Exploration

Planck Radiation Formula for Massive Photons

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Abstract

Having demonstrated earlier that massive photons should, *in principle*, exist without any such problems as (1) violation of gauge symmetry, (2) short lifetime, (3) short range, (4) extra degree of freedom, and (5) speed less than the speed of light, we compute the corresponding *Planck Radiation Law* for these massive photons. We find that these massive photons aught to obey modified *Stefan-Boltzmann Law* that is readily testable in the laboratory. This interesting finding maybe helpful in solving the long-standing issue of photon mass.

Keywords: Massive photon, Planck radiation, gauge symmetry, Stefan-Boltzmann Law.

1 Introduction

Since it was first conceived and understood that the photon mass is of fundamental importance to physics, a number of dedicated experiments have been designed and conducted to determine this mass (see *e.g.*, Tu et al. 2005, Tu & Luo 2004). In this paper – *via*, a modified Stefan-Boltzmann Law applicable to the hypothetical massive photon, we propose an experiment for determining the mass of photon. The Stefan-Boltzmann Law is given by:

$$\varepsilon = \epsilon\sigma_0 T^4, \tag{1.1}$$

where $\sigma_0 = 5.670373 \times 10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$ is constant of proportionality known as the Stefan-Boltzmann constant and ε is the total energy density radiated per unit time by a black-body radiating at a steady temperature T and ϵ is the emissivity of the black-body and this is a measure of an object's ability to emit thermal energy. A perfect black-body has ($\epsilon \equiv 1$). Quantitatively, emissivity is the ratio of the thermal radiation from a surface to the radiation from an ideal black surface at the same temperature as given by the Stefan-Boltzmann law and the ratio varies from 0 to 1. The Stefan-Boltzmann constant σ_0, is derived from other known *Constants of Nature*, *i.e.*:

$$\sigma_0 = \frac{2\pi^2 k_B^4}{15c^2h^3}, \tag{1.2}$$

where [$k_B = 1.38064852(79) \times 10^{-23}\,\mathrm{JK^{-1}}$] is the Boltzmann constant, ($c = 2.99792458 \times 10^8\,\mathrm{ms^{-1}}$) is the speed of light in *vacuo* and [$h = 6.626070040(81) \times 10^{-34}\,\mathrm{Js}$].

As for our proposed experiment, we demonstrate that if the photon is endowed with a mass (zero or non-zero), the Stephan-Boltzmann Law should read:

$$\varepsilon_0 = \epsilon\sigma_0 T^4 - \eta T^2, \tag{1.3}$$

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where the new extra-term ηT comes in as a result of the photon mass and this term is such that:

$$\eta = \left(\frac{\pi^3 c^2 k_B^2}{3h^3}\right) \mathrm{m}_0^2. \tag{1.4}$$

In equation (1.4) above, m_0 is the rest-mass of the photon. In-principle – from equation (1.3), the mass of the photon can be determined from the y-intercept of the graph of ε/T^2 *vs* T^2. In-practice, one will need to take into account heat losses, thus resulting in a set of different variables to be measured. We will come this in §(7).

We have sought earlier in Nyambuya (2017, 2014) to address the issue of photon mass. We will discuss these issues in §(5). In §(2), we shall present the *Planck Radiation Law* as it is typically presented in any good textbook of *Modern Physics*. Thereafter in §(3), we will present the main theme of this work by deriving the modified Stefan-Boltzmann (*i.e.*, equation 1.3); this is the equation which forms the central theme of this work. In §(4.2), we shall – for completeness and instruction purposes – give a brief exposition of the Maxwell-Proca Electrodynamics and thereafter, in §(5), we shall present the fundamental issues associated with massive photons and therein present our proposed solutions to these problems associated with massive photons. Lastly, in (6) and (7) we give the conclusion drawn thereof and the recommendation, respectively.

2 Planck's Radiation Law for Massless Photons

As already alluded – in this section, we will present Max Planck (1901, 1900*a*,*b*)'s radiation law as it usually presented in any good textbook of *Modern Physics* where the assumption ($\mathrm{m}_0 \equiv 0$) is made. As is well known, the number of quantum states dN in the momentum volume space d^3p and physical volume space V, is given by:

$$dN = \frac{2Vd^3p}{h^3}, \tag{2.1}$$

where h is Planck's constant. The factor 2 in equation (2.1) represents the number of degrees of freedom of the photon *i.e.*, 1 traverse and 1 longitudinal – meaning the photon has two polarization states, hence the factor 2.

