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Bianchi Type-II Modified Holographic Ricci Dark Energy Model in Lyra Manifold

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Work in this paper is, mainly, concerned with the investigation of Bianchi type-II cosmological models in Lyra (Math. Zeitschrift 54:52,1951) manifold with matter and anisotropic modified holographic Ricci dark energy as source. We have obtained exact solutions of the gravitational field equations using hybrid expansion law and a power law for the time dependent displacement vector field in Lyra manifold and some physical properties of the model are discussed in detail.

Keywords: Lyra manifold, Bianchi model, Ricci dark energy, holographic dark energy.

1. Introduction

Even today the concept of cosmic acceleration of the universe (Riess et al 1998; Perlmutter et al1999) is an active subject of investigation because the reason for this is still remains a mystery. An exotic type of energy with negative pressure dubbed as dark energy is supposed to be responsible for the accelerated expansion of the universe. It is believed that the universe is filled with two dark components dark matter, without pressure and dark energy with negative pressure which is causing cosmic acceleration. Einstein's theory of gravitation is a beautiful and sophisticated physical theory which has been successfully used to construct cosmological models to explain the origin and evolution of the universe. However, Einstein's theory does not explain late time acceleration of modern cosmology. Hence modifying Einstein's theory is one important way of explaining the scenario of dark energy by constructing dark energy models in modified theories of gravitation. Some important modifications of Einstein's theory are scalar –tensor theories of gravitation proposed by Brans and Dicke(1961), Saez and Ballester(1986),and Lyra (1951) . Also, F(R) gravity (Odintsov and Nojiri 2003) and f(R,T) (Harko et al.2011) gravity are the other important modified theories of gravitation. Here we are interested in the construction of dark energy cosmological models in modified theory formulated by Lyra (1951).

It is well known that Einstein's theory is a geometric theory which explains the gravitational phenomena. Weyl (1918) has proposed a more general geometric theory which incorporates both gravitation and electromagnetism. Later Lyra (1951) modified Riemannian geometry by introducing gauge function which removed the non- integrability condition of the length of a

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vector under parallel transport. This is known as Lyra's geometry. The connection in this geometry is metric preserving as in Riemannian geometry and length transfers are also integrable. Also, theories of gravitation constructed in the framework of Lyra geometry with both a constant and time dependent displacement vector field involve scalar and tensor fields which are intrinsic to geometry and they predict the same effects as in general relativity within the observational limits as far as the classical solar system(Sen 1951, 1970; Sen and Dunn 1971; Halford 1972). The models obtained in this theory are free of big bang singularity and will be useful to solve the entropy and horizon problem which are present in the standard models based on Riemannian geometry. The field equations in normal gauge in Lyra's manifold given by Sen (1957) are

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2} \varphi_i \varphi_j - \frac{3}{4} g_{ij} \varphi_k \varphi^k = -T_{ij} \tag{1}$$

where φ_i is a displacement field and the other symbols have their usual meaning as in Riemannian geometry (Here we have chosen gravitational units so that $8\pi G = c = 1$) . We assume φ_i to be time-like so that

$$\varphi_i = (0,0,0,\beta) \tag{2}$$

where β is a function of cosmic time t.

Cosmological models in this theory with different energy momentum sources have been investigated by several authors. Beesham(1988),Singh and Singh (1991, !992) and Singh and Desikan(1997) have studied cosmological models in Lyra geometry with time dependent displacement vector field. It may be noted that the displacement vector field of this geometry has the similar behavior as that of cosmological constant of general relativity. However, the displacement vector field arises naturally from the geometry while the introduction of cosmological constant was an adhoc assumption. Singh(2003), Pradhan and Viswakarma(2004), Rahaman et al.(2005), Reddy(2005), Mohanty et al.(2005), Kumar and Singh(2008), Singh and Kale (2009), Chaubey (2012) , Ali and Rahaman(2013) and Sahu et al.(2015)are some of the authors who have investigated several cosmological models, in this particular geometry, with different physical sources in FRW and Bianchi type space-times.

