Article

Bianchi Type-VI$_0$ Cosmological Model with Barotropic Perfect Fluid in Creation Field Theory with Time Dependent Λ

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Abstract

Bianchi type-VI$_0$ cosmological model for barotropic fluid distribution in creation field cosmology with varying cosmological term Λ is investigated. To get deterministic solution we assume that $\Lambda = \frac{1}{R^2}$ as considered by Chen and Wu, where R is a scale factor and A = B, where A and B are metric potentials. We find that creation field (C) increase with time and $\Lambda \sim \frac{1}{t^2}$ which matches with the result of HN theory. We have also discussed special cases of model (32) like dust filled universe (γ = 0), stiff fluid universe (γ = 1) and radiation dominated Universe $\left(\gamma = \frac{1}{3}\right)$.

Keywords: Cosmology, cosmological constant, C-field, perfect fluid.

1. Introduction

Cosmology is the scientific study of origin, evolution, large scale structures & dynamics of the universe. It involves the formation of theories or hypothesis about the universe which makes specific predictions for phenomenon. These predictions can be tested with observations. Einstein's theory of general relativity is a very successful gravitational theory in describing the gravitational phenomena which also served as a basis for the models of the universe. All the investigations dealing with physical process are successfully explained by Einstein's field equations based big-bang model. The phenomenon of expanding universe, Primordial nucleo synthesis & the observed isotropy of Comic Microwave Background Radiations (CMBR) are important observations in astronomy which were successfully explained the big-bang model. This model is described by FRW line element and a matter density source which obeys equation of state $\rho = 3p$, where p and ρ are the fluid pressure and matter density respectively. But this model has various problems like; singularity in the past and may possibly in the future; no remarkable predictions in the big-bang model that explain the origin, evolution and characteristic

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of structures in the universe; the conservation of the energy is violated; the flatness and horizon problem explanations given from big-bang model of the universe.

However Smoot et al. [1] revealed that the astronomical predictions of the FRW type of models do not always exactly meet our expectations. The theoretical explanations given from big -bang type model were contradicted by some puzzling results regarding the red-shifts from extra-galactic objects. The gravitational collapse of massive objects is an unavoidable consequence of general relativity [2, 3]. Also CMBR discovery did not prove it to be an outcome of big-bang theory. Therefore alternative theories of gravitation were proposed time to time to overcome the drawbacks of big-bang model. Bondi & Gold [4] proposed a steady state theory in which the universe does not have any singular beginning or an end on the cosmic time scale where the matter density is throughout constant.

Also they state that the statistical properties of the large scale features of the universe do not change. Further the constancy of mass density has been accounted by continuous creation of matter going on in contrast to the one time infinite and explosive creation of matter at t = 0 as in earlier standard model. But the principle of conservation of matter was violated in this formalism. This difficulty was overcome by Hoyle & Narlikar [5-7] by adopting a field theoretical approach and introducing a massless and chargeless scalar field C in the Einstein-Hilbert action to explain creation ofmatter. In Hoyle-Narlikar C-field theory there is no big bang type singularity as in steady state theory by Bondi & Gold. Narlikar [8]has shown that the matter creation is accomplished at the expense of negative energy C-field in which he solves horizon and flatness problem faced by big-bang model. Narlikar & Padmanabhan [9] have obtained a solution of Einstein field equations admitting radiation with a negative energy massless scalar field C. In fact Narlikar et al. [10] have proved the possibilities of non-relic interpretation of CMBR.

Chatterjee & Banerjee [11] have investigated higher dimensional cosmology in C-field theory. Singh & Chaubey [12] have studied Bianchi type I, III, V, VI_0 and Kantowski-Sachs universes in C-field Cosmology. Adhav et al.[13,14] have investigated higher dimensional Bianchi Type VI_0 and Bianchi type-I string cosmological models in Creation field cosmology. Katore [15] has studied plane symmetric universe in C-field cosmology. Bali and Tikekar [16] have investigated C–field cosmological model for dust distribution in FRW space-time with variable gravitational constant. Recently Bali and Kumawat [17, 18] have investigated C-field cosmological models for dust and barotropic fluid distribution in non flat FRW space-time with variable gravitational constant. Bali and Saraf[19, 20] have investigated Bianchi type I dust field universe with decaying vacuum energy in C–field cosmology.