Now, given that ($d^3p = 4p^2dp$), it follows that:

$$dN = \frac{8\pi Vp^2dp}{h^3}, \tag{2.2}$$

and further, given that for a photon of momentum p, energy E and frequency ν, its energy is such that ($p = E/c = h\nu/c$), it follows from this, that the number of modes in the frequency interval ($\nu, \nu + d\nu$), is:

$$dN = \left(\frac{8\pi V}{c^3}\right)\nu^2 d\nu. \tag{2.3}$$

The photon energy-momentum equation ($p = E/c = h\nu/c$) assumes ($\mathrm{m}_0 \equiv 0$).

Now, the actual number of occupied states dn is such that $dn = f_{\mathrm{BE}}(\nu, T)dN$, where:

$$f_{\mathrm{BE}}(\nu, T) = \frac{1}{e^{h\nu/k_BT} - 1}, \tag{2.4}$$

is the Bose-Einstein probability function which for a temperature T, it gives the probability of occupation of a quantum state whose energy is ($E = h\nu$). From the foregoing, it follows that:

$$dn = \frac{8\pi V}{c^3}\frac{\nu^2 d\nu}{e^{h\nu/k_BT} - 1}, \tag{2.5}$$

hence, the energy density $[u(\nu,T)d\nu = h\nu dn/V]$ is given by:

$$u(\nu,T)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{e^{h\nu/k_BT}-1}. \tag{2.6}$$

Setting $(x = h\nu/k_BT)$: this implies $(d\nu = k_BTdx/h)$, thus substituting this into the above, we will have:

$$u(\nu,T)d\nu = \frac{8\pi k_B^4T^4}{h^3c^3}\frac{x^3dx}{e^x-1}. \tag{2.7}$$

Further, the total energy density ε is such that:

$$\varepsilon = \frac{c}{4}\int_0^\infty u(\nu,T)d\nu = \frac{2\pi k_B^4T^4}{h^3c^2}\int_0^\infty\frac{x^3dx}{e^x-1}, \tag{2.8}$$

and given that:

$$\int_0^\infty\frac{x^2dx}{e^x-1} = \frac{\pi^4}{15}, \tag{2.9}$$

the Stefan-Boltzmann Law (1.1) follows directly from this and hence, one easily deduces that σ_0 is as given in (1.2). All the above presented is standard textbook material. In the next section, we shall proceed to drop the assumption of a zero-mass for the photon.

However, before we depart this section, we will briefly consider the issue of massive photons insofar as the Planck Radiation Law with regard to the supposed extra degree of freedom of the massive photon. It is well known that a massive photon will add a degree of freedom to the photon (*e.g.*, Lehnert & Roy 2012*b*, Bass & Schrödinger 1955); instead of the usual 2, the 1 traverse and 1 longitudinal, it will have 3 degrees of freedom *i.e.*, 2 traverse and 1 longitudinal (*e.g.*, Lehnert & Roy 2012*b*, Bass & Schrödinger 1955). This will modify the resultant Planck Radiation Law. That is, in the case of equation (2.1), we will have:

$$dN = \frac{3Vd^3p}{h^3}, \tag{2.10}$$

and in the case of equation (2.4), we will have:

$$f_{\mathrm{BE}}(\nu,T) = \frac{1}{e^{3h\nu/2k_BT}-1}, \tag{2.11}$$

hence, these equations (2.10) and (2.11), will ultimately lead to equation (2.6) now being given as:

$$u(\nu,T)d\nu = \frac{12\pi h}{c^3}\frac{\nu^3 d\nu}{e^{3h\nu/2k_BT}-1}. \tag{2.12}$$

Such a radiation law will result in the Stefan-Boltzmann Law (1.1) attaining a factor 8/27, *i.e.*:

$$\varepsilon = \frac{8}{27}\epsilon\sigma_0T^4. \tag{2.13}$$

This factor 8/27 and as-well the 3/2 factor in (2.12); these have not be observed, thus, leading a very strong scepticism against the existence of massive photons.

