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Bianchi Type-II Oscillating String Cosmological Model in $f(R,T)$ Gravity

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Abstract

A spatially homogeneous and anisotropic Bianchi type-II cosmological model is obtained in $f(R,T)$ theory of gravity (Harko et al., Phys. Rev. D 84, 024020, 2011) when the source for energy momentum tensor is one-dimensional cosmic strings. We obtained a string cosmological model using a time periodic varying deceleration parameter proposed by Shen and Jiang (2014). Some physical and kinematical properties of the model are also discussed.

Key words: Bianchi type-II, string model, oscillating universe, deceleration parameter.

1 Introduction

The study of string theory has received considerable attention in cosmology. String cosmological models are attracting more and more attention of research workers since cosmic strings are important in the early stages of evaluation of the universe before the particle creation. Spontaneous symmetry breaking in elementary particle physics has given rise to topological defects known as cosmic strings. These are line like structures which arise due to spontaneous symmetry breaking during phase transition in the early universe. The gravitational effects of such objects are of practical interest since they are considered as possible seeds for galaxy formation and gravitational lenses. Letelier (1983), Krori et al. (1990), Mahanta and Mukherjee (2001), Battacharjee and Baruah (2001) have studied several aspects of string cosmological models in general relativity. Several important aspects of strings in scalar-tensor theories of gravitation have been investigated in Bianchi type space times by Reddy (2003), Rao et al. (2008) and Tripathy et al. (2009). Sharma and Singh (2014) have presented a Bianchi type-II string cosmological model in addition to the magnetic field in the presence of $f(R,T)$ gravity. Yadav (2014) has shown that massive string dominated the Bianchi type-V universe at the early times but it does not survive long in $f(R,T)$ gravity.

It has been confirmed through various well-known cosmological tests (Riess et al. 1998; Perlmutter et al. 1999; Miller et al. 1999; Astier et al. 2006) that our universe currently undergoes accelerated expansion. In order to understand the nature of this accelerated expansion phenomenon, various approaches have been adopted. The first approach consists of the presence of dark energy (DE) which contains the repulsive force to push the matter apart in the universe. In this respect, many reviews have been presented (Copeland et al. 2006; Sami 2009; Frieman et

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al. 2008; Bamba et al. 2012). The second approach is the modification of Einstein's gravity into different theories. The basic idea behind this way is to present the gravitational description of DE (Brevik et al. 2005). These modified gravities has some important features. One of them is that it has ability to explains both scenarios i.e., early inflation and late time accelerated expansion (Caramsa and de Mellob 2009). Different classes of modified gravity have been reviewed in the references (Nojiri and Odintsov 2005, 2011; Olmo 2011).

Recently, Harko et al. (2011) proposed a new modified theory of gravity $f(R,T)$ theory of gravity, wherein the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and the trace of the stress-energy tensor T. The field equations of $f(R,T)$ gravity are derived from the Hilbert-Einstein type variation principle. The action for the $f(R,T)$ gravity is

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x, \tag{1.1}$$

where $f(R,T)$ is an arbitrary function of Ricci scalar R and T be the trace of stress-energy tensor (T_{ij}) of the matter. L_m is the matter Lagrangian density. The energy momentum tensor T_{ij} is defined as

$$T_{ij} = -\left(\frac{2}{\sqrt{-g}}\right)\frac{\delta(\sqrt{-g})L_m}{\delta g^{ij}}. \tag{1.2}$$

Here we assume that the dependence of matter Lagrangian is merely on the metric tensor g_{ij} rather than its derivatives, we obtain

$$T_{ij} = g_{ij}L_m - \frac{\partial L_m}{\partial g^{ij}}. \tag{1.3}$$

The $f(R,T)$ gravity field equations are obtained by varying the action S with respect to metric tensor g_{ij}

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij}, \tag{1.4}$$

where

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{\alpha\beta}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{\alpha\beta}}. \tag{1.5}$$

