Higher Dimensional Plane Symmetric Solutions in *f*(*R*) Theory of Gravitation

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Abstract

This paper is devoted to study of higher dimensional plane symmetric solutions in *f(R)* theory of gravitation. We extend the work in [63] on Plane symmetric solution in *f(R)* gravity which is in four dimension to higher five dimensions. Using assumption of non-constant and constant scalar curvature, field equations in five dimensions in *f(R)* theory of gravity are solved and obtained some well-known results which support the recent observations about nature of the universe.

Keywords: f(R) gravity, higher dimensional, plane symmetric.

1. Introduction

General theory of relativity is the most successful theory of gravitation which plays an important role in modern physics. Many authors done their remarkable work in the general theory of relativity. There are many problems which are very difficult to solve by this theory. This theory fails to answer the issue of accelerating expansion of the universe and dark energy problem. All the observations from the different sources such as supernovae type-Ia experiments performs by different authors [1-4],cosmic microwave Back ground fluctuations[5-7], large scale structure [8] and X- rays experiments [9] suggest that the universe is expanding with an accelerating rate. Different models have been proposed for dark energy and dark matter.

It has been found that energy of universe contains seventy six percent dark energy and twenty percent dark matter. Dark matter has the same properties as ordinary matter but cannot be detected from laboratory. Einstein gave the concept of dark energy and introduced the cosmological constant in the field equation while studding the static cosmological model and was later discarded by him saying that "it is the greatest blunder of my life" However in recent years researchers in general relativity attracted towards this cosmological constant and believed that the cosmological constant may be suitable candidate for dark energy.

Recent observations by Perlmutter et al [10] and Riess et al [11]strongly favor a significant and a positive value of cosmological constant.Kalita et al [12], Chawala et al [13] & Pradhan et al [14] have studied the time dependent cosmological constant in different context. Dark energy can be better understand through an equation of state (EOS) parameter ω defined as the ratio $\omega = \frac{p}{\rho}$ where p is the pressure and ρ is the energy density. It has been established that If $\omega = -1$ then the expansion of universe is accelerating [15 , 16, 17]. It is found that the universe with phantom like dark energy when $\omega < -1$ caused a finite time future singularity called as big rip singularity [18,19] and $w > -1$, universe have a quintessence dark era.

The existence of dark matter in the universe is justified by some observations like gravitational lensing of back ground objects by galaxy rotational velocities of galaxies and the observed fluctuations in the cosmic microwave background radiations. In order to explain the accelerated expansion of the universe, number of cosmological models has been proposed by different authors. The modified version of Einstein theory of relativity is considered as most suitable for tackling the problem of dark energy, dark matter and acceleration expansion of the universe. *f(R)* theory of gravity gives answer to such type of problems. And therefore nowadays *f(R)* theory of gravity has attracted much attention of the researchers. The *f(R)* theory of gravity considered as most suitable theory. In *f(R)* theory of gravity the cosmic acceleration can be attained by replacing Ricci scalar R with *f(R)* in Einstein-Hilbert action of general relativity. The dark matter problems can also be addressed using viable *f(R)* gravity models [20]. *f(R)* actions were first studied by Weyl and Edington[21 , 22] respectively.

In case of non-singular oscillating cosmologies, Buchdhal [23] studied these actions rigorously. Bertolami et al [24] have proposed a generalization of *f(R)* gravity by incusing an explicit coupling of an arbitrary function of the ricci scalar R with the matter Langangian densityLm. Nojiri S. and Odintsov S. D. [25, 26, 27] proved that the f(R) theory of gravity provides very natural unification of the early time inflation and late time acceleration. Also many authors [28 – 46] have done a remarkable work in f(R) gravity in different context. Capozziello et al [47] used Noether symmetries to study spherically symmetric solutions in f(R) theory of gravity. Multamaki and Vilja [48 , 49] investigated static spherically symmetric vacuum solutions of the field equations and non-vacuum solutions by taking fluid respectively in f(R) theory of gravity. Lukas Hollenstein and Franscisco S. N. Lobo [50] discussed exact solutions of f(R) gravity coupled to non-linear electrodynamics. Azadi A. et al [51] study cylindrical solutions in metric f(R) gravity and Momeni D. [52] explained constant curvature solutions in cylindrical symmetric metric f(R) gravity.