Einstein's basic cosmological model was a static, homogeneous with sphericalgeometry. Since atthat time universe was not known to be expanding, it is considered as the gravitational effect of matter caused acceleration in the model. In 1917, Einstein introduced a cosmological constantΛ in his equation as the universal repulsion or anti-gravity effect to make the universe static in accordance with generally accepted picture of that time. Hubble showed that the universe was expanding by his study about nearby galaxies. Then Einstein regretted modifying his elegant theory and viewed the cosmological constant as his greatest mistake. Recent cosmological observations by the High-Z Supernovae Team and Supernovae Cosmological Project suggested the existence of a positive cosmological constant Λ with the magnitude $\Lambda\ (G\dot{h} / c^3) \approx 10^{-123}$. Zel'dovich [21] has tried to visualize the meaning of cosmological constant from the theory of elementary particles. Bergmann [22] has interpreted the cosmological constant Λ in terms of Higgs scalar field. In quantum field theory, thecosmological constant is considered as the vacuum energy density. Linde [23] had shown that thecosmological term arises from spontaneous symmetry breaking and also suggested that the cosmological term is not a constant but a function of temperature. Dolgov[24] has focused that cosmological constant remains constant in the absence of any interaction with matter and radiation. Pavon [25] have studied model with cosmological constant problem as the discrepancy between the negligible values for the present universe. Glashow-Salam-Weinberg model [26] expected 10^{50} larger value whereas grand unified theory [27] expected 10^{107} larger value whereas of cosmological constant.

Bertolami [28] was the first who consider cosmological models with a variable cosmological constant of theform$\Lambda \sim t^{-2}$. Berman and Som [29] have shown that $\Lambda \sim t^{-2}$ plays a very important role in cosmology. Chen and Wu [30] have also solved the problem by considering $\Lambda \sim R^{-2}$, where R is the scale factor inthe Robertson-Walker space time. Carvalho et al. [31] generalized the proposed model of Chen and Wu by including a term proportional to H^2 on the time dependence of Λ. Recent observations indicate that $\Lambda \sim 10^{-56}cm^{-2}$ but the theory of physics of elementary particles predicts that the value of Λ must have been 10^{120} times larger in the past. It is worth noting that cosmological models based on Einstein field equations with a time-dependent cosmological constant Λ had been the subject of numerous papers in recent years.

Tyagi and Singh [32] have investigated time-dependent Λ in C-field Theory with LRS Bianchi type III universe and Barotropic perfect fluid. LRS Bianchi type V perfect fluid cosmological model in C-field theory with variable Λ are also investigated by Tyagi and Singh [33].Recently Patil et al. [34] have obtained Bianchi type-IX dust filled universe with ideal fluid distribution in Creation field theory.

In this paper, we have investigated Bianchi type VI$_0$ cosmological model for barotropic fluid distribution in C-field cosmology with time dependent term Λ. For deterministic model, we assumed$\Lambda = \frac{1}{R^2}$, where R is scale factor. We find that creation field (C) increase with time and $\Lambda \sim \frac{1}{t^2}$. The physical and geometrical parameters of the model are also discussed.

2. The Metric and Field Equation

We have considered Bianchi type VI$_0$ metric of the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2x}dy^2 + C^2e^{-2x}dz^2 \tag{1}$$

in which A, B and C are functions of t alone.

Hoyle and Narlikar modified the Einstein's field equation by introducing C-field with time dependent cosmological term as:

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G\left[T_i^{\ j}{}_{(m)} + T_i^{\ j}{}_{(c)}\right] + \Lambda g_i^j \tag{2}$$

The energy-momentum tensor $T_i^j{}_{(m)}$ for perfect fluid and creation field $T_i^j{}_{(c)}$ are given by

$$T_i^{\ j}{}_{(m)} = (p+\rho) v_i v^j + p g_i^j \tag{3}$$

$$T_i^{\ j}{}_{(c)} = -f\left(c_i c^j - \frac{1}{2} g_i^j c_\alpha c^\alpha\right) \tag{4}$$

where f > 0 is coupling constant between the matter and creation field and $C_i = \frac{dC}{dx^i}$.