3 Planck's Radiation Law for Massive Photons

We now will present Planck's radiation law for the case ($\mathrm{m}_0 \neq 0$). Our massive photon model is based on our earlier work (Nyambuya 2017, 2014). In §(5), we will present a discussion of the afore-stated work (Nyambuya 2017, 2014) on massive photons, where we shall once again argue (convincingly) that massive photons should – *in-principle* – pause no problems as they will exhibit all the good behaviour expected of a massless photon – such as, their speed – which is c; their range – which is infinite; their life time – which infinite too, and; their gauge invariant nature – they are gauge invariant. In simple terms, our massive photons exhibit very good behaviour.

Now, in the case ($\mathrm{m}_0 \neq 0$), the momentum is no longer given by ($p = E/c = h\nu/c$) , but is now given by ($p^2 = h^2\nu^2/c^2 - \mathrm{m}_0^2c^2$). According to Nyambuya (2017, 2014), in-order for the proposed massive photons to travel at the speed of light c, it is absolutely necessary that their rest mass be assumed to depend on the massive photon's momentum *i.e.* [$\mathrm{m}_0 = \mathrm{m}_0(p)$] – this assumption implies that:

$$pdp = \left(\frac{h^2}{c^2}\right)\nu d\nu - \mathrm{m}_0c^2d\mathrm{m}_0. \tag{3.1}$$

We shall assume that while $d\mathrm{m}_0$ is not identically equally to zero, it is extremely small – so small that – for all intents and purposes – one can ignore this term; hence [$\mathrm{m}_0c^2d\mathrm{m}_0 \lll (h^2\nu/pc^2)d\nu$], thus – to first order approximation – this leads to [$dp \sim (h\nu/pc)hd\nu/c$]. Again, while ($E \neq pc$) for massive photons, we can – for the good purposes of simplifying the expression [$dp \sim (h\nu/pc)hd\nu/c$], assume that ($E \sim pc$), so that ($dp \sim hd\nu/c$).

Now, substituting ($p^2 = h^2\nu^2/c^2 - \mathrm{m}_0^2c^2$) and ($dp \sim hd\nu/c$) into equation (2.2), it follows that to first order approximation – we will have:

$$dN = \frac{8\pi V}{c^3}\frac{\nu^2 d\nu}{e^{h\nu/k_BT}-1} - \frac{8\pi\mathrm{m}_0^2cV}{h^2}\frac{d\nu}{e^{h\nu/k_BT}-1}. \tag{3.2}$$

hence:

$$u(\nu,T)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{e^{h\nu/k_BT}-1} - \frac{8\pi\mathrm{m}_0^2c}{h}\frac{\nu d\nu}{e^{h\nu/k_BT}-1}. \tag{3.3}$$

As before, setting ($x = h\nu/k_BT$) and substituting this into the above, we will have:

$$u(\nu,T)d\nu = \frac{8\pi k_B^4T^4}{h^3c^3}\frac{x^3dx}{e^x-1} - \frac{8\pi\mathrm{m}_0^2ck_B^2T^2}{h^3}\frac{xdx}{e^x-1}. \tag{3.4}$$

Now, from this – it follows that the total energy density ε, will be such that:

$$\varepsilon = \frac{c}{4}\int_0^\infty u(\nu,T)d\nu = \frac{2\pi k_B^4T^4}{h^3c^2}\int_0^\infty\frac{x^3dx}{e^x-1} - \frac{2\pi\mathrm{m}_0^2c^2k_B^2T^2}{h^3}\int_0^\infty\frac{xdx}{e^x-1}. \tag{3.5}$$

Given that:

$$\int_0^\infty\frac{x^3dx}{e^x-1} = \frac{\pi^4}{15} \qquad \text{and,} \qquad \int_0^\infty\frac{xdx}{e^x-1} = \frac{\pi^2}{6}, \tag{3.6}$$

the modified Stefan-Boltzmann Law (1.3) follows directly from equation (3.5) and hence, one easily deduces that σ_0 and η are as given in (1.2) and (1.4) respectively. Equation (3.3), is the sought-for *Planck Radiation Law for Massive Photons*.

Now, before we go on to discuss how the mass-term in the modified SBL (1.3) can be measured to yield the mass of the photon, we need to discuss the serious problems associated with massive photons and – if possible, demonstrate solutions to these problems. This way, we pave the way for a smooth investigation of massive photons. So, in §(4), we discuss the Maxwell-Proca equations which constitute the most fundamental basis and point of departure for most – if not all – endeavours to probing massive photons. Thereafter in §(5), we shall present the problems of massive photons and a brief exposition of our proffered solution this these problems.