The idea of space-time should be extended from four to higher five dimensions was introduced by Kaluza [53] and Klein [54]. In the recent years many researcher showed keen interest in higher dimensional space-time and solved Einstein field equations in general theory of relativity. Higher dimensional cosmological models play a vital role in many aspects of early stage of cosmological problems. The study of higher dimensional space-time provides an idea that our universe is much smaller at early stage of evolution as observed today. Wesson [55,56] have studied several aspects of five-dimensional space-time in variable mass theory and biometric theory of relativity. Ghosh and Dadhich [57] have investigated the end static of gravitation collapse of null fluid in higher dimensional Vaidya space-time in general relativity. Authors [58 – 62] have studied the multi-dimensional cosmological models in general relativity and other modified theories of gravitation

Here we obtained the solutions of the field equations in five dimensional space-time in f(R) theory of gravity using non-constant curvature and constant curvature assumption and obtained the results similar to [63] which support the established nature of the universe.

2. $f(R)$ Theory of gravity

The action for $f(R)$ theory of gravity are given by

$$S = \int\left(\frac{1}{16\pi G} f(R) + L_m\right)\sqrt{-g}\ d^5x, \tag{1}$$

Where $f(R)$ is general function of Ricci scalar R and L_m is the matter Lagrangian.

Now by varying the action S with respect to g_{ij}, we obtain the field equations in $f(R)$ theory of gravity as

$$F(R)R_{ij} - \frac{1}{2} f(R)g_{ij} - \nabla_i \nabla_j F(R) + g_{ij} \square F(R) = kT_{ij}, (\text{i,j}=1,2,3,4,5) \tag{2}$$

Where $F(R) \equiv \frac{df(R)}{dR}, \square \equiv \nabla^i \nabla_i$

With ∇_i is the covariant derivative and T_{ij} is the standard matter energy momentum tensor.

If we take $f(R) = R$, the field equation (2) in *f(R)* theory of gravity reduce to the field equation of general theory of relativity which is proposed by Einstein.

Contracting the above field equations (2), we have

$$F(R)R - \frac{5}{2} f(R) + 4 \square F(R) = kT \tag{3}$$

For non-vacuum, we have

$$F(R)R - \frac{5}{2} f(R) + 4 \,\square\, F(R) = 8\pi T \quad (4)$$

This gives an important relation between $F(R)$ and $f(R)$, which may be used to simplify the field equations and to evaluate $f(R)$.

From (4), we get

$$f(R) = \frac{2}{5}\left[-8\pi T + 4 \,\square\, F(R) + F(R)R\right]. \quad (5)$$

Using equations (2) and (5), the field equations take the form

$$\frac{F(R)R_{ij} - \nabla_i \nabla_j\, F(R) - 8\pi T_{ii}}{g_{ij}} = \frac{1}{5}[F(R)R - \square\, F(R) - 8\pi T]. \quad (6)$$

It follows that the equation (6) is not depending on the index i, hence above equation can be express as

$$K_i = \frac{F(R)R_{ij} - \nabla_i \nabla_j F(R) - 8\pi T_{ij}}{g_{ij}} \quad (7)$$

And hence $K_i - K_j = 0$ for all i and j.

3. Metric & the Field equations

The line element in plane symmetric form in five dimension is

$$ds^2 = A(x)dt^2 - C(x)dx^2 - B(x)\left(dy^2 + dz^2 + du^2\right) \quad (8)$$

For simplicity take $C(x) = 1$

$$ds^2 = A(x)dt^2 - dx^2 - B(x)\left(dy^2 + dz^2 + du^2\right) \quad (9)$$

The Ricci scalar for the line element has value

$$R = \frac{\ddot{A}}{A} + \frac{3\ddot{B}}{B} - \frac{\dot{A}^2}{2A^2} + \frac{3\dot{A}\dot{B}}{2AB} \quad (10)$$

Where overhead$(\bullet)$ Represents derivative with respect to$x$

The combination equation is

$$K_i = \frac{F(R)R_{ii} - \nabla_i \nabla_i F(R)}{g_{ii}} \tag{11}$$

is independent of the index i and hence $K_i - K_j = 0$ for all i & j

For $K_5 - K_1 = 0$

$$\left[\frac{3\dot{A}\dot{B}}{2AB} + \frac{3\dot{B}^2}{2B^2} - \frac{3\ddot{B}}{B}\right]F + \frac{\dot{A}\dot{F}}{A} - 2\ddot{F} + 2k(p+\rho) = 0 \tag{12}$$

For $K_5 - K_2 = K_5 - K_3 = K_5 - K_4 = 0$

$$\left[\frac{2\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} + \frac{2\dot{A}\dot{B}}{AB} - \frac{2\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right]F + \frac{2\dot{A}\dot{F}}{A} - \frac{2\dot{B}\dot{F}}{B} + 2k(p+\rho) = 0 \tag{13}$$

Thus we obtain two equations (12) and (13) with five unknowns A, B, F, P & ρ . These equations are highly nonlinear and very difficult to solve. However using the concept of non-constant and constant curvature assumption, we investigate some solutions .