Here $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$, $f_T(R,T) = \frac{\partial f(R,T)}{\partial T}$ and $\Box = \nabla^\mu\nabla_\mu$, where ∇_μ denotes the covariant derivative. T_{ij} is the standard matter energy-momentum tensor derived from the Lagrangian L_m. Here, we assume that the energy-momentum tensor of the matter is given by

$$T_{ij} = (\rho + p)u_iu_j - pg_{ij} - \lambda x_ix_j \tag{1.6}$$

where ρ and p are the energy density and pressure of the string respectively, λ is the tension in the string, u^i is the four velocity vector and x^i is a space-like vector which represents the anisotropic direction of the string. We have u^i and x^i satisfying the conditions

$$g_{ij}u^iu^j = 1, \quad g_{ij}x^ix^j = -1 \quad and \quad u^ix_i = 0 \tag{1.7}$$

Here the matter Lagrangian can be taken as $L_m = -p$ since, there is no unique definition of the matter Lagrangian. This yields

$$\Theta_{ij} = -2T_{ij} - pg_{ij}. \tag{1.8}$$

On the physical nature of the matter field, the field equations also depend through the tensor Θ_{ij}. Hence in the case of $f(R,T)$ gravity depending on the nature of the matter source, we obtain several theoretical models corresponding to different matter contributions for $f(R,T)$ gravity. However, Harko et al. (2011) gave three classes of these models:

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases}$$

Here, we consider $f(R,T) = f_1(R) + f_2(T)$ with $f_1(R) = \mu_1 R$ and $f_2(T) = \mu_2 T$, then the field equations (1.4) can be written as

$$R_{ij} - \frac{1}{2}Rg_{ij} = \left(\frac{8\pi + \mu_2}{\mu_1}\right) T_{ij} + \frac{\mu_2}{\mu_1}\left(p + \frac{T}{2}\right) g_{ij}, \tag{1.9}$$

where R_{ij} is Ricci tensor, T_{ij} is energy-momentum tensor, T is trace of energy-momentum tensor, μ_1 and μ_2 are arbitrary constants. Rao and Sireesha (2013), Rao and Neelima (2013), Sahoo et al. (2014), Reddy et al. (2014a), Mishra and Sahoo (2014), Rao and Prasanthi (2015, 2016), Rao et al. (2015) and Aditya et al. (2016) have discussed some cosmological models in $f(R,T)$ theory of gravity. Rao and Rao (2015) have investigate string cosmological model for the Bianchi type-V space-time in $f(R,T)$ gravity. Sahoo et al. (2016) have constructed string cosmological models in $f(R,T)$ theory of gravity. Very recently, Kanakavalli et al. (2017) explored LRS Bianchi type-II massive string and Takabyasi string models in $f(R,T)$ theory, also they have shown that geometric string does not exist in this theory.

Inspired by the above discussion, we have obtained spatially homogeneous Bianchi type-II string cosmological model in $f(R,T)$ theory of gravity for the particular choice of function $f(R,T) = f_1(R) + f_2(T)$. Plan of the paper as follows: Field equations and Bianchi type-II oscillating string cosmological model in $f(R,T)$ gravity are obtained in section 2. Section 3 contains the physical properties of the model. The last sections is devoted to summary and conclusions.

2 Field equations and the model

We consider a spatially homogeneous Bianchi type-II metric of the form

$$ds^2 = dt^2 - A^2[dx^2 + dz^2] - B^2[dy + xdz]^2, \tag{2.1}$$

where A and B are the functions of cosmic time t only.

Using comoving coordinate system, the field equations (1.9) for the metric (2.1) with the help of (1.6) can be written as

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{B^2}{4A^4} = \left(\frac{16\pi + 3\mu_2}{2\mu_1}\right)p - \frac{\mu_2}{2\mu_1}(\lambda + \rho) \tag{2.2}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3B^2}{4A^4} = \left(\frac{16\pi + 3\mu_2}{2\mu_1}\right)(p - \lambda) - \frac{\mu_2}{2\mu_1}\rho \tag{2.3}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} - \frac{B^2}{4A^4} = -\left(\frac{16\pi + 3\mu_2}{2\mu_1}\right)\rho + \frac{\mu_2}{2\mu_1}(p - \lambda) \tag{2.4}$$

here the overhead dot denotes ordinary differentiation with respect to cosmic time t.