The co-moving coordinates are chosen such that $v_i = (0,0,0,1)$

The non-vanishing components of energy-momentum tensor for matter are given by

$$T^1_{1\ (m)} = p = T^2_{2\ (m)} = T^3_{3\ (m)}$$

$$T^4_{4\ (m)} = -\rho \tag{5}$$

The non-vanishing components of energy-momentum tensor for creation field are given by

$$T^1_{1\ (c)} = T^2_{2\ (c)} = T^3_{3\ (c)} = -\frac{1}{2} f\dot{c}^2$$

$$T^4_{4\ (c)} = \frac{1}{2} f\dot{c}^2 \tag{6}$$

Hence, the Einstein's field equation (2) for the metric (1) and EMT (5) and (6) takes the form

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{1}{A^2} = 8\pi G\left(-p + \frac{1}{2} f\dot{c}^2\right) + \Lambda \tag{7}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = 8\pi G\left(-p + \frac{1}{2} f\dot{c}^2\right) + \Lambda \tag{8}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{C} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} = 8\pi G\left(-p + \frac{1}{2} f\dot{c}^2\right) + \Lambda \tag{9}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = 8\pi G\left(\rho - \frac{1}{2} f\dot{c}^2\right) + \Lambda \tag{10}$$

$$\frac{1}{A}\left(\frac{C_4}{C}-\frac{B_4}{B}\right)=0 \tag{11}$$

The suffix 4 by the symbols A and B denotes differentiation w.r.t. t.

3. Solution of Field Equations

Equation (11) leads to

$$C = lB \tag{12}$$

where l is constant of integration.

To get determinate solution of equations (7) to (10), we take condition between the metric potentials i.e.

$$A = B \tag{13}$$

Using equations (12) and (13) in equations (7) to (10), we get

$$\frac{2B_{44}}{B}+\frac{B_4^{\ 2}}{B^2}+\frac{1}{B^2}=8\pi G\left(-p+\frac{1}{2}f\dot{c}^2\right)+\Lambda \tag{14}$$

$$\frac{2B_{44}}{B}+\frac{B_4^{\ 2}}{B^2}-\frac{1}{B^2}=8\pi G\left(-p+\frac{1}{2}f\dot{c}^2\right)+\Lambda \tag{15}$$

$$\frac{2B_{44}}{B}+\frac{B_4^{\ 2}}{B^2}-\frac{1}{B^2}=8\pi G\left(-p+\frac{1}{2}f\dot{c}^2\right)+\Lambda \tag{16}$$

$$\frac{3B_4^2}{B^2}-\frac{1}{B^2}=8\pi G\left(\rho-\frac{1}{2}f\dot{c}^2\right)+\Lambda \tag{17}$$

The conservation equation of energy momentum tensor is

$$\left(8\pi G T_j^i+\Lambda g_j^i\right)_{;i}=0 \tag{18}$$

which leads to

$$8\pi G\left[\dot{\rho}-f\dot{c}\ddot{c}+\{(\rho+p)-f\dot{c}^2\}\left(\frac{A_4}{A}+\frac{2B_4}{B}\right)\right]+\dot{\Lambda}=0 \tag{19}$$

Following Hoyle and Narlikar theory, the source equation of C-field i.e. $c^i_{;i}=n/f$ leads to $c=t$ thus $\dot{c}=1$.