3.1)Solutions with the non-constant curvature assumption

Using non-constant curvature assumption, we explore the field equations in f(R) gravity.

Subtracting equation (12) from (13), we get

$$\left[\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} - \frac{4\ddot{B}}{B} - \frac{2\ddot{A}}{A} + \frac{4\dot{B}^2}{B^2}\right]F - \frac{2\dot{B}\dot{F}}{B} + 4\ddot{F} = 0 \tag{14}$$

This equation is of highly nonlinear nature. To solve this equation we assume $B = A^n$, we get

$$\frac{\dot{A}^2}{A^2}(5n+1) - 2(2n+1)\frac{\ddot{A}}{A} - \frac{2n\dot{A}\dot{F}}{AF} + 4\frac{\ddot{F}}{F} = 0 \tag{15}$$

And Ricci scalar is

$$R = \frac{-1}{2}\left[(6n^2 - 3n - 1)\frac{\dot{A}^2}{A} + (6n+2)\frac{\ddot{A}}{A}\right] \tag{16}$$

We assume $F(R) = f_0 R^m$ where f_0 an arbitrary constant.

Using equations (15) and (16), and performing very tedious calculations , we obtain,

$$
\begin{aligned}
&\begin{bmatrix}\left(180n^5-144n^4-51n^3+27n^2+11n+1\right)+\left(144mn^5-144mn^4+24mn^2+4mn\right)+\\ \left(576m^2n^4-576m^2n^3-48m^2n^2+96m^2n+16m^2\right)+\left(-576mn^4+576mn^3+48mn^2-96mn-16m\right)\\ +\left(864mn^4-864mn^3-72mn^2+144mn-24m\right)\end{bmatrix}\dot{A}^6\\
&+\begin{bmatrix}\left(-144n^5+432n^4+96n^3-150n^2-60n-6\right)+\left(-144mn^5+360mn^4-96mn^2-24mn^3+16mn\right)\\ +\left(-1152m^2n^4+1728m^2n^3-384m^2n-64m^2\right)+\left(1152mn^4-1728mn^3+384mn+64m\right)+\\ \left(-1440mn^4-256mn^3-72mn^2-624mn-104m\right)\end{bmatrix}\dot{A}\ddot{A}A\\
&+\begin{bmatrix}\left(-288n_4+84n^3+276n^2+108n+12\right)+\left(-144mn^4+96mn^3+96mn^2+16mn\right)+\\ \left(-576mn^4+1132mn^3-192mn^2-192mn-64m\right)+\left(288mn^4-1872mn^3-528mn^2+768mn+128m\right)\\ +\left(576m^2n^4-1152m^2n^3+192m^2n^2+192m^2n+64m^2\right)\end{bmatrix}\dot{A}^2\ddot{A}^2A^2\\
&+\left[\left(-144n^3-168n^2-64n-8\right)+\left(288mn^3-192mn^2-192mn-32m\right)\right]\ddot{A}^3A^3\\
&+\begin{bmatrix}\left(-72mn^4+12mn^3+24mn^2+4mn\right)+\left(-576m^2n^3+96m^2n^2+384m^2n+32m^2\right)+\\ \left(576mn^3-96mn^2-192mn-32m\right)+\left(288mn^4-576mn^3+24mn^2+144mn+24m\right)\end{bmatrix}\dot{A}^3\dddot{A}A^2\\
&+\begin{bmatrix}\left(-72mn^3+48mn^2+8mn\right)+\left(576m^2n^3-384m^2n^2-384m^2n-64m^2\right)+\\ \left(-576mn^3+384mn^2+384mn+64m\right)+\left(288mn^3-336mn^2-288mn-48m\right)\end{bmatrix}\dot{A}\ddot{A}\dddot{A}A^3\\
&+\left[\left(144m^2n^2+144m^2n+16m^2\right)+\left(-144mn^2-96mn-16m\right)\right]\dddot{A}^2A^4\\
&+\left[288mn^3-48mn^2-96mn-16m\right]\dot{A}^2\ddddot{A}A^3+\left[144mn^2+96mn+16m\right]\ddot{A}\ddddot{A}A^2=0
\end{aligned}
\tag{17}
$$

Using equation (17) many solutions can be reconstructed .