The set of field equations (2.2)-(2.4) is a system of three independent equations with five unknowns A, B, p, λ and ρ. In order to solve the above system completely we consider the following physically plausible conditions:

(i) The shear scalar is proportional to expansion scalar, which leads to the relationship between the metric potentials (Collins et al. 1983)

$$A = B^m \tag{2.5}$$

(ii) We assume the varying deceleration parameter as (Shen and Jiang 2014; Shen 2016) as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = ncos(kt) - 1 \tag{2.6}$$

where n and k are positive constants. The deceleration parameter is an important observational quantity. The universe exhibits decelerating expansion if $q > 0$, a constant expansion rate if $q = 0$, accelerating expansion if $-1 < q < 0$, exponential expansion if $q = -1$ and for $q < -1$ super-exponential expansion. Thus, the universe starts with decelerating expansion with $q = n - 1 (n > 1)$; and evolves into super-exponential expansion with $q = -n - 1$, continually; then the q begins to continuously increase. Finally, the universe ends with decelerating expansion, which is the beginning of every cycle.

By solving the above equation (2.6) using some suitable assumptions the average scale factor obtained as

$$a(t) = \left[tan\left(\frac{kt}{2}\right)\right]^{\frac{1}{n}} \tag{2.7}$$

Now using equations (2.5) and (2.7), we obtain the expressions for metric potentials

$$\begin{aligned} A &= \left[tan\left(\frac{kt}{2}\right)\right]^{\frac{3m}{n(2m+1)}} \\ B &= \left[tan\left(\frac{kt}{2}\right)\right]^{\frac{3}{n(2m+1)}}. \end{aligned} \tag{2.8}$$

Now the metric (2.1) can be written as

$$ds^2 = dt^2 - \left[tan\left(\frac{kt}{2}\right)\right]^{\frac{6m}{n(2m+1)}}[dx^2 + dz^2] - \left[tan\left(\frac{kt}{2}\right)\right]^{\frac{6}{n(2m+1)}}[dy + xdz]^2 \tag{2.9}$$

Now from the field equations (2.2)-(2.4) and (2.8), we get the following:
the pressure of string as

$$\begin{aligned} p \;=\; & \frac{\mu_1}{2(8\pi+\mu_2)(4\pi+\mu_2)}\left\{\frac{3k^2cot(kt)csc(kt)\left[4\pi(m-m^2-3)-3\mu_2\right]}{n(2m+1)}\right. \\ & \left.+\frac{9k^2\left[4\pi(3+7m-m^2)+\mu_2(3+4m+4m^2)\right]}{n^2(2m+1)^2sin^2(kt)} - \mu_1(\mu_2-2\pi)\left[tan\left(\frac{kt}{2}\right)\right]^{\frac{6(1-2m)}{n(1+2m)}}\right\} \end{aligned} \tag{2.10}$$

the energy density of string as

$$\rho = \frac{\mu_1}{2(8\pi+\mu_2)(4\pi+\mu_2)}\left\{\frac{9k^2\left[4\pi(5+7m-12m^2)+\mu_2(5+4m-2m^2)\right]}{n^2(2m+1)^2 sin^2(kt)} - \frac{3k^2\mu_2 cot(kt)csc(kt)\left[4\pi(3m-3m^2-5)+\mu_2(2m-2m^2-5)\right]}{n(2m+1)} + \mu_1(\mu_2-\pi)\left[tan\left(\frac{kt}{2}\right)\right]^{\frac{6(1-2m)}{n(1+2m)}}\right\}, \quad (2.11)$$

the tension in the string as

$$\lambda = \frac{\mu_1}{8\pi+\mu_2}\left\{\frac{9k^2(1+m-2m^2)}{n^2(2m+1)^2 sin^2(kt)} - \frac{3k^2(1+m-m^2)cot(kt)csc(kt)}{n(2m+1)} + \left[tan\left(\frac{kt}{2}\right)\right]^{\frac{6(1-2m)}{n(1+2m)}}\right\} \quad (2.12)$$

Thus the metric (2.9) together with equations (2.10)-(2.12) constitutes a Bianchi type-II string cosmological model in $f(R,T)$ theory of gravity for the particular choice of function $f(R,T) = f_1(R) + f_2(T)$ with time periodic varying deceleration parameter.

