Article

Bianchi Type-I Inhomogeneous String Cosmological Model with Electromagnetic Field in General Relativity

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Abstract

In this paper, we have investigated an inhomogeneous string cosmological model in the presence of electromagnetic field. We have assumed that F_{12} is the only non-vanishing component of electromagnetic field tensor F_{ij}. The metric potentials are functions of x and t both. To get the deterministic solution, it has been assumed that the expansion (θ) in the model is proportional to the eigenvalue $\sigma^1{}_1$ of the shear tensor $\sigma^i{}_j$. The derived model represents the inflationary scenario as the proper volume increases exponentially with cosmic time. It is observed that the model has a Barrel type singularity. The behaviour of the electromagnetic field tensor together with physical and geometrical aspects of the model is also discussed.

Keywords: Cosmic string, Electromagnetic field, Inhomogeneous universe.

1. Introduction

In recent years, there has been considerable interest in string cosmology because cosmic strings play an important role in the study of the early universe. These strings arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories. Moreover, the investigation of cosmic strings and their physical processes near such strings has received wide attention because it is believed that cosmic strings give rise to density perturbations which lead to formation of galaxies. These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings by using Einstein's equations. The general treatment of strings was initiated by Letelier and Stachel [3]. Letelier [1] obtained the general solution of Einstein's field equations for a cloud of strings with spherical, plane and a particular case of cylindrical symmetry. Letelier [2] also obtained massive string cosmological models in Bianchi type-I and Kantowski-Sachs space-times.

Benerjee et al. [4] have investigated an axially symmetric Bianchi type I string dust cosmological model in presence and absence of magnetic field using a supplementary condition $\alpha = a\beta$ between metric potential where $\alpha = \alpha(t)$ and $\beta = \beta(t)$ and a is constant. Exact solutions of string cosmology for Bianchi type-Il, VI_0 ,VIII and IX space-times have been studied by Krori et al. [5]

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and Wang [6]. Bali et al. [7,8,9,10] have obtained Bianchi type I, III, V and type IX string cosmological models in general relativity.

The string cosmological models with a magnetic field are discussed by Chakraborty [11], Tikekar and Patel [12,13]. Singh and Singh [14] investigated string cosmological models with magnetic field in the context of space-time with G_3 symmetry. Singh [15] has studied string cosmology with electromagnetic fields in Bianchi type II, VIII and IX space-time. Bali and Upadhaya [16] investigated LRS Bianchi type- I string dust magnetized cosmological models. Bali and Tyagi [17,18] also obtained cylindrically symmetric inhomogeneous cosmological model and stiff fluid universe with electromagnetic field in general relativity. Sharma et al. [19] have obtained inhomogeneous Bianchi type VI_0 string cosmological model for stiff fluid distribution. The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out Zel'dovich [20]. Also Harrison [21] has suggested that magnetic field could have a cosmological origin.

As a natural consequence, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than Robertson-Walker model [22]. The presence of primordial magnetic fields in the early stages of the evolution of the universe has been discussed by several authors (Misner, Thorne and Wheeler [23]; Asseo and Sol [24]; Pudritz and Silk [25]; Kim, Tribble, and Kronberg [26]; Perley and Taylor [27]; Kronberg, Perry and Zukowski [28]; Wolfe, Lanzetta and Oren [29]; Kulsrud, Cen, Ostriker and Ryu [30]; Barrow [31]). Melvin [32], in his cosmological solution for dust and electromagnetic field suggested that during the evolution of the universe, the matter was in a highly ionized state and was smoothly coupled with the field, subsequently forming neutral matter as a result of universe expansion. Hence the presence of magnetic field in string dust universe is not unrealistic.

Patel and Maharaj [33] investigated stationary rotating world model with magnetic field. Ram and Singh [34] obtained some new exact solution of string cosmology with and without a source free magnetic field for Bianchi type I space-time in the different basic form considered by Carminati and McIntosh [35]. Lidsey, Wands and Copeland [36] have reviewed aspects of super string cosmology with the emphasis on the cosmological implications of duality symmetries in the theory. Bali et al. [37] have investigated Bianchi type I magnetized string cosmological models. Pradhan et al. [38] have investigated string cosmological model in cylindrically symmetric inhomogeneous universe with electromagnetic field.