In recent years, holographic dark energy (HDE) models have received considerable attention to describe dark energy cosmological models. According to the holographic principle, the number of degrees of freedom in a bounded system should be finite and is related to the area of its boundary (Hoof't 1995). It is argued that this model may solve the cosmological constant problem and some other issues. Several aspects of holographic dark energy have been investigated by Cohen et al. (1999) and Hsu (2014), Gao et al. (2009) have proposed a HDE model, where the future event horizon is replaced by the inverse of the Ricci scalar curvature, and thismodel is named as "Ricci dark energy model" (RDE), that is, a holographic Ricci dark energy model, whose length scale is the inverse of the Ricci curvature scalar, i.e. $L \approx |R|^{\frac{-1}{2}}$. Granda and Oliveros (2008) suggested a new holographic Ricci dark energy model. Later, Chen and Jing (2009) modified this model by assuming the density of dark energy contains the Hubble parameter H, the first order and the second order derivatives (i.e., $\dot{H}$ and $\ddot{H}$).The expression of the energy density of dark energy is given by

$$\rho_\Lambda = 3\eta_1 \ddot{H} H^{-1} + 3\eta_2 \dot{H} + 3\eta_3 H^2 \tag{3}$$

where η_1, η_2 and η_3 are three arbitrary dimensionless parameters. Recently, Sarkar (2014), Adhav et al. (2015), Kiran et al.(2014,2015a), Umadevi and Ramesh (2015) have investigated minimally interacting and interacting holographic dark energy Bianchi models in general relativity and in scalar-tensor theories of gravitation. Very recently, Kiran et al.(2015b) and Reddy et al. (2015) have discussed Bianchi type minimally interacting holographic dark energy models using linearly varying deceleration parameter proposed by Akarsu and Dereli (2012). Das and Sultana (2015,2016) have studied Bianchi type anisotropic modified holographic Ricci dark energy cosmological models. Rahman and Ansari (2014a) have investigated interacting holographic polytropic gas model of dark energy with hybrid expansion law in Bianchi type-VI_0 space time while Rahman and Ansari (2014) have studied interacting generalized ghost polytropic gas model of dark energy with a specific Hubble parameter in LRS Bianchi type-II space time. Hova(2013) discussed a dark energy model in Lyra manifold. Recently, Singh et al.(2016) have studied Bianchi type-I cosmological model with Chaplygin gas in Lyra geometry. Very recently, Reddy (2016) obtained Bianchi type-III modified holographic Ricci dark energy model in this manifold.

In the present study, motivated by the above investigations, we intend to study Bianchi type-II modified holographic Ricci dark energy model in modified theory of gravitation in Lyra manifold.

2. Metric and field equations

We consider the locally rotationally symmetric (LRS) Bianchi type-II space-time in the form

$$ds^2 = -dt^2 + A^2(dx^2 + dz^2) + B^2(dy + xdz)^2 \tag{4}$$

where A and B are functions of cosmic time t only.

The energy momentum tensors for pressure less matter and holographic dark energy are, respectively, given by

$$T_{ij} = \rho_M u_i u_j, \quad i, j = 1,2,3,4 \tag{5}$$

$$\overline{T}_{ij} = (\rho_\Lambda + p_\Lambda) u_i u_j + g_{ij} p_\Lambda$$

where ρ_M is matter energy density, ρ_Λ is the energy density of the modified holographic Ricci dark energy. In this particular case, Barber's field equations (1) and (2) take the form

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4} g_{ij}\phi^k\phi_k = -(T_{ij} + \bar{T}_{ij}) \tag{6}$$

Now, parameterising, we have, from Eq. (5)

$$\bar{T}_i^{\ j} = diag[-1, \omega_x, \omega_y, \omega_z]\rho_\Lambda = diag[-1, \omega_\Lambda, (\omega_\Lambda + \delta), (\omega_\Lambda + \gamma)]\rho_\Lambda \tag{7}$$

Here we have used the EoS parameter ω given by

$$\omega_\Lambda \rho_\Lambda = p_\Lambda \tag{8}$$

and $\omega_x, \omega_y, \omega_z$ are the directional EoS parameters along *x, y, z* axes respectively. For the sake of simplicity we choose $\omega_x = \omega_\Lambda$ and the skewness parameters δ and γ are the deviations from ω on *y* and *z* axes respectively.

