin s]^2 \\ & + [-2\sqrt{2} + (2\sqrt{2} - 4s)\cos s + (4 + 2\sqrt{2}s)\sin s]^2, \end{aligned}$$

$$\begin{aligned} r_2 &= 24 + (12\sqrt{2} - 8s + 4\sqrt{2}s)\cos s + (-12\sqrt{2} + 8s + 4\sqrt{2}s)\sin s, \\ n_1 &= 12 - (4 - 4\sqrt{2})\cos s - (4 + 4\sqrt{2})\sin s, \\ n_2 &= 2\sqrt{2} + (4 + 2\sqrt{2}s)\cos s + (-2\sqrt{2} + 4s)\sin s, \\ n_3 &= -2\sqrt{2} + (2\sqrt{2} - 4s)\cos s + (4 + 2\sqrt{2}s)\sin s. \end{aligned}$$

Then, the rational ruled bisector surface $\mathfrak{B}(s,t)$ can be constructed as follows:

$$\mathfrak{B}(s,t) = \mathfrak{B}(s) + t\mathcal{N}(s)\,;\ \text{for } s,t \in IR.$$



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