In this paper, we have investigated an inhomogeneous string cosmological model in the presence of electromagnetic field. We have assumed that F_{12} is the only non-vanishing component of electromagnetic field tensor F_{ij} . The metric potentials are functions of x and t both. To get the deterministic solution, it has been assumed that the expansion (θ) in the model is proportional to the eigen value $\sigma^1{}_1$ of the shear tensor $\sigma^i{}_j$. The derived model represents the inflationary scenario as the proper volume increases exponentially with cosmic time. It is observed that the model has a Barrel type singularity. The behaviour of the electromagnetic field tensor together with physical and geometrical aspects of the model are also discussed.

2. The Metric and Field Equations

We consider the metric in the form

$$ds^2 = dx^2 - dt^2 + B^2 dy^2 + C^2 dz^2 \tag{1}$$

where B and C are both functions of x and t. The energy-momentum tensor for the string with electromagnetic field has the form,

$$T_i^j = \rho \, v_i \, v^j - \lambda x_i x^j + E_i^j \tag{2}$$

With $\rho = \rho_p + \lambda$ and v_i and x_i satisfy conditions,

$$v_i \, v^i = -1 = -x_i \, x^{i.} \tag{3}$$

and $$v^i \, x_i = 0 \tag{4}$$

Here ρ is the rest energy density of strings with massive particles attached to them $\rho = \rho_p + \lambda$, ρ_p being the rest energy density of particles attached to the strings and λ is the density of tension that characterizes the strings. The unit space like vector x^i represents the string direction, i.e. the direction of anisotropy and the unit time like vector v^i describes the four velocity vector of the matter satisfying the following conditions,

$$g_{ij} v^i v^j = -1 \tag{5}$$

In equation (2) , E^i_j is the electromagnetic field given by Lichnerowicz [39]

$$E^i{}_j = \bar{\mu}\left[h_l h^l \left(v_i v^j + \frac{1}{2} g^i{}_j \right) - h_i h^j \right] \tag{6}$$

Where $\bar{\mu}$ is the magnetic permeability and h_i is the magnetic flux vector defined by

$$h_i = \frac{1}{\bar{\mu}} * F_{ji} v^j \tag{7}$$

Where the dual electromagnetic field tensor $*F_{ij}$ is defined by Synge [40]

$$* F_{ji} = \frac{\sqrt{-g}}{2} \varepsilon_{ijkl} F^{kl} \tag{8}$$

Here F_{ij} is the electromagnetic field tensor and ε_{ijkl} is the Levi-Civita tensor density. The components of electromagnetic field are obtained as

$$E_1^1 = E_2^2 = E_4^4 = \frac{F_{12}^{\ \ 2}}{2\bar{\mu}B^2},$$
$$E_3^3 = -\frac{F_{12}^{\ \ 2}}{2\bar{\mu}B^2} \tag{9}$$

In present scenario, the comoving coordinates are taken as,

$$v^i = (0,0,0,1) \tag{10}$$

We choose the direction of string parallel to x- axis so that

$$x^i = (1,0,0,0) \tag{11}$$

We consider the current as flowing along the z-axis so that F_{12} is the only Non - vanishing component of F_{ij} . Maxwell's equations

$$F_{[ij;k]} = 0 \tag{12}$$

$$\left[\frac{F^{ij}}{\bar{\mu}}\right]_{;j} = 0 \tag{13}$$

require that F_{12} is the function of x and t both and the magnetic permeability is the functions of x and t both. The semicolon represents a covariant differentiation.

The Einstein's field equation in the geometrized unit (c = 1, 8πG = 1)

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j \tag{14}$$

for the line-element (1) lead to the following system of equations are

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} - \frac{B_1C_1}{BC} = \lambda - \frac{F_{12}^{\ \ 2}}{2\bar{\mu}B^2} \tag{15}$$

$$\frac{C_{44}}{C} - \frac{C_{11}}{C} = -\frac{F_{12}^{\ \ 2}}{2\bar{\mu}B^2} \tag{16}$$

$$\frac{B_{44}}{B} - \frac{B_{11}}{B} = \frac{F_{12}^{\ \ 2}}{2\bar{\mu}B^2} \tag{17}$$

$$\frac{B_4C_4}{BC} - \frac{B_{11}}{B} - \frac{C_{11}}{C} - \frac{B_1C_1}{BC} = \rho - \frac{F_{12}^{\ \ 2}}{2\bar{\mu}B^2} \tag{18}$$

$$\frac{B_{14}}{B}+\frac{C_{14}}{C}=0 \tag{19}$$

Where the sub-indices 1 and 4 in B, C and elsewhere denote ordinary differentiation with respect to x and t respectively.

