Article

Generator of the Lanczos Spinor in Type-D Vacuum Spacetime

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Abstract

Andersson-Edgar proved that the Lanczos spinor can be generated via $L_{ABC\dot{D}} = f\, \nabla^{E}{}_{\dot{D}}\, T_{ABCE}$. Here we determine the potential T_{ABCE} for any type D empty spacetime.

Keywords: Lanczos spinor, spin coefficients.

1. Introduction

In [1] was obtained the Lanczos generator [2, 3] for an arbitrary vacuum spacetime of Petrov type D, with the following Newman-Penrose components:

$$\Omega_1 = \psi_2^{-\frac{2}{3}}\,\frac{\rho}{2}\,,\ \Omega_2 = \psi_2^{-\frac{2}{3}}\,\frac{\pi}{2}\,,\ \Omega_5 = \psi_2^{-\frac{2}{3}}\,\frac{\tau}{2}\,,\ \Omega_6 = \psi_2^{-\frac{2}{3}}\,\frac{\mu}{2}\,,\ \ \Omega_r = 0,\ r = 0, 3, 4, 7, \qquad (1)$$

in terms of the spin coefficients [4, 5] associated to the canonical null tetrad [6]:

$$l^{\mu} \ \leftrightarrow\ o^{A} o^{\dot{B}}\,, \qquad n^{\mu} \ \leftrightarrow\ \iota^{A} \iota^{\dot{B}}\,, \qquad m^{\mu} \ \leftrightarrow\ o^{A} \iota^{\dot{B}}\,, \qquad \overline{m}^{\mu} \ \leftrightarrow\ \iota^{A} o^{\dot{B}}\,; \qquad (2)$$

hence for the type D the Lanczos spinor [7-9] is given by:

$$L_{ABC\dot{D}} = \frac{1}{2}\,\psi_2^{-\frac{2}{3}}\,[(o_A o_B \iota_C + (o_A * \iota_B) o_C)(-\mu\, o_{\dot{D}} + \pi\, \iota_{\dot{D}}) + (\iota_A \iota_B o_C + (o_A * \iota_B)\iota_C)(\tau\, o_{\dot{D}} - \rho\, \iota_{\dot{D}})]. \qquad (3)$$

On the other hand, Andersson-Edgar [10-12] proved that any Lanczos spinor can be generated via the relation:

$$L_{ABC\dot{D}} = f\, \nabla^{E}{}_{\dot{D}}\, T_{ABCE}\,, \qquad T_{ABCE} = T_{(ABC)E}\,. \qquad (4)$$

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where f is certain scalar function. In Sec. 2 we give an explicit expression for the potential T_{ABCE} in terms of the dyad (o^F, ι^G), for the Lanczos spinor (3), with $f = \psi_2^{-\frac{2}{3}}$.

2. Potential for the Lanczos spinor

In the spin coefficients formalism [5, 13] are very known the following relations:

$$\nabla^A{}_{\dot{B}}\, o_A = (\tau - \beta) o_{\dot{B}} + (\varepsilon - \rho)\iota_{\dot{B}}, \qquad \nabla^A{}_{\dot{B}}\, \iota_A = (\gamma - \mu) o_{\dot{B}} + (\pi - \alpha)\iota_{\dot{B}},$$

$$o_A \nabla^A{}_{\dot{B}}\, o_C = (-\beta\, o_{\dot{B}} + \varepsilon\, \iota_{\dot{B}}) o_C + (\sigma\, o_{\dot{B}} - \kappa\, \iota_{\dot{B}})\iota_C,$$

$$o_A \nabla^A{}_{\dot{B}}\, \iota_C = (-\mu\, o_{\dot{B}} + \pi\, \iota_{\dot{B}}) o_C + (\beta\, o_{\dot{B}} - \varepsilon\, \iota_{\dot{B}})\iota_C, \tag{5}$$

$$\iota_A \nabla^A{}_{\dot{B}}\, o_C = (-\gamma\, o_{\dot{B}} + \alpha\, \iota_{\dot{B}}) o_C + (\tau\, o_{\dot{B}} - \rho\, \iota_{\dot{B}})\iota_C, \;\; \iota_A \nabla^A{}_{\dot{B}}\, \iota_C = (-\nu\, o_{\dot{B}} + \lambda\, \iota_{\dot{B}}) o_C +$$

$$+ (\gamma\, o_{\dot{B}} - \alpha\, \iota_{\dot{B}})\iota_C,$$

hence:

$$\nabla^E{}_{\dot{D}}[(o_A o_B \iota_C + (o_A * \iota_B) o_C)\iota_E] = 3 o_A o_B o_C(-\nu\, o_{\dot{D}} + \lambda\, \iota_{\dot{D}}) +$$

$$+(o_A o_B \iota_C + (o_A * \iota_B) o_C)(-\mu\, o_{\dot{D}} + \pi\, \iota_{\dot{D}}) + 2(\iota_A \iota_B o_C + (o_A * \iota_B)\iota_C)(\tau\, o_{\dot{D}} - \rho\, \iota_{\dot{D}}),$$

$$\nabla^E{}_{\dot{D}}[(\iota_A \iota_B o_C + (o_A * \iota_B)\iota_C) o_E] = 3 \iota_A \iota_B \iota_C(\sigma\, o_{\dot{D}} - \kappa\, \iota_{\dot{D}}) + \tag{6}$$

$$+(\iota_A \iota_B o_C + (o_A * \iota_B)\iota_C)(\tau\, o_{\dot{D}} - \rho\, \iota_{\dot{D}}) + 2(o_A o_B \iota_C + (o_A * \iota_B) o_C)(-\mu\, o_{\dot{D}} + \pi\, \iota_{\dot{D}}),$$

then it is immediate that (4) implies (3) for [$\kappa = \sigma = \nu = \lambda = 0$ in the canonical null tetrad for any type D empty spacetime]:

$$T_{ABCE} = \frac{1}{6}[(o_A o_B \iota_C + (o_A * \iota_B) o_C)\iota_E + (\iota_A \iota_B o_C + (o_A * \iota_B)\iota_C) o_E], \tag{7}$$

where is evident the symmetry (4) in ABC.

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