Here, we discuss only three cases

Case I:

Here we try to recover Taubs solution. Substitute $A = x^1$ in equation (17) we get constraint equation

$$16m^2 + 4m + 5n - 4 = 0 \tag{18}$$

We can obtain for $B = x^{-1}$ for $n = -1$. Then Eq. (18) reduce to

$$16m^2 + 4m - 9 = 0 \tag{19}$$

Finding the roots ,we get $m=\frac{-1\pm\sqrt{37}}{8}$. Thus we have

$$F(R)=f_0 R^{\frac{-1\pm\sqrt{37}}{8}} \tag{20}$$

After integration we obtain

$$f(R)=\hat{f}_0\, R^{\frac{7+\sqrt{37}}{8}}+k_5 \tag{21}$$

$$f(R)=\check{f}_0\, R^{\frac{7-\sqrt{37}}{8}}+k_6 \tag{22}$$

where

$$\hat{f}_0=\frac{8f_0}{7+\sqrt{37}} \qquad \check{f}_0=\frac{8f_0}{7-\sqrt{37}}$$

And k_5 and k_6 are constant of integration. The corresponding Ricci scalar becomes

$$R=\frac{-4}{x^2} \tag{23}$$

Using $m=\frac{-1-\sqrt{37}}{8}$, EOS parameter ω and Eq. (12) and (13), the expression forenergy density

$$\rho=\frac{f_0}{k(1+\omega)}\left[\frac{(-4)^{\frac{-1-\sqrt{37}}{8}}}{x^{\frac{-1-\sqrt{37}}{4}}}+\frac{\sqrt{37}(-4)^{\frac{-9-\sqrt{37}}{8}}}{x^{\frac{7-\sqrt{37}}{4}}}\right] \tag{24}$$

Case II:

In this case we take $A=x^{\frac{-1}{2}}$ in Eq. (17) we get a constraint equation

$$8m^2-m-n-5=0 \tag{25}$$

With this equation we obtain

$$B=x^{\frac{-(8m^2-m-5)}{2}} \tag{26}$$

And the solution metric takes the form

$$ds^2 = x^{\frac{-1}{2}} dt^2 - dx^2 - x^{\frac{-(8m^2-m-5)}{2}} \left(dy^2 + dz^2 + du^2\right) \tag{27}$$

The corresponding Ricci scalar becomes

$$R = \frac{3(n^2 + 2n + 1)}{2x^2} \tag{28}$$

For different values of m, we can construct different $f(R)$ models. So for $m = -1$, logarithmic form of $f(R)$ models is obtained.

$$f(R) = f_0 \log R + k_7 \tag{29}$$

where k_7 is constant of integration such a logarithmic form was first introduced by Nojiri and Odintsov, In this case, the Ricci scalar becomes $R = \frac{-161}{8x^2}$ and the solution metric takes the form

$$ds^2 = x^{\frac{-1}{2}} dt^2 - dx^2 - \frac{1}{x^2}\left(dy^2 + dz^2 + du^2\right) \tag{30}$$

and the matter density turns out to be

$$\rho = \frac{-f_0}{2k(1+\omega)}\left(\frac{44}{161}\right) \tag{31}$$

Similarly, for $m = -2$ we obtain

$$f(R) = -f_0 R^{-1} + k_8 \tag{32}$$

where K_8 is integration constant. The negative power of the curvature support the cosmic acceleration.

Case III:

Here we consider $A = e^x$ in Eq. (17) in this case we obtain the constraints equation

$$(n-1)(6n^2 + 3n + 1) = 0 \tag{33}$$

This equation does not involve parameter m , hence this case yields a solution for any $f(R)$ model in a power law or logarithmic form. The roots of Eq. (33) are

$$n = 1, \frac{-3 \pm i\sqrt{15}}{12} \tag{34}$$

Here , we consider only real value of n. For n=1, we get, $R = -5$

Solution metric becomes

$$ds^2 = e^x dt^2 - dx^2 - e^x\left(dy^2 + dz^2 + du^2\right) \tag{35}$$

This corresponds to the well-known anti-de sitter space-time in GR

3.2 Solutions with the constant curvature assumption

Here, we assume constant curvature $R = R_0$

Then we have

$$\dot{F}(R_0) = 0 = \ddot{F}(R_0) \tag{36}$$

It is clear that any solution in GR must be found for $f(R)$ theory.