4 Maxwell-Proca Equations

The Maxwell-Proca equations (Proca 1930*a*,*b*,*c*, 1936*c*, 1931, 1936*a*,*b*, 1937, 1938) constitute the most fundamental basis and point of departure for most – if not all – endeavours to probing massive photons. Written in the rich Lagrangian formalism, the Maxwell-Proca Lagrangian $\mathscr{L}_{\mathrm{MPED}}$, is given by:

$$\mathscr{L}_{\mathrm{MPED}} = \overbrace{\underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J_{\mu}A^{\mu}}_{\textbf{Maxwell Term}}}^{\mathscr{L}_{\mathrm{MED}}} + \overbrace{\frac{1}{2}\kappa^2 A_{\mu}A^{\mu}}^{\textbf{Proca Term}}, \tag{4.1}$$

where $F_{\mu\nu}$ is the Maxwell electromagnetic field tensor, A_{μ} is the electromagnetic four vector potential, J_{μ} is four current density and κ^2 is non-zero mass-term of the photon. The resulting source coupled Maxwell-Proca field equations from this Lagrangian (4.1), are:

$$\partial^{\mu}F_{\mu\nu} - \kappa^2 A_{\mu} = J_{\nu}. \tag{4.2}$$

This equation (4.2) is the classical Maxwellian Electrodynamic field equation with it added the Proca mass term, $\kappa^2 A_{\mu}$.

The source coupled Maxwell-Proca equation (4.2) were – perhaps – first written down in their popular form as:

$$\boldsymbol{\nabla} \times \boldsymbol{E} = \varrho/\varepsilon_0 + \kappa^2\Phi_{\mathrm{em}}, \tag{4.3}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0\boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} + \kappa^2\boldsymbol{A}. \tag{4.4}$$

by the great Austrian physicist – Erwin Rudolf Josef Alexander Schrödinger (1887 – 1961), while conducting one of the early Solar system investigations of these equation (Schrödinger 1943*b*,*a*). In equation (4.3) and (4.4), $\boldsymbol{E}$ is electric field, $\boldsymbol{B}$ is the magnetic field, $\boldsymbol{A}$ the magnetic vector potential and Φ_{em} is the electric potential. In seeking a non-zero photon mass, both laboratory measurements (*e.g.*, Accioly et al. 2010, Spavieri & Rodriguez 2007, de Broglie & Vigier 1972, Franken & Ampulski 1971, Goldhaber & Nieto 1971*a*, Williams et al. 1971) and astronomical observations (*e.g.*, Goldhaber & Nieto 1968, Colafrancesco & Marchegiani 2014, Leverett et al. 1975, Goldhaber & Nieto 1971*b*, Accioly & Paszko 2004), focus on the two extra terms $\kappa^2\Phi_{\mathrm{em}}$ and $\kappa^2\boldsymbol{A}$. That is to say, measurements seek to detect the presence of these two terms: $\kappa^2\Phi_{\mathrm{em}}$ and $\kappa^2\boldsymbol{A}$. Their ($\kappa^2\Phi_{\mathrm{em}}$ and $\kappa^2\boldsymbol{A}$) positive detection implies ($\kappa^2 \neq 0$), hence a non-zero photon mass. In the theory developed in the reading Nyambuya (2014), a gauge condition is introduced that sweeps away these terms : $\kappa^2\Phi_{\mathrm{em}}$ and $\kappa^2\boldsymbol{A}$, thus making them non-detectable by means devised by the said laboratory and astronomical measurements thus leading to "misleading" results that the photon mass is zero when this may not be the case. We will talk about this gauge condition of Nyambuya (2014) in the subsequent section.

5 Fundamental Issues with Massive Photons

Massive photons have several issues associated with them rendering their existence almost unfavourable and highly unlikely. The "*Scientific Inquisition*" has long "*ruthlessly*" condemned them [massive photons] "*to suffer eternally in the peripheries of the unknown dungeons of the scientific wildness*". At the top of the list are the following five seemingly inescapable "*cardinal charges*" that have been levelled against massive photons:

1. **Gauge Invariance:** A non-zero photon mass term in Quantum Electrodynamics (QED) would break the highly regarded sacrosanct *Gauge Invariance Principle*, and this gauge invariance violation may very well spoil the renormalizability of the resulting massive QED theory, thus rendering the theory quantum-mechanically inconsistent (*e.g.*, Kouwn et al. 2016, Lehnert & Roy 2012*a*). Under an appropriate choice of the boundary conditions (*e.g.*, Srednicki 2007), the Maxwellian term in equation (4.1), *i.e.*, the term represented by the Maxwellian Electrodynamic Lagrangian $\mathscr{L}_{\rm MED}$ is invariant under the gauge transformation ($A_\mu \mapsto A_\mu + \partial_\mu \chi$), while the Proca-term $\kappa^2 A_\mu A^\mu/2$ is not readily invariant, thus violating (breaking) the gauge symmetry existing in the Maxwellian term. This is a problem.

