Article

The Interrelationship of Spin and Scales

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Abstract

A possible relationship between Spin, Scales and Supersymmetry is proposed, and an interesting application relevant to Grand Unified Theories is presented.

Key Words: spin, scale, interrelationship, supersymmetry, Grand Unified Theory.

Theory

Postulate 1 – Different intrinsic spins correspond to different scales.

Table 1 – The Interrelationship of Spin and Scales

Spin (in ħ)	Statistics	"Complexergy" Scale
0	Boson – Continuous	Sub-Quantum
½	Fermion – Discrete	Quantum
1	Boson – Continuous	Classical
3⁄2	Fermion – Discrete	Cosmic
2	Boson - Continuous	Super-Cosmic

The Minimal Supersymmetric Standard Model (MSSM) [1] is a model that solves the Hierarchy Problem (Why are observed masses so small in comparison to the GUT and Planck Scales?) of High-Energy Physics by introducing a new Weak-Scale Symmetry between fermions and bosons (Supersymmetry – SUSY). It is interesting to note that the Hierarchy Problem is a Scale Problem. The MSSM introduces $\text{spin-}\tfrac{1}{2}\hbar$ bosinos for each $\text{spin-}1\hbar$ vector boson, and thus connects the Classical "Field" scale to the Quantum "Fundamental Particle" scale and marries concepts of fundamental particles and fields with structures that duplicate the Direct and Reciprocal lattices of Solid State Physics. Likewise, the MSSM introduces spin-0 sfermions for each $\text{spin-}\tfrac{1}{2}\hbar$ fermion, and thus connects the Quantum "Fundamental Particle" scale to the Sub-Quantum "Hidden Variable" scale and introduces the origin of mass via

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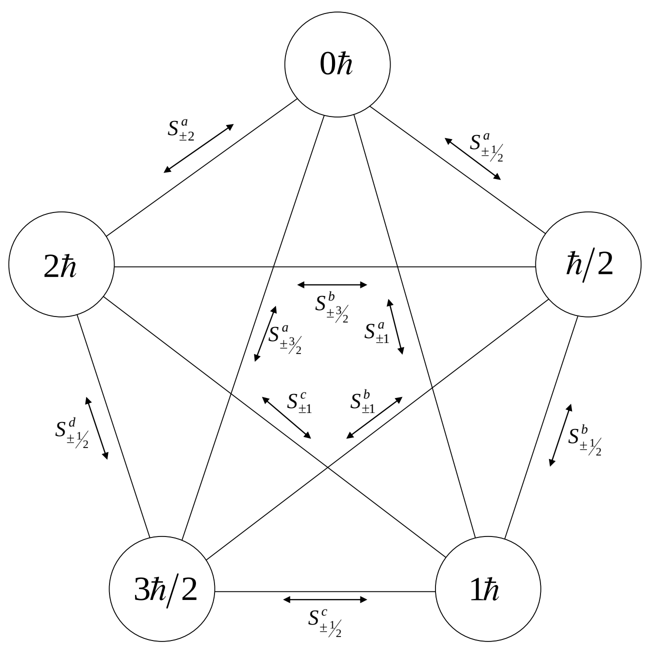
"scalar fermions" / tachyons / ghosts [2]. Supersymmetry (SUSY) also requires $\text{spin-}\frac{3}{2}\hbar$ gravitino partners to $\text{spin-}2\hbar$ gravitons, and thus begins to envelope the Cosmic scales. Laurent Nottale [3,4] expects two unobservable scales – one smaller than the smallest observable scale (the Sub-Quantum "Hidden Variable" scale is hidden within the Quantum "Fundamental Particle" scale), and one larger than the largest observable scale (the Super-Cosmic "Multiverse" scale is hidden beyond the Cosmic "Relativistic Universe" scale). Nottale's "Complexergy" implies that more information-rich scales would also contain more energy and be larger in size. This leads to a minimum of five distinct scales, and implies that a corrected version of SUSY (that the author calls Hyper-SUSY) must unite at least five different intrinsic spins (and corresponding scales): $(0, \frac{1}{2}, 1, \frac{3}{2}, 2)\hbar$, and is thus based on at least an $SU(5)_{S2} \to SO(5)_{S2+} \times SO(5)_{S2-}$ Lie Algebra [5]. The twenty off-diagonal components of this $SU(5)_{S2} \to SO(5)_{S2+} \times SO(5)_{S2-}$ are schematically represented in Figure 1, where the notation is a refinement of Table 15 (page 73) of Reference [5]. The assumption of a Weak-Scale SUSY may be valid for a simple SUSY, such as $S^b_{\pm 1/2}$, that unites the Classical and Quantum scales. However a Hyper-SUSY that unites all five scales of Table 1 need not necessarily obey those simple constraints, and may ultimately involve all of the coupling scales (color, electromagnetic, weak, gravitational, etc.?).