Case I:

With condition (36) equations (12) and (13) reduce to

$$\left[\frac{3\dot{A}\dot{B}}{2AB} + \frac{3\dot{B}^2}{2B^2} - \frac{3\ddot{B}}{B}\right]F_0 - 2k(p+\rho) = 0 \tag{37}$$

$$\left[\frac{2\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} + \frac{2\dot{A}\dot{B}}{AB} - \frac{2\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right]F_0 - 2k(p+\rho) = 0 \tag{38}$$

We can describe the dark energy with the EOS parameter $\omega = \dfrac{p}{\rho}$, and it is well known from literature that the expansion of the universe is accelerating when $\omega \approx -1$. In this case equations (37) and (38) reduce to

$$\left[\frac{3\dot{A}\dot{B}}{2AB} + \frac{3\dot{B}^2}{2B^2} - \frac{3\ddot{B}}{B}\right] = 0 \tag{39}$$

$$\left[\frac{2\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} + \frac{2\dot{A}\dot{B}}{AB} - \frac{2\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right] = 0 \tag{40}$$

To solve these equations, we use power law assumption

Let $A \propto x^a$ and $B \propto x^b$ i.e. $A = k_1 x^a$ and $B = k_2 x^b$

Where a, b are real numbers and k_1, k_2 are constant of proportionality

By above assumption, we get

$$a = -1 \; b = 1 \tag{41}$$

And hence the solution becomes

$$ds^2 = k_1 x^{-1} dt^2 - dx^2 - k_2 x\left(dy^2 + dz^2 + du^2\right) \tag{42}$$

And $R = 0$ for the values of $a \& b$

By defining parameters $\sqrt{k_1}t \to T$, $\sqrt{k_2}y \to Y$, $\sqrt{k_2}z \to Z$ and $\sqrt{k_2}u \to U$

above metric takes the form

$$ds^2 = x^{-1} dT^2 - dx^2 - x\left(dY^2 + dZ^2 + dU^2\right) \tag{43}$$

Which is the same as Taub's metric

Case II:

Now we assume that $B = A^n$. Then by substituting equations (37) and (38) we get

$$\left(2n^2 - n + 1\right)\dot{A}^2 - 2A\ddot{A} = 0 \tag{44}$$

This equation yields

$$A = k_3\left[\left(2n^2 - n - 1\right)x + k_4\right]^{\frac{-2}{2n^2 - n - 1}} \tag{45}$$

where k_3 and k_4 are constants of integration.Without loss of generality, we can choose $k_3 = 1$ and $k_4 = 0$

$$A = \left[\left(2n^2 - n - 1\right)x\right]^{-2/2n^2 - n - 1} \tag{46}$$

Hence

$$B = \left[\left(2n^2 - n - 1\right)x\right]^{-2n/2n^2 - n - 1} \tag{47}$$

By these values of A and B the solution metric takes the form

$$ds^2 = \left[\left(2n^2 - n - 1\right)x\right]^{-2/2n^2-n-1} dt^2 - dx^2 - \left[\left(2n^2 - n - 1\right)x\right]^{-2n/2n^2-n-1}\left(dy^2 + dz^2 + du^2\right) \tag{48}$$

This equationgives Taub's metric for $n = -1 \,\&\, \frac{3}{2}$.

4. Discussion &Conclusion

Like Spherical Symmetry plane symmetric space- time has also many properties. Number of researchers studied plane symmetric space time in different context. In this paper, we have studied five dimensional plane symmetric solutions in f(R) theory of gravity. Without relaxing the Conditions, we have extended the work in [63] which is in four dimensions to five dimensional space- time and obtained the similar results which support the recent observations about the nature of the universe. We find the solutions using non-constant and constant curvature condition.