Now using co moving coordinate system, the field equations (6) with the help of Eq.(7), for the metric given by Eq.(4) take the form

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{3}{4}\beta^2 = -\omega_\Lambda \rho_\Lambda \tag{9}$$

$$2\frac{\ddot{A}}{A} + (\frac{\dot{A}}{A})^2 + \frac{3}{4}\beta^2 = -(\omega_\Lambda + \delta)\rho_\Lambda \tag{10}$$

$$2\frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 + \frac{3}{4}\beta^2 = -(\omega_\Lambda + \gamma)\rho_\Lambda \tag{11}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \left(\frac{\dot{A}}{A}\right)^2 - \frac{3}{4}\beta^2 = (\rho_\Lambda + \rho_M) \tag{12}$$

We ,now, define the kinematical and physical parameters of cosmology which will be useful to solve the field equations.
The spatial volume of the space-time (4) is given by

$$V = a^3(t) = A^2 B \tag{13}$$

where *a(t)* is the scale factor of the universe.

The average Hubble parameter is

$$H = \frac{1}{3}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \tag{14}$$

The scalar expansion is given by

$$\theta = 2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \tag{15}$$

The shear scalar is defined as

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 \tag{16}$$

The anisotropy parameter is given by

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{17}$$

where H_i, $i = 1, 2, 3$ are directional Hubble parameters.

3. Solution and the model

From equations (10) and (11) we obtain

$$\gamma = \delta \tag{18}$$

Using Eq.(18) the field equations (9)-(12) reduce to a system of three independent equations in seven unknowns, $A, B, \rho_m, \rho_{\Lambda}, \beta, \delta, \omega_{\Lambda}$. Hence to find a determinate solution of the system we require four more conditions. We use the following physically significant conditions:

(i) The shear scalar σ^2 is proportional to expansion scalar θ so that we have (Collins1983)

$$A = B^n \tag{19}$$

where $n \neq 0$ is a positive constant which takes care of anisotropy of the space-time.

(ii) We consider the average scale factor as a combination of power law and exponential law, given by Akarsu et al.(2014), as

$$a(t) = a_0 t^{\alpha_1} e^{\alpha_2 t} \tag{20}$$

where α_1 and α_2 are non-negative constants . Here, when $\alpha_1 = 0$ we get the exponential law and when $\alpha_2 = 0$ we obtain power law. Thus, Eq. (20) gives the combination of exponential and power law which is usually known as hybrid expansion law. This choice of average scale factor leads to a time dependent deceleration parameter. The solution gives inflation and radiation dominance era with subsequent transition from decelerating to accelerating phase of the universe.

(iii) The energy density of modified holographic Ricci dark energy given by the equation (Chen and Jing 2009).

$$\rho_{\Lambda} = 3(\eta_1 H^2 + \eta_2 \dot{H} + \eta_3 \ddot{H} H^{-1}) \tag{21}$$

(iv)we assume that the displacement vector field β of Lyra manifold as some power of the average scale factor so that we have

$$\beta = \beta_0 a^m \tag{22}$$

where β_0 is a constant This type of power law has already been used By Johri and Sudharsan (1989) in the context of FRW Brans-Dicke cosmological models.

Now from Eqs. (13), (19) and (20) we obtain the metric potentials as

$$A = (a_0 t^{\alpha_1} e^{\alpha_2 t})^{\frac{3n}{2n+1}} \ , \ B = (a_0 t^{\alpha_1} e^{\alpha_2 t})^{\frac{3}{2n+1}} \tag{23}$$

Using Eq.(23), we can write the metric (4) in the form

$$ds^2 = -dt^2 + (a_0 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6n}{2n+1}} (dx^2 + dz^2) + (a_0 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6}{2n+1}} (dy + xdz)^2 \tag{24}$$

4. Physical discussion

This section is devoted to the discussion of some physical and kinematical properties of the model given by Eq.(24).This model represents a Bianchi type-II holographic modified Ricci dark energy model in Lyra manifold. The following are the physical and kinematical parameters which are useful in the discussion cosmology of the model (24).

