Article

A Cosmological Model Explained with Lambda Term

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Abstract

In this paper, we have studied the LRS Bianchi type-I cosmological model filled with viscous fluid and time varying cosmological term Λ. Exact solution of Einstein field equations are obtained by considering a condition between metric potentials. It is assumed that coefficient of viscosity of bulk viscous fluid is a power function of mass density whereas coefficient of shear viscosity is proportional to expansion scalar in the model of the universe. We have found that the cosmological term Λ is positive and is a decreasing function of time t. A careful analysis of all the physical parameters of the model has also been carried out.

Keywords: LRS Bianchi Type I cosmological model, viscous fluid, cosmological constant.

Introduction

The Bianchi type I cosmological models, which are spatially homogeneous and anisotropic, plays an important role in the description of the universe at its early stages of evolution. At early times in the evolution of the universe most of the matter and radiation dominated but currently observed and have been created during the inflation. Recent experimental data for the existence of an anisotropic phase approaching to isotropic phase lead to investigating the models of the universe with anisotropic background and observational data suggests that our universe in accelerating. Supernova 1A data gave the first indication of the accelerated expansion of the universe [1-4]. Astrophysical observations indicate that the accelerated expansion of the universe is driven by exotic energy with large negative pressure which is known as dark energy [5].In Einstein's theory of gravity, the cosmological term Λ is considered to be a fundamental constant. The problem of cosmological constant is salient yet unsettled in cosmology.

Now a days this cosmological constant is one of the major candidate of dark energy and which is responsible for expansion of universe. Adhav [6] has presented spatially homogeneous and anisotropic Bianchi type-I model whereas Bulk viscosity associated with phase transition may lead to an inflationary scenario [7-10]. Bianchi type-I model with a shear viscosity is the power function of energy density obtained by Banerjee et al. [11] whereas Bianchi type-I model with

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bulk viscosity is studied by Huang [12]. Some of the recent discussions on the cosmological constant have been studied by Dolgov [13,14], Tsagas and Maartens [15], Sahani and Starobinsky [16], Padmnabhan [17,18], Vishwakarma [19-22], Pradhan et al. [23-27] and Singh et al. [28]. Roy and Singh [29], have investigated LRS (Locally Rotationally Symmetric) Bianchi type-I space time with shear and bulk viscous fluid. Pradhan and Shrivastava [30], Bali et.al [31] have investigated Bianchi type-IX cosmological model filled with viscous fluid as a source of matter and shows that the models are expanding, shearing and non-rotating.

A variety of cosmological models with varying Λ term have been tested by researchers to overcome these cosmological puzzles and to explore the accelerating behaviour of the universe R.K. Tiwari et.al [32,37] . In cosmology, bulk viscosity arises an effective pressure to restore the system back to its thermal equilibrium and was broken when the cosmological model with fluid contract too fast [38]. Among the anisotropic Bianchi models, Bianchi type-I cosmological models are the simplest anisotropic universe models which are the generalization of FRW models but the rate of contraction depend on the direction [39,40]. In this paper, we obtain LRS Bianchi type-I cosmological model in presence of Λ. To obtain a solution of field equations, we consider a metric potential as $A = B^m$ .We study different cosmological parameters by assuming $p = \omega\rho$. Some physical and kinematical behaviours of the model are also studied.

The Metric & Field Equations

The spatially homogeneous LRS Bianchi type-I cosmological model is given by the metric

$$ds^2 = dt^2 - A^2 dx^2 - B^2\left(dy^2 + dz^2\right), \tag{1}$$

where A and B are functions of time t only.

Einstein's field equations with cosmological constant Λ (in gravitational units $c = 1, G = 1$) are given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} + \Lambda g_{ij} \tag{2}$$

where R_{ij}, R, g_{ij} are Ricci tensor, Ricci scalar and metric tensor respectively.