3 Some properties of the model

Spatial volume of the model is given by

$$V = \sqrt{-g} = \left[tan\left(\frac{kt}{2}\right)\right]^{3/n} \quad (3.1)$$

Directional Hubble's parameters and mean Hubble's parameter are given as

$$H_1 = H_3 = \frac{\dot{A}}{A} = \frac{3mk}{n(2m+1)\ sin(kt)}, \qquad H_c m2 = \frac{\dot{B}}{B} = \frac{3k}{n(2m+1)\ sin(kt)} \quad (3.2)$$

$$H = \frac{\dot{a}}{a} = \frac{k}{n\ sin(kt)} \quad (3.3)$$

Expansion scalar and shear scalar are given by

$$\theta = 3H = \frac{3k}{n\ sin(kt)} \quad (3.4)$$

$$\sigma^2 = \frac{1}{2}\sum_{i=1}^{3} H_i^2 - \frac{1}{6}\theta^2 = \frac{3k^2(m-1)^2}{n^2(2m+1)^2\ sin^2(kt)} \quad (3.5)$$

Anisotropic parameter is

$$A_h = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2(m-1)^2}{(2m+1)^2} \quad (3.6)$$

where H_i $(i = 1, 2, 3, 4)$ are directional Hubble's parameters.

The statefinder parameter pair $\{r, s\}$ can effectively differentiate various forms of dark energy models and provide simple diagnosis regarding whether a particular model fits into the basic observational data. They are defined as

$$r = \frac{\dddot{a}}{aH^3} \qquad s = \frac{r-1}{3(q-\frac{1}{2})} \tag{3.7}$$

and for $(r, s) = (1, 0)$ the model represents spatially flat ΛCDM cosmological model. In our case the statefinder parameters are

$$r = 1 + n^2 + n^2 cos^2(kt) - 3n\ cos(kt) \tag{3.8}$$

$$s = \frac{n(n + ncos^2(kt) - 3\ cos(kt))}{3(n\ cos(kt) - \frac{3}{2})}. \tag{3.9}$$

Our model reduce to ΛCDM model at some time if there exists a value of n satisfying the following equation

$$cos^2(kt) - \frac{3}{n} cos(kt) + 1 = 0. \tag{3.10}$$

In view of $n > 0$ and $\mid coskt \mid \geq 1$, it can be obtained from equation (3.10) that

$$\frac{9}{4n^2} \geq 1 \tag{3.11}$$

$$\frac{3}{2n} + \sqrt{\frac{9}{4n^2} - 1} \leq 1 \tag{3.12}$$

Equations (3.11) and (3.12) lead to $n = \frac{3}{2}$. Under the case of $n = \frac{3}{2}$, the relations (3.8) and (3.9) reduce to

$$r = 1 + \frac{9}{4}\left(cos(kt) - 1\right)^2 \tag{3.13}$$

$$s = \frac{cos(kt) - 1}{2}. \tag{3.14}$$

Therefore, from the statefinder diagnostic pair we can observe that our model resemble the standard ΛCDM model at the time $t = \frac{2l\pi}{k}$ where l is an integer. Moreover, the deceleration parameter satisfies $-2.5 \leq q \leq 0.5$ under the condition of $n = \frac{3}{2}$, which is consistent with observational results (Akarsu and Dereli 2012).

4 Summery and conclusions

In recent years modified theories of gravitation have attracted much attention, because of the discovery of the accelerated expansion of the universe. So, the investigation of cosmological models with anisotropic background in these theories are gaining importance. Here, we have studied Bianchi type-II string cosmological model in $f(R, T)$ theory of gravity using a time periodic varying deceleration parameter. It is observed that as time $t \to \frac{l\pi}{k}$ ($l \epsilon \mathbf{Z}$), the Hubble's parameter H, the expansion scalar θ, energy density ρ, absolute value pressure $\mid p \mid$ and tension in string λ all are tend to infinity, and the volume V tends to infinity as $t \to \frac{(2l+1)\pi}{k}$. Thus, the model (2.9) has future Big rip singularity at $t = t_s = \frac{(2l+1)\pi}{k}$. It can be, also, seen that since $\frac{\sigma^2}{\theta^2} \neq 0$, the model remains anisotropic for $m \neq 1$. But, for $m = 1$, it may be observed that the model becomes isotropic and shear free. Our model represents the standard ΛCDM model at the time $t = \frac{2l\pi}{k}$. Since the deceleration parameter of obtained model is within the limit $-2.5 \leq q \leq 0.5$ under the conditions $n = \frac{3}{2}$ and $t = \frac{2l\pi}{k}$, which is consistent with observational results.

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