The spatial volume of the universe is

$$V = (a_0 t^{\alpha_1} e^{\alpha_2 t})^3 \tag{25}$$

The average Hubble parameter is

$$H = (\frac{\alpha_1}{t} + \alpha_2) \tag{26}$$

The scalar expansion in the universe is

$$\theta = 3\,(\frac{\alpha_1}{t} + \alpha_2) \tag{27}$$

The shear scalar is

$$\sigma^2 = \frac{3(n-1)^2}{(2n+1)^2} (\frac{\alpha_1}{t} + \alpha_2)^2 \tag{28}$$

The anisotropy parameter is

$$\Delta = \frac{4}{3} (\frac{n-1}{2n+1})^2 \tag{29}$$

From the equations (21) and (26) the energy density of modified holographic Ricci dark energy in the universe is obtained as

$$\rho_\Lambda = \frac{3}{t^2}[\eta_1(\alpha_1+\alpha_2 t)^2 - \eta_2\alpha_1 + 2\eta_3\alpha_1(\alpha_1+\alpha_2 t)^{-1}] \tag{30}$$

Using Eq.(20) in Eq.(22) the displacement vector field in this model is

$$\beta(t) = \beta_0(a_0 t^{\alpha_1} e^{\alpha_2 t})^m \tag{31}$$

From the Eqs. (9),(23), (30)and (31) we get the EoS parameter in the model as

$$\omega_\Lambda = -\left[\frac{12(\alpha_1+\alpha_2 t)^2(n^2+n+1) - 4\alpha_1(2n^2+3n+1) + (2n+1)^2\beta_0{}^2 t^2(a_0 t^{\alpha_1} e^{\alpha_2 t})^{2m}}{(2n+1)^2[\eta_1(\alpha_1+\alpha_2 t)^2 - \eta_2\alpha_1 + 2\eta_3\alpha_1(\alpha_1+\alpha_2 t)^{-1}}\right] \tag{32}$$

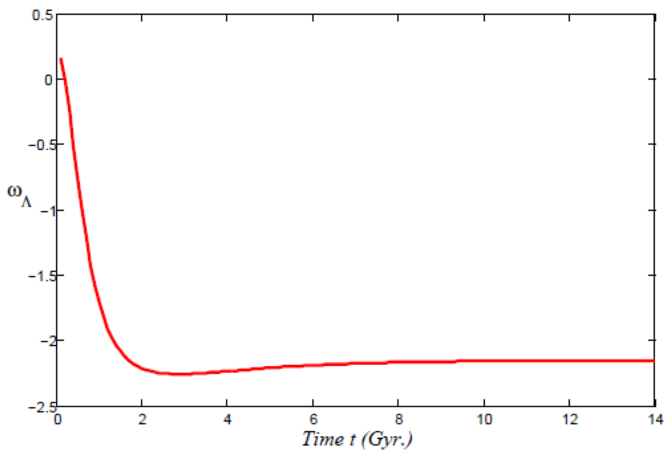

Fig. 1: Plot of EoS parameter versus time for $\alpha_1 = 0.3, \alpha_2 = 0.7$, $\beta_1 = 1.8, \beta_2 = 0.3, \beta_3 = 0.8$, $m = 0.3$ and $n = 1.1$.