Here T_{ij} is the energy momentum tensor for a viscous fluid and has the form

$$T_{ij} = (\rho + \bar{p})\upsilon_i \upsilon_j + \bar{p} g_{ij} - 2\eta\sigma_{ij} \tag{3}$$

where

$$\bar{p} = p - \left(\xi - \frac{2}{3}\eta\right)\upsilon_{i;i} = p - (3\xi - 2\eta)H \tag{4}$$

Here ρ is the energy density, p is pressure, η and ξ are coefficient of shear and bulk viscosity respectively and υ^i is the flow vector satisfying

$$\upsilon_i \upsilon^i = -1 \tag{5}$$

The semicolon denotes covariant differentiation. We choose the coordinates to be commoving, so that

$$\upsilon^1 = 0 = \upsilon^2 = \upsilon^3, \upsilon^4 = 1 \tag{6}$$

The field equations (2) for the metric (1) becomes

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - 2\eta\frac{\dot{A}}{A} = -p + \left(\xi - \frac{2}{3}\eta\right)\theta + \Lambda \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - 2\eta\frac{\dot{B}}{B} = -p + \left(\xi - \frac{2}{3}\eta\right)\theta + \Lambda \tag{8}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = \rho + \Lambda \tag{9}$$

where the dot means ordinary differentiation with respect to t and θ is the scalar of expansion given by

$$\theta = \upsilon_{i;i} \tag{10}$$

The average scale factor (a) for metric (1) is defined by

$$a = \left(AB^2\right)^{1/3} \tag{11}$$

A volume scale factor V for metric is given by

$$V = a^3 = AB^2 \tag{12}$$

The generalized Hubble's parameter H is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(H_1 + H_2 + H_3\right) \tag{13}$$

where $$H_1 = \frac{\dot{A}}{A}, H_2 = H_3 = \frac{\dot{B}}{B} \tag{14}$$

The expansion scalar θ, shear scalar σ, anisotropy parameter A_m and deceleration parameter q are defined as

$$\theta = 3H = \frac{\dot{a}}{a} \tag{15}$$

$$\sigma^2 = \frac{b^2}{3a^6} \tag{16}$$

$$A_m = \frac{2\sigma^2}{3H^2} \tag{17}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \tag{18}$$

Equations (7)-(9) can also be written in terms of Hubble's parameter H, shear scalar σ and deceleration parameter q as

$$p - \Lambda = H^2(2q-1) - \sigma^2 \tag{19}$$

$$\rho + \Lambda = 3H^2 - \sigma^2 \tag{20}$$

From equations (19) and (20), we have

$$\dot{H} + 3H^2 - \frac{1}{2}(\rho - p) - \frac{3}{2}\xi H - \Lambda = 0 \tag{21}$$

This equation is known as Raychaudhari's equation [41].

Solution of the Field Equations

Equations (7)-(9) are three independent equations in seven unknowns A, B, ρ, p, ξ, η and Λ. For the complete determinacy of the system, we need four extra conditions.

Firstly, we assume a metric potential as

$$A = B^m \tag{22}$$

where m is a constant and secondly we assume that the coefficient of shear viscosity is proportional to scalar of expansion, i.e.,

$$\eta \alpha \theta \tag{23}$$

which gives
$$\eta = 3\eta_0 \frac{\dot{a}}{a} \tag{24}$$

where η_0 is a proportionality constant.

We assume that matter content obeys equation of state (EoS) given by

$$p = \omega\rho \quad ; 0 \le \omega \le 1 \tag{25}$$

From equations (7) and (8), we obtain

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{\ddot{A}}{A} - \frac{\dot{A}\dot{B}}{AB} - 2\eta\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0 \tag{26}$$

If $A = B^m$, then from above equation, we get

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - m\frac{\ddot{B}}{B} - \left(m^2 - m\right)\left(\frac{\dot{B}}{B}\right)^2 - m\frac{\dot{B}^2}{B^2} - 2\eta(m-1)\frac{\dot{B}}{B} = 0 \tag{27}$$

Using equation (24) in equation (27)and then integrating eq. (27), we get

$$B = \left[k_1(m+2)(1+2\eta_0)t + k_2\right]^{1/(m+2)(1+2\eta_0)} \tag{28}$$

where k_1 and k_2 are constant of integration.