Figure 1 – Petrie Polygon of a 4-D *SU*(5) Pentachoron of Hyper-SUSY



Application

Previously, the author attempted to use a new Quartic-Energy (Energy-to-the-fourth-power) form of Quantum Statistical Mechanics to describe the observed force couplings in terms of the average occupation value of a proposed Grand Unified Mediating (GUM) boson [5]. It was assumed that the various quantum states of the GUM boson were gluons, photons, W's, Z's and gravitons; and that the GUM "boson" must therefore obey Bose-Einstein statistics. However, if quantum-scaled photons obey Bose-Einstein statistics, then hidden sub-quantum-scaled GUM "bosons" should obey Fermi-Dirac statistics. A bounded Fermi-Dirac $[0,1]$ distribution actually fits the observed asymptotic freedom of Quantum Chromo-Dynamics (QCD – the strongest force and therefore most sensitive to the applied statistics) better than an unbounded Bose-Einstein $[0,\infty)$ distribution. This bears similarities to the Cooper pair [6] phenomena of Solid State Physics – whereby a pair of electrons (quantum-scaled fermions) behaves like a composite, classical-scaled boson. Now an even number of hidden sub-quantum scaled fermions behaves like a quantum-scaled GUM "boson".

Following the steps of Reference [5], the use of Quartic-Energy Statistics is justified by the phenomenology of Table 2 (with ratios of quartic integers), but requires replacing Bose-Einstein statistics with Fermi-Dirac statistics so that we may obtain the coupling strengths / occupation values for the GUM "boson".

Table 2 - A Thermodynamic Pattern

Force	S = Strength	$-\ln(S)$	Ratios of ln's vs. Quartic Integers
Strong Nuclear	~ 1	$\sim \pm 0$	$\begin{cases} 4.9202/(\sim \pm 0) \\ \sim \pm\infty \neq 16. = 16/1 \end{cases}$
Electromagnetic	7.29735×10^{-3}	4.92024	$\begin{cases} 26.517/4.9202 \\ = 5.39 \approx 5.06 = 81/16 \end{cases}$
Weak Nuclear	3.0456×10^{-12}	26.5173	$\begin{cases} 88.025/26.517 \\ = 3.32 \approx 3.16 = 256/81 \end{cases}$
Gravitational	5.9061×10^{-39}	88.025	

Postulate 2: We assume that all force-carrying bosons are different states of the Grand-Unified Mediating (GUM) boson.

Postulate 3: We assume that the coupling strengths of the fundamental forces are proportional to the respective density of states of the GUM boson, which can be modeled with the appropriate type of Quantum-Statistical Mechanics.

Using quartic-energy statistics to model the density of states for GUM bosons will build a framework that will be suitable for both bradyons and tachyons, and thus model known particles and the vev associated with the undiscovered Higgs boson. We will assume that the only variable that makes these different types of GUM bosons behave differently is their quantum number, *n*. The corresponding quartic-energy is denoted Q_n.