In section 3.1, we study five dimensional non-vacuum plane symmetric solution in f(R) theory of gravity using non-constant curvature assumption and this gives fifth order highly non- linear differential equations which are very difficult to solve. So we consider only three cases. In case I, substituting= x^1 , we recovered Taub's solution and obtained the value of Ricci Scalar R and energy density ρ .In case II, We obtained the logarithmic form of f(R) models for m=-1 and for m = -2, we obtained a model with negative power of curvature which support the cosmic acceleration.

In case III, we consider $A = e^x$ which gives a solution for any f(R) model in a power law or logarithmic form. In this case non-zero Ricci scalar and solution metric which corresponds to anti- de-sitter space –time in general relativity is obtained. In section 3.2, we obtained a solutions with the constant curvature assumption. In case I, using power law assumption , we obtained a metric which is same as Taub's metric. In case II ,by taking $B = A^n$, a metric is obtained from which Taub's metric can be recovered for $n = -1 \,\&\, \frac{3}{2}$.

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References

1)Perlmutter, S. et al (1998)Discovery of supernovae explosion at half the age of the universe. Nature, 391, 51-54 .http : || dx.doi.org /10.1038 / 34124.
2) Perlmutter, set al (1999)Measurement from 42 high –red shift supernovae. The Astrophysical journal 517, 565-586.http : || dx.doi.org /10.1086 / 307221.
3) Riess A. G. et al (1998) Observational evidence fromfor super-novae for an accelerating universe and a cosmological constant. The Astrophysical journal 116, 1009-1038.
4) RiessA.G. et al (2004)The Astrophysical journal 607, 665.
5) Bennett C. L. et al(2003) Astrophys J. Suppl. 148,1
6) Spergel D. N. et al (2003) Astrophys J. Suppl. 148 , 175
7) Cadwell R. R. and Doran M. (2004) Cosmic microwave background and supernovae constraints on quintenssence concordance region and target models. Physics review D., 69, 103517.http : || dx.doi.org /10.1103.
8) Tegmartk N. et al(2004)Physics Review D., 69, 103501
9) Allen S. W. et al (2004),Mpn. Not. R. Astron Soc. **353,** 457
10)Perlmutter S.,et al (1997)Measurement of the cosmological parameters Ω and $\wedge$ from the first seven supernovae at $z > 0.35$. The Astrophysical Jounal,483,565http://dx.dvi.org/10.1086/304265
11) Riess A. G. , et al (2000) Publ. Astronsoc. Pac 114, 1284
12) Kalita .S. et al(2010) Late time cosmic acceleration of a flat matter dominated with constant vacuum energy *Indian J. Phys.* ***84****, 629*
13) Chawla C, et al string cosmological model from early deceleration to current acceleration phase with varying G and $\wedge$, *Eur. Phys. J. plus 127, 137 (2012)*
14) Pradhan, et al Bianchi type I transit cosmological models with time dependent gravitational and cosmological constant ,*Indian J phys 88(7), 757 (2014)*
15)Hogan J, Nature **448,**240(2007)
16)Corasaniti P. S. et al. phys. Rev. D **70**. 083006(2004)
17) Weller J, A.M. Lewis, Mon. Not. Astron. Soc**. 346**, 987 (2003)
18) Caldwell R. R. , Phys. Lett. B545, 23(2002)
19) Cadwell R. R. ,Kamiokowski, Weinberg N. N. Phys. Lett. B91,071301 . (2003)
20)S. Nojori and S.D. Odintsov, Int. J. Geom, Meth Mod. Phys .4 ,115 (2007)
21) Weyl H. Ann physics 59 (1919) 101
22) Eddington A.S. The mathematical theory of relativity (Cambridge university press, Cambridge, 1923)
23) Buchdahl H.A. Mon.Not.Astr.soc.150(1970)1.
24) Bertolami o, et al "Extra force in f(R) modified theories of gravity .Phys.RevD75(2007)104016 arXiv:0704-1733 $\left[gr-qc\right]$
25) Nojiri, S. & Odintsov S.D. physical Review D 68, 123512(2003)
26) Nojiri S and Odintsov S.D. P 266-285 (TSPU publishing Tomsk) arxiv, 0807.0685
27) Nojiri S and OdintsovS.D . Phys. Rev. D 78(2008)046006
28)T. P. Sotiriou , Class. Quantum Grav.**23**, 5117 (2006)
29) L. Amendola, D. Polarski, and S. Tsujikawa, Phys. Rev. Lett. **98**, 131302 (2007)
30) M. Sharif and M. F. Shamir , Class Quantum Grav. **26** , 235020 (2009)
31) M. Sharif and M. F. Shamir , Gen. Relativ. Gravil.**42**, 2643 (2010)
32) M. F. Shamir and Z. Raza , Can. J. Phys. **93**, 37 (2015)
33) T. P. Sotiriou and V. Faraoni , Rev. Mod. Phys. **82**, 451 (2010)
34) E. Elizalde, S. Nojiri, S. D. Odinstov ,and D. Saez-GomezEur. Phys. J. C **70** 351 (2010)
35) K. Bamba, C. Geng, S. Nojiri,and S. D. Odinstov Mod. Phys. Lett.A **25** , 900 (2010)
36) S. Nojiri and S. D. Odinstov Phys. Rep. **505** , 59 (2011)
37)S. Capozziello and S. Vignolo, Int. J. Goem. Meth .Mod. Phys**. 8**, 167 (2011)