and using condition (22), we get

$$A = \left[k_1(m+2)(1+2\eta_0)t + k_2\right]^{m/(m+2(1+2\eta_0))} \tag{29}$$

Therefore metric (1) takes the form

$$ds^2 = dt^2 - \left[k_1(m+2)(1+2\eta_0)t + k_2\right]^{\frac{2m}{(m+2)(1+2\eta_0)}} dx^2 - \left[k_1(m+2)(1+2\eta_0)t + k_2\right]^{\frac{2}{(m+2)(1+2\eta_0)}}\left(dy^2 + dz^2\right) \tag{30}$$

Physical & Geometrical Properties

The mean generalized Hubble parameter has the value given by

$$H = k_1\left(\frac{m+2}{3}\right)\frac{k_1}{\left[k_1(m+2)(1+2\eta_0)t+k_2\right]} \tag{31}$$

while the spatial volume 'V' has the form

$$V = \left[k_1(m+2)(1+2\eta_0)t+k_2\right]^{1/1+2\eta_0} \tag{32}$$

The expansion scalar θ is given by

$$\theta = 3H = \frac{k_1(m+2)}{\left[k_1(m+2)(1+2\eta_0)t+k_2\right]} \tag{33}$$

Deceleration parameter q is given as

$$q = 2(1+3\eta_0) \tag{34}$$

However, shear scalar σ is given below as

$$\sigma = \frac{b}{\sqrt{3}\left[k_1(m+2)(1+2\eta_0)t+k_2\right]^{1/1+2\eta_0}} \tag{35}$$

Anisotropy parameter A_m has the value

$$A_m = \frac{2b^2}{k_1^2(m+2)^2\left[k_1(m+2)(1+2\eta_0)t+k_2\right]^{\frac{2(m+1)}{(m+2)(1+2\eta_0)}-2}} \tag{36}$$

Solution for $\xi = \xi_0\theta$

In this case, we obtain energy density ρ, pressure p and cosmological constant Λ as follows:

$$\xi = \xi_0\frac{k_1(m+2)}{\left[k_1(m+2)(1+2\eta_0)t+k_2\right]} \tag{37}$$

$$\rho=\frac{1}{1+\omega}\left[\left(\frac{2k_1^2(m+2)^2(1+2\eta_0)}{3[k_1(m+2)(1+2\eta_0)t+k_2]^2}\right)-\frac{2b^2}{3[k_1(m+2)(1+2\eta_0)t+k_2]^{2/1+2\eta_0}}+\frac{\xi_0k_1^2(m+2)^2}{[k_1(m+2)(1+2\eta_0)t+k_2]^2}\right] \tag{38}$$

$$p=\frac{\omega}{1+\omega}\left[\left(\frac{2k_1^2(m+2)^2(1+2\eta_0)}{3[k_1(m+2)(1+2\eta_0)t+k_2]^2}\right)-\frac{2b^2}{3[k_1(m+2)(1+2\eta_0)t+k_2]^{2/1+2\eta_0}}+\frac{\xi_0k_1^2(m+2)^2}{[k_1(m+2)(1+2\eta_0)t+k_2]^2}\right] \tag{39}$$

$$\Lambda=\frac{k_1^2(m+2)^2}{3[k_1(m+2)(1+2\eta_0)t+k_2]^2}-\frac{k_1^2(m+2)^2(1+2\eta_0)}{3[k_1(m+2)(1+2\eta_0)t+k_2]^2}-\frac{k_1^2(m+2)^2\xi_0}{2[k_1(m+2)(1+2\eta_0)t+k_2]^2}$$
$$-\frac{(1-\omega)}{2(1+\omega)}\left[\left(\frac{2k_1^2(m+2)^2(1+2\eta_0)}{3[k_1(m+2)(1+2\eta_0)t+k_2]^2}\right)-\frac{2b^2}{3[k_1(m+2)(1+2\eta_0)t+k_2]^{2/2\eta_0+1}}+\frac{\xi_0k_1^2(m+2)^2}{[k_1(m+2)(1+2\eta_0)t+k_2]^2}\right] \tag{40}$$

Particular Cases

CaseI: when $m=0$, then the geometry of space time (30) reduces to the form

$$ds^2=dt^2-dx^2-[2k_1(1+2\eta_0)t+k_2]^{\frac{1}{(1+2\eta_0)}}(dy^2+dz^2) \tag{41}$$