2. **Speed of Propagation:** If photons are massive, they can not travel at the speed of light, c. Paradoxically, the speed c is the speed with which they (photons) are observed to propagate with in *vacuo*. That is to say, all indications are that photon travel in the *vacuo* at the *vacuo* speed of light c, thus, pointing to a massless photon. From Einstein's momentum energy equation ($E^2 = p^2c^2 + {\rm m}_0^2c^4$) and the experimentally verified fact that for photons ($E = pc$); from these bare facts alone, mathematical logic directly points to (${\rm m}_0 \equiv 0$). The group velocity ($v_{\rm g} = \partial E/\partial p$) for waves packets whose energy is given by ($E = pc$) is naturally c, *i.e.* ($v_{\rm g} = c$) for waves whose energy packets is ($E = pc$) while for energy packets whose energy is ($E^2 = p^2c^2 + {\rm m}_0^2c^4$): ($v_{\rm g} \neq c$).

3. **Stability:** As shall be seen in the proposal that we make herein, in the case of ($\kappa \neq 0$), the *Special Gauge Condition* introduced in Nyambuya (2014), is going to whip away these two terms ($\kappa^2\Phi_{\rm em}$ and $\kappa^2\boldsymbol{A}$) leaving the usual Maxwell's source coupled equations with a vanishing mass term. What this means is that if one tried to use MPED-model to decipher a non-zero mass for the photon, they will not detect a non-zero mass but a vanishing mass because it has been 'whipped' away despite it being non-zero. Therefore, the special gauge condition that we shall introduce renders it difficult if not impossible to detect a non-vanishing photon mass – this obviously puts the question of a non-zero mass into a serious anti-juxtaposition because, in the end, laboratory and astronomical observations that employ the MPED-model to detect a non-vanishing photon mass *via* the terms $\kappa^2\Phi_{\rm em}$ and $\kappa^2\boldsymbol{A}$, these are here made inadequate to discern if the photon truly has a vanishing mass. In-closing, let us call the photon(s) described by MPED-model as presented here – MPED-photon(s).

4. **Range:** Just like the the mediating gauge bosons of the Weak nuclear force, a massive photon can not travel an infinite distance because its range is (must) be limited. All indications are that photons have infinite range thus pointing to a massless photon.

5. **Degrees of Freedom:** A massive photon will have an extra-degree of freedom (2 transverse modes and 1 longitudinal mode – whereas, a massless photon has 1 transverse mode and 1 longitudinal mode) and this degree of freedom will add to the total energy of the photon and must manifest in the Planck radiation law (*e.g.*, Lehnert & Roy 2012*b*, Greiner & Reinhardt 1996, Bass & Schrödinger 1955). All indications are that this hypothetical degree of freedom does not exist because measurement of the Planck radiation law does not support this hypothesis: logically, this obviously and most strongly points to a massless photon.

In the readings Nyambuya (2017, 2014), we have made efforts to address all of the above problems in such a manner that, massive photons should – *in-principle* – not pause any problems – at least for the above mentioned problems. Below we present in brief, an exposition of our proffered solution(s):

1. **Proposed Solution to the Gauge Invariance Problem:** Gauge invariance is a very important symmetry in physics – it is so important that nearly all physicists throughout all of the World are not readily

willing to consider theories that violent this principle. This gauge invariance principle was first (Weyl 1918, 1928, 1929*b*,*a*,*c*) introduced by the great German mathematician, mathematical physicist and philosopher – Professor Herman Klaus Hugo Weyl (1885 − 1955); plays a central role – not only in field theory, but in all of physics – it is a principle without which, modern field theories could not be. However, if one abandons this (gauge symmetry), they can – as the great Romanian physicist Professor Alexandru Proca (1897 − 1955) did (Proca 1930*a*,*b*,*c*, 1936*c*, 1931, 1936*a*,*b*, 1937, 1938); construct – for themselves – an electrodynamic theory were the photon has a non-zero mass.