3. Solution of Field Equations

To find the deterministic solution of line element (1), we assume that

$$B = f(x)\, g(t) \tag{20}$$

and

$$C = h(x)\, k(t) \tag{21}$$

Using equations (16) and (17) we have

$$\frac{B_{44}}{B}-\frac{C_{44}}{C}+\frac{C_{11}}{C}-\frac{B_{11}}{B}=\frac{{F_{12}}^{2}}{\bar{\mu}B^{2}} \tag{22}$$

and

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}-\frac{C_{11}}{C}-\frac{B_{11}}{B}=0 \tag{23}$$

Using equations (20) , (21) in (19), we obtain

$$\frac{f_1 g_4}{fg}+\frac{h_1 k_4}{hk}=\alpha(const.) \tag{24}$$

which leads to

$$\frac{f_1}{f}=\alpha\frac{h_1}{h} \tag{25}$$

and

$$\frac{k_4}{k}=-\alpha\frac{g_4}{g} \tag{26}$$

From Equations (25) and (26), we obtain

$$f = Lh^{\alpha} \quad \text{and} \quad k = Ng^{-\alpha} \tag{27}$$

where L and N are integrating constants.

Using equations (20) , (21) in (23), we have

$$\frac{g_{44}}{g}+\frac{k_{44}}{k}=\frac{h_{11}}{h}+\frac{f_{11}}{f}=q(const.) \tag{28}$$

Equations (27) and (28), lead to

$$gg_{44}+\ell g_4^2=mg^2 \tag{29}$$

where $\ell=\frac{\alpha(\alpha+1)}{1-\alpha}$, $m=\frac{q}{1-\alpha}, \alpha\neq 1$

Integrating equation (29), we obtain

$$g=\beta\sinh^{\frac{1}{\ell+1}}(bt+t_0) \tag{30}$$

where $\beta=C_2^{1/\ell+1}$, $b=\sqrt{m(\ell+1)}$ and t_0 ,C_2 are constants of integration.

Thus from equation (27) and (30) , we get

$$k=N\beta^{-\alpha}\sinh^{\frac{-\alpha}{\ell+1}}(bt+t_0) \tag{31}$$

Again From equation (27) & (28), we have

$$hh_{11}+sh_1^2=nh^2 \tag{32}$$

where $s=\frac{\alpha(\alpha-1)}{1+\alpha}$, $n=\frac{q}{1+\alpha}$

Integrating equation (31), we obtain

$$h=\delta\sinh^{\frac{1}{s+1}}(cx+x_0) \tag{33}$$

where $\delta=C_4^{1/s+1}$, $c=\sqrt{n(s+1)}$ and x_0 ,C_4 are constants of integration.

Hence from equation (27) and (33) , we get

$$f=L\delta^{\alpha}\sinh^{\frac{\alpha}{s+1}}(cx+x_0) \tag{34}$$

Hence, we obtain the value of metric potential

$$\begin{aligned} B &= \left[f(x) g(t) \right] \\ &= P \sinh^{\frac{\alpha}{s+1}}(cx + x_0) \sinh^{\frac{1}{\ell+1}}(bt + t_0) \end{aligned} \tag{35}$$

$$\begin{aligned} C &= \left[h(x) k(t) \right] \\ &= Q \sinh^{\frac{-\alpha}{\ell+1}}(bt + t_0) \sinh^{\frac{1}{s+1}}(cx + x_0) \end{aligned} \tag{36}$$

where $P = L\beta\delta^{\alpha}$ and $Q = N\delta\beta^{-\alpha}$

Therefore after using suitable transformation of coordinates, metric (1) reduces to

$$\begin{aligned} ds^2 &= (dX^2 - dT^2) + P^2 \sinh^{\frac{2\alpha}{s+1}}(cX) \sinh^{\frac{2}{\ell+1}}(bT) dY^2 \\ &+ Q^2 \sinh^{\frac{-2\alpha}{\ell+1}}(bT) \sinh^{\frac{2}{s+1}}(cX) dZ \end{aligned} \tag{37}$$

where $cX = cx + x_0$, $bT = bt + t_0$, $Y=Py$ *and* $Z=Qz$

4. Some Physical And Geometrical Features

The physical and geometrical properties of the model (37) are given as follows:

The magnitude of rotation ω is zero i.e.