The pressure, density and cosmological constant for the model (44) are given by

$$\rho=\frac{1}{1+\omega}\left[\left(\frac{8k_1^2(1+2\eta_0)}{3[2k_1(1+2\eta_0)t+k_2]^2}\right)-\frac{2b^2}{3[2k_1(1+2\eta_0)t+k_2]^{2/1+2\eta_0}}+\frac{4\xi_0k_1^2}{[2k_1(1+2\eta_0)t+k_2]^2}\right] \tag{42}$$

$$p=\frac{\omega}{1+\omega}\left[\left(\frac{8k_1^2(1+2\eta_0)}{3[2k_1(1+2\eta_0)t+k_2]^2}\right)-\frac{2b^2}{3[2k_1(1+2\eta_0)t+k_2]^{2/1+2\eta_0}}+\frac{4\xi_0k_1^2}{[2k_1(1+2\eta_0)t+k_2]^2}\right] \tag{43}$$

$$\Lambda = \frac{4k_1^2}{3[2k_1(1+2\eta_0)t+k_2]^2} - \frac{4k_1^2(1+2\eta_0)}{3[2k_1(1+2\eta_0)t+k_2]^2} - \frac{2k_1^2\xi_0}{[2k_1(1+2\eta_0)t+k_2]^2}$$
$$-\frac{(1-\omega)}{2(1+\omega)}\left[\left(\frac{8k_1^2(1+2\eta_0)}{3[2k_1(1+2\eta_0)t+k_2]^2}\right) - \frac{2b^2}{3[2k_1(1+2\eta_0)t+k_2]^{2/2\eta_0+1}} + \frac{4\xi_0 k_1^2}{[2k_1(1+2\eta_0)t+k_2]^2}\right] \quad (44)$$

CaseII: when $m = 1$, then the geometry of space time (30) reduces to the form

$$ds^2 = dt^2 - [3k_1(1+2\eta_0)t+k_2]^{\frac{2}{3(1+2\eta_0)}}(dx^2+dy^2+dz^2) \quad (45)$$

The pressure, density and cosmological constant for the model (44) are given by

$$\rho = \frac{1}{1+\omega}\left[\left(\frac{6k_1^2(1+2\eta_0)}{[3k_1(1+2\eta_0)t+k_2]^2}\right) - \frac{2b^2}{3[3k_1(1+2\eta_0)t+k_2]^{2/1+2\eta_0}} + \frac{9\xi_0 k_1^2}{[3k_1(1+2\eta_0)t+k_2]^2}\right] \quad (46)$$

$$p = \frac{\omega}{1+\omega}\left[\left(\frac{6k_1^2(1+2\eta_0)}{3[3k_1(1+2\eta_0)t+k_2]^2}\right) - \frac{2b^2}{3[3k_1(1+2\eta_0)t+k_2]^{2/1+2\eta_0}} + \frac{9\xi_0 k_1^2}{[3k_1(1+2\eta_0)t+k_2]^2}\right] \quad (47)$$

$$\Lambda = \frac{3k_1^2}{[3k_1(1+2\eta_0)t+k_2]^2} - \frac{3k_1^2(1+2\eta_0)}{[3k_1(1+2\eta_0)t+k_2]^2} - \frac{9k_1^2\xi_0}{2[3k_1(1+2\eta_0)t+k_2]^2}$$
$$-\frac{(1-\omega)}{2(1+\omega)}\left[\left(\frac{6k_1^2(1+2\eta_0)}{[3k_1(1+2\eta_0)t+k_2]^2}\right) - \frac{2b^2}{3[3k_1(1+2\eta_0)t+k_2]^{2/2\eta_0+1}} + \frac{9\xi_0 k_1^2}{[3k_1(1+2\eta_0)t+k_2]^2}\right] \quad (48)$$

Conclusion

We have studied Einstein's field equations for LRS Bianchi type-I cosmological model filled with viscous fluid as a source of matter. Here we have assumed that the fluid obeys an equation of state $p = \omega\rho$ and bulk viscosity is assumed to be $\xi = \xi_0\theta$

The particular models for the values $m=0$, 1 are obtained. In both cases we observed that the positive cosmological term Λ is a decreasing function of time t and approaches to small value in the present epoch. Further, it is observed that the expansion scalar θ, is a decreasing function of time t and approaches to 0 as $t \to \infty$. Since $\lim_{t\to\infty} \frac{\sigma}{\theta} =$constant, the model is not isotropic for large value of t. Thus we conclude that the cosmological constant is decreasing function of time t which corresponds the recent observations.