The barotropic fluid condition leads to

$$p = \gamma\rho \qquad \text{where } 0 \le \gamma \le 1 \tag{20}$$

Using equation (20) in equation (16) we have

$$\frac{2B_{44}}{B} + \frac{B_4^{\ 2}}{B^2} - \frac{1}{B^2} = 8\pi G\left(-\gamma\rho + \frac{1}{2} f\dot{c}^2\right) + \Lambda \tag{21}$$

Now equations (17) and (21) together with $\dot{c} = 1$ leads to

$$\frac{2B_{44}}{B} + (3\gamma+1)\frac{B_4^2}{B^2} - (1+\gamma)\frac{1}{B^2} = 4\pi f G(1-\gamma) + \Lambda(1+\gamma) \tag{22}$$

To get deterministic solution of equation (22) we also assume that

$$\Lambda = \frac{1}{R^2} = \frac{1}{B^2} \tag{23}$$

Using equation (23) in equation (22) we have

$$\frac{2B_{44}}{B} + (3\gamma+1)\frac{B_4^2}{B^2} = 4\pi f G(1-\gamma) + \frac{2}{B^2}(1+\gamma) \tag{24}$$

Now put $B_4 = f(B)$ which leads to $B_{44} = ff'$ (25)

Now equation (24) with the help of (25) becomes

$$\frac{df^2}{dB} + (3\gamma+1)\frac{f^2}{B} = 4\pi f G(1-\gamma)B + \frac{2}{B}(1+\gamma) \tag{26}$$

Equation (26) leads to

$$f^2 . B^{(3\gamma+1)} = 4\pi f G\frac{(1-\gamma)}{3(\gamma+1)}B^{3\gamma+3} + 2(1+\gamma)\frac{B^{3\gamma+1}}{(3\gamma+1)} \tag{27}$$

which gives

$$f^2 = \alpha B^2 + \beta \tag{28}$$

where $\alpha = \dfrac{4\pi f G(1-\gamma))}{3(1+\gamma)}$, $\qquad \beta = 2\dfrac{(1+\gamma)}{(3\gamma+1)}$ (29)

Equation (28) leads to

$$\frac{dB}{\sqrt{\alpha B^2+\beta}}=dt \tag{30}$$

Hence, equation (30) gives

$$\mathrm{B}=\sqrt{\frac{\beta}{\alpha}}\sinh\sqrt{\alpha}\mathrm{t} \tag{31}$$

Also the metric (1) reduces to

$$ds^2=-dt^2+\left(\frac{\beta}{\alpha}\sinh^2\sqrt{\alpha t}\right)\left[dx^2+e^{2x}dy^2+l^2e^{-2x}\,dz^2\right] \tag{32}$$

Now using $p=\gamma\rho$ and equations (12) and (13) in equation (19) we have

$$8\pi G\left[\dot{\rho}-f\dot{c}\ddot{c}+\{(1+\gamma)\rho-f\dot{c}^2\}\left(\frac{3B_4}{B}\right)\right]+\dot{\Lambda}=0 \tag{33}$$

Equation (33) leads to

$$4\pi fG\frac{d\dot{c}^2}{dt}+8\pi fG3\sqrt{\alpha}\coth\sqrt{\alpha}t\dot{c}^2$$

$$=\left(3\alpha-\frac{\alpha}{4\beta}\right)2\sqrt{\alpha}\operatorname{cos}ech^2\sqrt{\alpha t}\coth\sqrt{\alpha t}+(1+r)9\alpha\sqrt{\alpha}\coth\sqrt{\alpha t}$$

$$+4\pi fG(1+r)3\sqrt{\alpha}\coth\sqrt{\alpha t}+\frac{2\alpha\sqrt{\alpha}}{\beta}\operatorname{cos}ech^2\sqrt{\alpha t}\coth\sqrt{\alpha t}$$

$$-(1+r)3\sqrt{\alpha}\left(3\alpha-\frac{\alpha}{4\beta}\right)\mathrm{cosech}^2\sqrt{\alpha t}\coth\sqrt{\alpha t} \tag{34}$$

To obtain the solution of equation (34) we assume $\alpha=1$ and $4\pi f\mathrm{G}=3$ so equation (34) leads to

$$\frac{d\dot{c}^2}{dt^2}+6(\coth)\dot{c}^2=6(\coth\mathrm{t}) \tag{35}$$

Equation (35) gives

$$\dot{c}^2=1 \tag{36}$$

So, we have $\dot{c}=1$ (37)

which agrees with the value used in source equation. Thus, creation field is proportional to time t.