$\omega = 0$

String Tension λ of the model is given by,

$$\begin{aligned} \lambda &= \frac{b^2(1-\alpha)}{(1+\ell)} + \frac{\{\alpha(\alpha+1) - \ell\} b^2}{(\ell+1)^2} \coth^2(bT) - \frac{b^2\alpha}{(s+1)^2} \coth^2(cX) \\ &+ \frac{F_{12}{}^2}{\bar{\mu} P^2 \sinh^{2\alpha/s+1}(cX) \sinh^{2/1+\ell}(bT)} \end{aligned} \tag{38}$$

The Energy density ρ of the model is given by,

$$\begin{aligned} \rho &= -\frac{c^2(1+\alpha)}{(1+s)} + \frac{\{\alpha(s-\alpha) + s\} c^2}{(s+1)^2} \coth^2(cX) - \frac{b^2\alpha}{(\ell+1)^2} \coth^2(bT) \\ &+ \frac{F_{12}{}^2}{\bar{\mu} P^2 \sinh^{2\alpha/s+1}(cX) \sinh^{2/1+\ell}(bT)} \end{aligned} \tag{39}$$

The particle density ρ_p of the model is given by,

$$\rho_p = \rho - \lambda = \frac{[\alpha + \alpha(s-\alpha) + s]c^2}{(s+1)^2}\coth^2(cX) - \frac{c^2(1+\alpha)}{(s+1)} - \frac{b^2(1-\alpha)}{(\ell+1)} - \frac{[2\alpha + \alpha^2\ell - \ell]b^2}{(\ell+1)^2}\coth^2(bT) \tag{40}$$

where

$$F_{12}{}^2 = \bar{\mu}\sinh^{\frac{2\alpha}{s+1}}(cX)\sinh^{\frac{2}{\ell+1}}(bT)\left\{\begin{aligned}&\frac{b^2(1-\alpha)}{(1+\ell)} + \frac{c^2(1-\alpha)}{(1+s)} - \frac{[\ell + \alpha(\alpha+\ell+1)]b^2}{(\ell+1)^2}\coth^2(bT)\\ &+ \frac{[s+\alpha(\alpha-s-1)]c^2}{(s+1)^2}\coth^2(cX)\end{aligned}\right\} \tag{41}$$

The Scalar expansion θ of the model is given by,

$$\theta = \frac{b(1-\alpha)}{1+\ell}\coth(bT) \tag{42}$$

The Shear Scalar σ of the model is given by,

$$\sigma^2 = \frac{1}{3}\left[\frac{(1-\alpha)^2 + 3\alpha}{(1+\ell)^2}\right]b^2\coth^2(bT) \tag{43}$$

The proper volume V of the model is given by,

$$V^3 = PQ\sinh^{1-\alpha/\ell+1}(bT)\sinh^{\alpha+1/s+1}(cX) \tag{44}$$

The Deceleration parameter q of the model is given by,

$$q = -1 + \frac{3(1+\ell)}{(2+\alpha+3\ell)}\tanh^2(bT) \tag{45}$$

From equation (42) and (43) we obtain

$$\frac{\sigma}{\theta} = \sqrt{\frac{1}{3}\left[\frac{(1-\alpha)^2 + 3\alpha}{1-\alpha}\right]} = const. \tag{46}$$

5. Conclusion

The model (37) starts with big bang at T = 0 and goes on expanding till $T \to \infty$ when θ becomes zero. It is clear that as T increases, the ratio of the shear scalar σ and expansion θ tends to finite value i.e. $\frac{\sigma}{\theta} \to$ constant. Hence the model does not approach isotropy for large value of T. We also observe that ρ, λ, ρ_p tend to ∞ when $X \to 0$, $T \to 0$. The energy density (ρ) and string tension density (λ) increases as electromagnetic field component (F_{12}) increases. The proper volume V^3 increases as time increases. The model (37) has a Barrel type singularity at T=0. Since the deceleration parameter $q < 0$, hence the model (37) represents an accelerating universe. In general the model represents expanding, shearing and non-rotating universe.

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