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References

1. A G Riess, et al.: Astrophys. J. 116, 1009 (1998)
2. A G Riess, et al.: Astrophys. J. 607, 665 (2004)
3. S Perlmutter, et al.: Astrophys. J. 517, 565 (1999)
4. C Bennet, et al.: Astrophys. J. Suppl. Ser. 148, 1 (2003)
5. J P Singh, Astrophys. Space Sci. 318, 103-107 (2008)
6. K S Adhav, Astrophys. Space Sci. 339, 365 (2012)
7. P Langacker, Phys. Rep., 72, 185 (1981)
8. L Waga, R C Falcan, R Chanda, Phys. Rev., D33, 1839 (1986)
9. T Pacher, J A Stein-Schabas, M S Turner, Phys. Rev., D36, 1603 (1987)
10. A Guth, Phys. Rev., D23, 347 (1981)
11. A Banerjee, S B Duttachaudhari, Sanyal, Math., Phys., 26, 3010 (1985)
12. W Huang, J Math., Phys., 31, 1456 (1990)
13. A D Dolgov, Phys. Rev.D 48, 249 (1993)
14. A D Dolgov, Phys. Rev.D 55, 5881 (1997)
15. C G Tsagas, R Maartens, gr-qc/9912022 (2000)
16. V Sahni, AStarobinsky, Int. J. Mod. Phys. D q, 373 (2000)
17. T Padmnabhan, Phys. Rep. 380, 235; hep-th/02122990 (2003)
18. T Padmnabhan, S M Chitre, Phys. Lett. A. 120, 43 (1987)
19. R G Vishwakarma, Abdussattar, Phys. Rev.D 60, 063507 (1999)
20. R G Vishwakarma, Class. Quant. Grav.17, 3833 (2000)
21. R G Vishwakarma, Gen. Rel. Grav.33, 1973 (2001)
22. R G Vishwakarma, Class. Quant. Grav., **19**, 474 (2002)
23. A Pradhan, V K Yadav, I Chakraborty, Int. J. Mod. Phys.D 10, 339 (2001)
24. A Pradhan, V K Yadav, N NSaste, Int. J. Mod. Phys.D 11, 857 (2002)
25. A Pradhan, K Yadav, Int. J. Mod. Phys.D 11, 839 (2002)
26. A Pradhan, H R Pandey, Int. J. Mod. Phys.D 12, 941(2003)
27. A Pradhan, A K Singh, S Otarod, Rom. Journ. Phys.52, 445 (2007)
28. G P Singh, S Kotambkar, D Srivastava, A Pradhan, Rom. Journ. Phys. 53, 607 (2007)
29. S R Roy and J P Singh, Astrophys. Space Sci. 96, 303 (1983)
30. A Pradhan, S K Shrivastava: gr-qc/030/039 (2006)
31. R Bali, J P Singh, Int. J. Theor. Phys. D 10, 1007/15/0773 -008-9764-4 (2008)
32. R K Tiwari, Astrophys. Astron. 10, 4 (2010)

33. R K Tiwari, Astrophys. Space Sci. 318, 243, 2008
34. R KTiwari Singh, R.: Eur. Phys. J. Plus **129**, 253 (2014)
35. R KTiwari Singh, R.: Int. J. Theor. Phys. **54**, 1417 (2015)
36. R K Tiwari Singh, R.: Int. J. Theor. Phys. **54**, 2652 (2015)
37. R.K.Tiwari Singh, R.: Astrophys. Space Sci. **357**, 130 (2015)
38. S Weinberg, "The cosmological constant problem", Review of Modern Physics, vol. 61, no.1, (1989).
39. G P Singh and A Beesham, Australian Journal of Physics52, 6 (1999).
40. J M Overduin, Astrophysical Journal, 517, (1999).
41. A K Raychaudhuri, Phys. Rev. 108, 171 (1967).