In a massless wave-particle model, these GUM bosons are quanta with energy-momentum vectors of $\hbar k_\alpha$ and two allowable spin polarization states. If we further impose an ad-hoc condition in the form of periodic boundary conditions (such as Kaluza-Klein momentum or winding-mode excitations on a string) on our GUM bosons, then we will also have quantized momentum vectors. Thus, using a massless wave-particle model for the density of states for GUM bosons is similar to the problem of blackbody radiation, only done in quartic-energy intervals with quantized string momenta and a Fermi-Dirac distribution.

Imagine a blackbody cavity with *D* string membrane dimensions, each of length *L*, and at temperature *T* that is so dense that it is opaque to GUM boson radiation. Also assume that GUM boson division and recombination occurs at the walls of the cavity without any loss or gain of quartic-energy. This assumption allows us to approximate the "quartic chemical potential" μ^4 as zero. There will be an infinite number of standing wave GUM boson modes with momenta $p_\alpha = n_\alpha \hbar k_\alpha$, and a uniform momentum space separation of $\hbar k_\alpha$ on a string membrane. The "hyper-surface-area" that encloses a *D*-dimensional hypersphere of radius *r* is given by:

$$A(D,r) = 2\pi^{(D/2)} r^{(D-1)} / \Gamma(D/2) \tag{1}$$

where $\Gamma(z) = (z-1)!$ with $\Gamma(1/2) = \sqrt{\pi}$ and $\Gamma(1) = 1$ is the gamma function [7]. This leads to a string momentum space density of states:

$$g(p)\,dp = \sum_{\alpha,spin}^{hypersurface} n_\alpha dn = 4\left(L^2/4\pi\hbar^2\right)^{(D/2)} p^D\left(p^{-1}dp\right)/\Gamma(D/2) \tag{2}$$

Assuming periodic boundary conditions, $Q_n = n^4 Q_1 = n^4(\pi\hbar c/L)^4$, and using a change of variables, $Q = p^4 c^4$, so that $Q^{-1}dQ = 4p^{-1}dp$, and transforming Equation (2) into a quartic-energy density of states gives:

$$g(n,D,Q_1)\,dQ = n^{(D-4)}(\pi/4)^{(D/2)} Q_1^{-1} dQ \,/\Gamma(D/2) \tag{3}$$

Note that we are using Quantum Statistical Mechanics because our GUM bosons have integral quantized spins. This extra quantization condition, $Q_n = n^4 Q_1$, could be due to Kaluza-Klein momentum excitations on a string combined with $Q = p^4c^4$.

Now the number density of GUM bosons with quartic-energy values between $Q_n - dQ/2$ and $Q_n + dQ/2$ is given by:

$$N\left(n,D,\beta^4,Q_1\right)dQ = CM_n g(n,D,Q_1) f_{FD}\left[\beta^4\left(n^4Q_1 - \mu^4\right)\right]dQ \tag{4}$$

where C is a normalization constant, and M_n is a degrees-of-freedom multiplicity factor to account for eight gluons versus one photon versus three Intermediate Vector Bosons $\left(IVB's = W^{\pm}, Z^0\right)$.

If we assume a large number of GUM bosons, and that the coupling strength of the $n^{\underline{th}}$ force in this non-interacting independent GUM boson model (denoted by α_n) is proportional to the number density of GUM bosons in the $n^{\underline{th}}$ state, then:

$$\alpha_n = C_1 M_n n^{(D-4)} / \left\{\exp\left[\beta^4\left(n^4Q_1 - \mu^4\right)\right]+1\right\} \tag{5}$$

where $C_1(D)$ is a modified normalization constant containing dimensional factors from Equation (4). These generic GUM bosons condense to yield the properties of gluons, photons, IVB's or gravitons based on Thermodynamic occupation probabilities, and this is the fundamental reason for different valued couplings and charges.

Using the 2008 recommended values of fundamental physical constants [8], the fine structure constant is $\alpha_2 = e^2/4\pi\varepsilon_0\hbar c = 7.297\ 352\ 5376(50)\times 10^{-3}$ where the digits in parenthesis represent the propagated error in the last digit or two. However, the coupling constants for the gravitational (α_4) and weak nuclear (α_3) forces are both mass dependent, and mass is not quantized. Thus, we must allow the masses to equal any reasonable mass (such as electron, pion or proton rest mass) and vary the number of string membrane dimensions, D, over integer values. Coincidentally, the best fit has dimension, D = 3 and gives the modeling results in Table 3.