In a very simple and trivial manner – as did Stückelberg (1938*a*,*c*,*b*), the MPED-Lagrangian can be modified to include a Stüeckelberg scalar Ψ as follows:

$$\mathscr{L}_{\text{MPSED}} = \overbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J_\mu\left(A^\mu - \partial^\mu\Psi\right)}^{\text{Maxwell}-\text{Stückelberg Term}} + \overbrace{\frac{1}{2}\kappa^2\left(A_\mu - \partial_\mu\Psi\right)\left(A^\mu - \partial^\mu\Psi\right)}^{\text{Stückelberg}-\text{Proca Term}}. \tag{5.1}$$

This modified MPED Lagrangian $\mathscr{L}_{\text{MPSED}}$, we have (in the reading, Nyambuya 2014) called the Maxwell-Proca-Stückelberg (MPSED) Lagrangian. This Lagrangian ($\mathscr{L}_{\text{MPSED}}$), is invariant (cf., Adelberger et al. 2007) under the following pair of gauge transformations:

$$\begin{array}{lcl} A_\mu & \mapsto & A_\mu + \dfrac{1}{\kappa}\partial_\mu\chi \\ \Psi & \mapsto & \Psi + \chi \end{array}. \tag{5.2}$$

The resulting field equations (Nyambuya 2014) from this Lagrangian $\mathscr{L}_{\text{MPSED}}$, are:

$$\partial^\mu F_{\mu\nu} - \kappa^2 A_\mu + \kappa^2\partial_\mu\Psi = J_\nu. \tag{5.3}$$

As noted in the reading Nyambuya (2014); because of the existence of the mass-term $\kappa^2 A_\mu$, this equation (5.3) leads to short ranged and short lived (massive) photons – something that is at odds with physical and natural reality as we currently understand. In this same reading Nyambuya (2014), the said problem is overcome by introducing a *Special Gauge Condition* (SGC), a condition that is attained by sacrificing the Lorenz (1867) gauge condition ($\partial^\mu A_\mu = 0$); *i.e.*, the said SCG assumes ($\partial^\mu A_\mu \neq 0$). To see how this SGC works, we shall first unpack the Maxwellian Electromagnetic field tensor $F_{\mu\nu}$, so that this equation (5.3) is now written equivalently as:

$$\Box A_\nu \underbrace{-\partial_\nu\left(\partial^\mu A_\mu\right) - \kappa^2 A_\nu + \kappa\partial_\nu\Psi}_{\textbf{SGC Sets these Terms to Zero}} = J_\nu. \tag{5.4}$$

Now, the proposed SGC that is introduced (in, Nyambuya 2014) so as to attain the desired theory of massive photons that are gauge invariant, long ranged and long lived; is to set the terms in the *under-brace* [in (5.4)] to identically equal zero, *i.e.*:

$$\partial_\nu\left(\partial^\mu A_\mu\right) + \kappa^2 A_\nu - \kappa\partial_\nu\Psi \equiv 0. \tag{5.5}$$

The resulting *Massive Photon Electromagnetic Equations* after the introduction of the SGC, are:

$$\Box A_\nu = J_\nu. \tag{5.6}$$

Equation (5.6) is the same equation as Maxwell (1865)'s source coupled equations of electrodynamics for a massless photons under the Lorenz (1867) gauge ($\partial^\mu A_\mu = 0$). Therefore, with the SGC (5.5) in place, we achieve the attainment of long range, long lived and gauge invariant massive photons. The SGC is actually a *panacea* to the problems that one can ever think or conceive of that can be associated with massive photons because this SGC causes these massive photons to have the exact same behavioural properties that are expected of a massless photon. In this way, we have shown that massive photons can exist without the three major problems that are normally associated with massive photons, that is, the problems of them being short ranged, short lived and non-gauge invariant. This achievement has come at a "severe" cost, namely that the Lorenz gauge ($\partial^\mu A_\mu = 0$) is to be sacrificed at the SGC-alter. If A_μ and Ψ are assumed to be related by:

$$\partial^\mu A_\mu = \left(\frac{\mathrm{g}_\mathrm{s}^2 \kappa}{1+\mathrm{g}_\mathrm{s}^2}\right)\Psi, \tag{5.7}$$