38) S. Capozziello and F. Darabi and D. Vernieri Mod.Phys.Lett. A **26** , 65 (2011)
39) S. Capozziello, M. D. Laurentis, S. D. Odinstov, andA. Stabile .*Phys. Rev. D.* ***83***,*064004 (2011).*
40) K. Bamba , S. Nojiri, S. D. Odinstov Phys. Lett. B **698**, 451 (2011)
41) A. D. Felice and S. Tsujikawa , Living Rev. Rel. **13**, 3 (2010)
42) T. Clifton T. G. Ferreira, A. Padilla, and C. S kordis, Phys. Rep. 513, 1 (2012)
43) K. Bamba , A. N. Makarenko, A. N. MyagkyS. Nojiri and S. D. Odinstov *JCAP 01, 008, (2014)*
44) K. Bamba ,S. Nojiri , S. D. Odinstov and D. Saez-Gomez, Phys. Lett. B 370, 136,(2014)
45)M. F. Shamir ,Zh. Eksp. Teor .Fiz.**146**, 281 (2014)
46)M. F. Shamir and Z. Raza, Comm. Theor. Phys.**62**, 348 (2014)
47)S. Capozziello, A. Stabile and Troisi ,Spherically Symmetric solution in f(R)theory of gravity via the Noether symmetry approach. Class. Quantum Grav.**24** (2007) 2153
48)Multamaki and Vilja I. : Static spherically symmetric solution of modified field equations in f(R)theory of gravity: Phys. Rev. D.**74** (2006)064022
49)Multamaki and Vilja I. : Static spherically symmetric fluid solutions in f(R)theories of gravity: Phys. Rev. D.**76**(2007)064021.
50) L. Hollenstein and F. S. N. Lobo, Phys. Rev. D. **78** ,124007 (2008)
51) A. Azadi , D. Momeni and M. Nouri – Zonoz, Phys. Lett. B 670 (2008) 210
52) D. Momeni et al:A note on constant curvature solution in cylindrically symmetric metric f(R) gravity. Int. J. Mod. Phys. D 18 , 1 (2009)
53) Kaluza T.:*Sitz-preuss.Akad.Wiss.* D **33**, 966-972.
54). Kelin O.: Z. Phys. **37**,895-906. (1926)

55)Wesson, P.S. ;Astro. andAstrophys.119, 145 (1983).
56) Wesson, P.S. ;An embedding for general relativity with variable rest mass, Gen. Relativ. Grav.**16**, 193 (1984).
57) Ghose and Dadhich : arxiv: gr-qc | 0005085vz (2001)
58) Lorentz and Petzold ; Higher-dimensional Brans-Dicke cosmologies, Gen. Relativ. Grav.**17**, 1189, (1985).
59)Ibanez, Z. and Verdaguer,E**.**: Radiative isotropic cosmologies with extra dimensions, Phys. Rev. D, 34, 1202, (1986).
60) Reddy, D.R.K. and Venkateswara Rao,N. ; Some Cosmological Models in Scalar-Tensor theory of Gravitation, Astro. phys. Space Sci. 277, 461, (2001).
61) Adhav. K.S. *et al*.: Astrophysics and Space Science, **310**(3-4), 231, (2007).
62) K. D. Patil: Indian Academy of Sciences Vol. 60 , No 3 pp 423- 431
63) M. F. Shamir , Exploring plane symmetric solutions in f(R) gravity . TOM 149, BbIII. 2, CTP. 382-388, (2016)