Table 3 – Results of Quantum Statistical Modeling of Force Strengths

n	$n^{\underline{th}}$ Force	α_n	$n^{\underline{th}}$ Charge	$n^{\underline{th}}$ GUM Boson
1	Strong Nuclear	$11.837\ 839(8)$	Color	8 gluons
2	Electromagnetic	$7.297\ 352\ 5376(50)\times 10^{-3}$	Electric	1 photon
3	Weak Nuclear	$2.965\ 813(9)\times 10^{-12}$	Isospin	3 $W^{\pm}, Z^0$
4	Gravitational	$5.906\ 13(6)\times 10^{-39}$	Mass	1 graviton
5	Fifth (WIMP-Gravity?)	$6.596\ 9(2)\times 10^{-93}$	WIMP-Mass?	15 WIMP-gravitons?

Here, $\mu^4 \approx 0$ is assumed, and $C_1 = 3.565\ 7491(24)$ and $\beta^4 Q_1 = 0.343\ 398\ 077(43)$ are obtained by fitting to the fine structure constant, α_2, and the gravitational force strength between two protons, $\alpha_4 = G_N m_p^2 / \hbar c$. The Fifth Force (WIMP-Gravity of Reference [5]) may ultimately play a greater role in the Super-Cosmic Scale of (assumed) greater complexergy.

It is interesting to note that the sum of coupling strengths $\sum \alpha_n = 11.845$ is close to 4π – as we might expect from the surface area of our sphere $A = 4\pi r^2$ in Equation (1). We can adjust our normalization with a negative "quartic chemical potential" μ^4 such that we obtain the right-most column of Table 4 with $\beta^4 \mu^4 = -0.15045$, $C_1 = 4.142\ 309$ and $\beta^4 Q_1 = 0.343\ 398$.

Table 4 – The Effect of "Quartic Chemical Potential" on Normalization

n	$n^{\underline{th}}$ Force	α_n for $\beta^4 \mu^4 = 0$	α_n for $\beta^4 \mu^4 = -0.15045$
1	Strong Nuclear	11.838	12.559
2	Electromagnetic	$7.297\ 352\ 538 \times 10^{-3}$	$7.297\ 352\ 538 \times 10^{-3}$
3	Weak Nuclear	$2.965\ 8 \times 10^{-12}$	$2.964\ 1 \times 10^{-12}$
4	Gravitational	$5.906\ 13 \times 10^{-39}$	$5.906\ 13 \times 10^{-39}$
5	Fifth	$\sim 10^{-92}$	$\sim 10^{-92}$
…	Sum	11.845…	$4\pi = 12.566\ldots$

The modeled α_3 is slightly less than the best value for weak leptonic (muon) decay expected at momentum transfers equal to the electron rest mass, $\alpha_3 = G_F m_e^2 / (\hbar c)^3 = 3.045\ 624(27) \times 10^{-12}$, but is very close to the expected value for weak nucleonic (neutron) decay – which is weaker because quark states mix. The α_3 variance in Table 3 may be due to effects from the Cabibbo angle and the CKM matrix [9,10], and may imply that $V_{ud} = 0.9737947(85) = \dfrac{2.965\ 813(09)\ e-12}{3.045\ 624(27)\ e-12}$ vs. the accepted value of $V_{ud} = 0.97418(27)$ [8] – a variance between this simple theory (not even accounting for radiative corrections) and the best data fit of 1.4σ – close enough to take seriously.

Conclusion

Realistically, many of these numbers fit better with this unexpected use of Fermi-Dirac, rather the Bose-Einstein (as used in Reference [5]), Statistics. The only apparent explanation for this anomaly is that an unobservable Sub-Quantum scale exists, wherein GUM bosons are composite systems of an even number of fermions. Spin, Scales and SUSY may be more closely related than we previously realized.

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