where ($\mathrm{g}_\mathrm{s}^2 \neq 0$) is dimensionless constant – this constant is a fundamental and universal constant just as is the Planck constant $\hbar$, Newton's universal constant of gravitation G *etc*; then, Ψ satisfies the Klein-Gordon equation (Klein 1926, Gordon 1926). With equation (5.7) as given, it follows that taking the four divergence of the new gauge condition (5.5), one will have:

$$\Box\left(\partial^\mu A_\mu\right) + \kappa^2\left(\partial^\mu A_\mu\right) - \kappa\Box\Psi \equiv 0, \tag{5.8}$$

and given (5.7), it follows that the resulting equation is an equation for the field Ψ and this equation is the Klein-Gordon equation (Klein 1926, Gordon 1926) for the Stückelburg scalar Ψ, *i.e.*:

$$\Box\Psi = \mathrm{g}_\mathrm{s}^2\kappa^2\Psi, \tag{5.9}$$

If this constant (g_s^2) is equal to zero, then we are back to the usual Lorenz (1867) gauge and, this leads us to the normal MPSED theory.

2. **Proposed Solution to the Stability Problem:** Despite the endowment of a non-zero mass to the photon, the resulting theory obtained in the Maxwell-Stückelburg-Proca Electrodynamics under the special gauge condition are exactly the massless electrodynamic equations of Maxwell, hence the resulting photon as exactly to exhibit the behaviour of massless photon insofar as is stability is concerned.

3. **Proposed Solution to the Range Problem:** Just as in the solution to the stability problem of massive photons, despite the endowment of a non-zero mass to the photon, as stated above – the resulting theory obtained in the Maxwell-Stückelburg-Proca Electrodynamics under the special gauge condition are exactly the massless electrodynamic equations of Maxwell, hence the resulting photon is expected to exhibit the usual behaviour of massless photon insofar as range is concerned – this photon will be long ranged.

4. **Proposed Solution to the Degrees of Freedom Problem:** The real reason why a massive photon will have an extra-degree of freedom is because it [massive photon] should have a frame of reference in which it is at rest (*e.g.*, Bass & Schrödinger 1955). What this means is that if a massive photon where to have no frame of reference in which it is at rest, then, the issue of the extra degree of freedom would forthwith drop by the wayside. This is exactly what is demonstrated in the readings Nyambuya (2017, 2014). That is to say, in-order for the photon to travel at the speed of light c in *vacuo*, the hypothesis is made – *therein* – that its rest mass is to have a functional dependence on its momentum *i.e.* [$\mathrm{m}_0 = \mathrm{m}_0(p)$]. Once this hypothesis is made and accepted, the group velocity ($c_\mathrm{g} = \partial E/\partial p$) of the massive photon is – for better or for worse – set so that it is identically equal to c in all the possible frames of reference (*i.e.*, $c_\mathrm{g} = c$), hence – there, is not a single frame of reference – *in all of the Universe* – in which this photon will be observed to be at rest. In this way, the degree of freedom problem is solved and the resulting energy for the photon is (Nyambuya 2017):

$$E = pc + \underbrace{\left[\frac{d\mathrm{m}_0(p)c}{dp}\right]\mathrm{m}_0(p)c^2}_{\epsilon_\gamma}, \tag{5.10}$$

where the additional new energy term ϵ_γ is expected to be extremely tiny so much that the current limits and capabilities of experimental measurements have not been able to flash this term out; in this way, one finds justification as to why currently experimental measurements obtain that the energy E of the photon is such that $(E = pc)$. The smallness of ϵ_γ should arise from $d\mathrm{m}_0(p)/dp$ being the small term and not $\mathrm{m}_0(p)$.

5. **Proposed Solution to the Speed Problem:** As stated above in the solution to the "problem of the degrees of freedom for a massive photon", the group velocity of a massive photon is the usual speed of light c in *vacuo* and this is possible if the rest mass of the photon is assumed to have a momentum dependence (Nyambuya 2017, 2014).

Clearly, from the above discussion, massive, long ranged, long lived and gauge invariant photons that travel at the speed of light c in *vacuo* should, *in principle* be feasible. These photons will have the exact same properties of Maxwell's massless photons!