4. Physical and Geometrical Properties

For the model (32), the mass density (ρ) is given by

$$8\pi G\rho = 4\pi Gf + 3\alpha \coth^2 \sqrt{\alpha}t - \frac{2\alpha}{\beta} \operatorname{cosech}^2 \sqrt{\alpha}t \tag{38}$$

The scale factor R is

$$R = \sqrt{\frac{\beta}{\alpha}} \sinh \sqrt{\alpha}t \tag{39}$$

The cosmological term (Λ) is

$$\Lambda = \frac{\alpha}{\beta} \operatorname{cosech}^2 \sqrt{\alpha}t \tag{40}$$

and the deceleration parameter (q) is

$$q = -\tanh^2 \sqrt{\alpha}t \tag{41}$$

Special Cases

Case I: Dust Filled Universe (γ= 0)

The metric (32) for the dust filled universe is given by

$$ds^2 = -dt^2 + \left(2\sinh^2(t+t_0)\right)\left[dx^2 + e^{2x}dy^2 + l^2e^{-2x}\,dz^2\right] \tag{42}$$

From equation (30)

$$\frac{dB}{\sqrt{B^2+2}} = dt \tag{43}$$

So, the mass density (ρ), scale factor (R), cosmological constant (Λ) and the decelerating parameter (q) for the model (42) are given by

$$8\pi G\rho = 2\coth^2(t+t_0) + 4 \tag{44}$$

$$R = \sqrt{2}\sinh(t+t_0) \tag{45}$$

$$\Lambda = \frac{1}{2}\cos ech^2(t+t_0) \tag{46}$$

and $$q = -\tanh^2(t+t_0) \tag{47}$$

Case II: Stiff Fluid University (γ = 1)

The metric (32) for stiff fluid universe is given by

$$ds^2 = -dt^2 + (t+t_0)^2\ \left[dx^2 + e^{2x}dy^2 + l^2e^{-2x}\ dz^2\right] \tag{48}$$

From equation (30)

$$\frac{dB}{1} = dt \tag{49}$$

Also the mass density (ρ), scale factor (R), cosmological constant (Λ) and declaration parameter (q) for the model (48) are given by

$$8\pi G\rho = 3 \tag{50}$$

$$R = t + t_0 \tag{51}$$

$$\Lambda = \frac{1}{(t+t_0)^2} \tag{52}$$

and $$q = 0 \tag{53}$$

Case III: Radiation Dominated Universe (γ = 1/3)

The metric (32) for radiation dominated universe becomes

$$ds^2 = -dt^2 + \left[\frac{8}{3}\sinh^2\left(\frac{t+t_0}{\sqrt{2}}\right)\right]\left[dx^2 + e^{2x}dy^2 + l^2e^{-2x}\ dz^2\right] \tag{54}$$

From equation (30)

$$\frac{dB}{\sqrt{\frac{1}{2}B^2 + \frac{4}{3}}} = dt \tag{55}$$

Also, the mass density (ρ), scale factor (R), cosmological constant (Λ) and deceleration parameter (q) for the model (52) are givenby

$$8\pi G\rho = \frac{3}{2}\coth^2\frac{(t+t_0)}{\sqrt{2}} + \frac{15}{4} \tag{56}$$

$$R = \frac{\sqrt{8}}{\sqrt{3}}\sinh\left(\frac{t+t_0}{\sqrt{2}}\right) \tag{57}$$

$$\Lambda = \frac{3}{8}\operatorname{cosech}^2\left(\frac{t+t_0}{\sqrt{2}}\right) \tag{58}$$

and $q = -\tanh^2(t+t_0)$ (59)

5. Conclusion

The creation field C increases with time and $\dot{c}$=1 which agrees with the value taken in source equation. The scale factor R for the model (32) increases with time and the cosmological term Λ decreases as time increases. The decelerating parameter i.e. q < 0 which represent that universe is accelerating.

Further for all special cases i.e. dust filled universe, stiff fluid universe and radiation dominated universe, scale factor R increases and cosmological term Λ decreases with time. In case of stiff fluid, model has uniform motion and the universe is accelerating for dust filled and radiation dominated cases.

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