6 Conclusion

In principle, if what has been presented herein is reasonable and acceptable, then it should be possible to measure – *albeit*, in a high precession experiments – the mass of the photon using *e.g.*, the usual simple experiment of the tungsten bulb to measure the Stefan-Boltzmann constant σ_0 and the exponent of the Stefan-Boltzmann Law. From this experiment, the value of η is to be inferred from the y-intercept of the graph of ε/T^2 *vs* T^2. The value of η, whether zero or non-zero, this will – according to the massive-photon model described here, yield the mass of the photon.

7 Recommendation

We here make a recommendation directed to the experimentalist. Experiments to measure the SBL has been conducted several times (Carlà 2013, Clauss et al. 2001, Prasad & Mascarenhas 1978, Edmonds 1968) and this experiment is actually a standard laboratory experiments for students (Ahmad et al. 2010, Wray 1975). In these experiments, the thrust is not so much to determine both the the Stefan-Boltzmann constant σ_0 and the exponent of this law but to determine the exponent – to confirm that – within acceptable experimental limits, this exponent is indeed 4. The experimental setup us takes into account the heat losses that the actual law to be tested is (*e.g.*, Carlà 2013):

$$P = \overbrace{\epsilon\sigma_0\mathcal{A}\left(T^\alpha - T^\alpha_{\mathrm{bg}}\right)}^{\textbf{Stephan-Boltzmann Term}} + \overbrace{\lambda_N\mathcal{A}\left(T - T_{\mathrm{bg}}\right)^\beta}^{\textbf{Newton's Cooling Term}} , \tag{7.1}$$

where λ_N is *Newton's Parameter for Cooling* and $\mathcal{A}$ is the radiating surface area of the lamp. This equation (7.1) can be written as:

$$P = a\left(T^\alpha - T^\alpha_{\mathrm{bg}}\right) + b\left(T - T_{\mathrm{bg}}\right)^\beta , \tag{7.2}$$

where $(a = \epsilon\sigma_0\mathcal{A})$ and $(b = \lambda_N\mathcal{A})$. In the experiments, what is being determined are the four parameters (a, α, b, β). In the same vein, if we are to test the modified SBL (1.3), the actual equation to be tested – following the above described – would be:

$$P = \epsilon \overbrace{\sigma_0\mathcal{A}\left(T^\alpha - T^\alpha_{\mathrm{bg}}\right)}^{\textbf{Stephan-Boltzmann Term}} + \overbrace{\lambda_N\mathcal{A}\left(T - T_{\mathrm{bg}}\right)^\beta}^{\textbf{Newton's Cooling Term}} - \overbrace{\eta\mathcal{A}\left(T^2 - T^2_{\mathrm{bg}}\right)}^{\textbf{Massive Photon Term}} . \tag{7.3}$$

In this experiment whose aim is to measure η, the SBL is to assumed as holding identically and the purpose of this is to minimize the number of free parameters and in addition to this, Newton's law of cooling is to be assumed *i.e.*, having an exponent of unity: therefore, we propose that the same typical experiments of the tungsten bulb be conducted – *albeit* – taking $(\alpha \equiv 4)$ and $(\beta \equiv 1)$. This experiment is to be conducted with much greater exactness in-order to get an as accurate value of η as is possible, hence – an accurate value for the rest mass of the photon. If the suggested assumptions are made [*i.e.*, $(\alpha \equiv 4)$ and $(\beta \equiv 1)$], then, the above law (7.3) will reduce to:

$$P = a\left(T^4 - T_{\rm bg}^4\right) + b\left(T - T_{\rm bg}\right) + c\left(T^2 - T_{\rm bg}^2\right), \tag{7.4}$$

where $(c = -\eta\mathcal{A})$. The law (7.4) to be tested has only three free parameters: (a, b, c).

We reiterate – that delicate and dedicated experiments be conducted to test (7.4) with the thrust of the experiment being to ascertain the value of c, that is, whether or not it is statistically significant from zero? If yes, then, the mass of the photon can surely be ascertained by more accurate follow-up experiments carefully designed to pin down the value of η hence the mass of the photon. Perhaps – we should say that, we have taken up Carlà (2013)'s data and processed it in-order to make a *prima facie* deduction of the photon mass and we are getting a non-zero value, thus implying a non-zero mass. We have this work at an advanced stage of preparation into a journal article and – we should say of it – that, this said work may stimulate searches for a non-zero rest mass using (7.4). The said results – in the journal paper under preparation – are only an pointer to a non-zero mass. As said, only a dedicated experiment to test equation (7.4) will be believable.

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