From Eqs. (12),(23),(30) and (31) we find the matter energy density in the model as

$$\rho_M = \left[\frac{36(n(n+2)(\alpha_1+\alpha_2 t)^2 - 3(2n+1)^2\beta_0{}^2 t^2(a_0 t^{\alpha_1} e^{\alpha_2 t})^{2m} - 3(2n+1)^2[\eta_1(\alpha_1+\alpha_2 t)^2 - \eta_2\alpha_1 + 2\eta_3\alpha_1(\alpha_1+\alpha_2 t)^{-1}}{(2n+1)^2 t^2}\right] \tag{33}$$

From Eqs.(10), (18), (23), (30) and(32) skewness parameters are determined as

$$\delta = \gamma = \frac{2\alpha_1(6n^2+7n+2) - (\alpha_1+\alpha_2 t)^2(21n^2+12n+12) - 2(2n+1)^2\beta_0{}^2 t^2(a_0 t^{\alpha_1} e^{\alpha_2 t})^{2m}}{(2n+1)^2[\eta_1(\alpha_1+\alpha_2 t)^2 - \eta_2\alpha_1 + 2\eta_3\alpha_1(\alpha_1+\alpha_2 t)^{-1}} \tag{34}$$

From Eqs.(14), (30) and (33) we find the total energy density in the universe is

$$\Omega = \frac{\rho_\Lambda + \rho_M}{H^2} = \frac{9(\alpha_1+\alpha_2 t)^2(n^2+2n) - 3(2n+1)^2\beta_0{}^2 t^2(a_0 t^{\alpha_1} e^{\alpha_2 t})^{2m}}{(2n+1)^2(\alpha_1+\alpha_2 t)^2} \tag{35}$$

The jerk parameter is defined as the third derivative of the scale factor with respect to the cosmic time and is given by (Chiba and Nakamura 1998)

$$j(t)=\frac{1}{H^3}\frac{\dddot{a}}{a}=q+2q^2-\frac{\dot{q}}{H} \quad (36)$$

To describe models close to Λ *CDM*, cosmic jerk parameter is used in cosmology. Cosmologists believe that transition of the universe from deceleration to acceleration occurs for models with positive value of jerk parameter and negative value of deceleration parameter. The jerk parameter for ΛCDM model have a constant jerk, *j=1*. For this model, we get

$$\text{j(t)}=1-\frac{3\alpha_1}{(\alpha_1+\alpha_2 t)^2}+\frac{2\alpha_1}{(\alpha_1+\alpha_2 t)^3} \quad (37)$$

The deceleration parameter is given by

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1=-1+\frac{\alpha_1}{(\alpha_1+\alpha_2 t)^2} \quad (38)$$

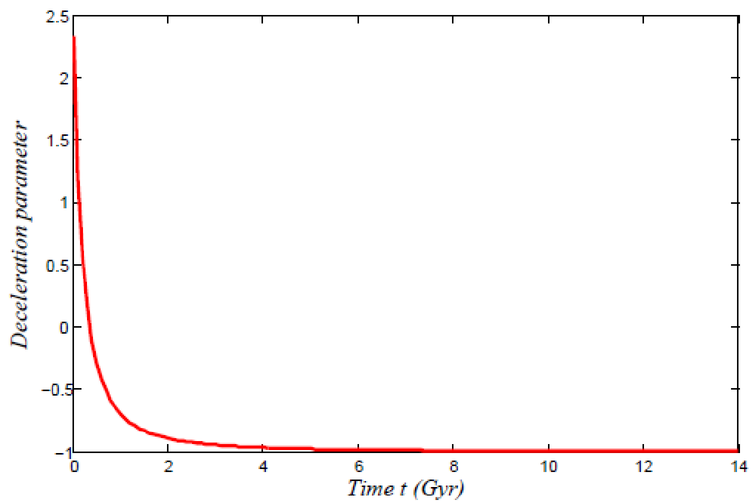

Fig. 2: Plot of deceleration parameter versus time for $\alpha_1 = 0.3$ and $\alpha_2 = 0.7$.

Sahni et al.(2003) has defined a new geometrical diagnostic pair called statefinder pair
{r, s} given by

$$r=\frac{\dddot{a}}{H^3 a},\ s=\frac{r-1}{3(q-\frac{1}{2})} \quad (39)$$

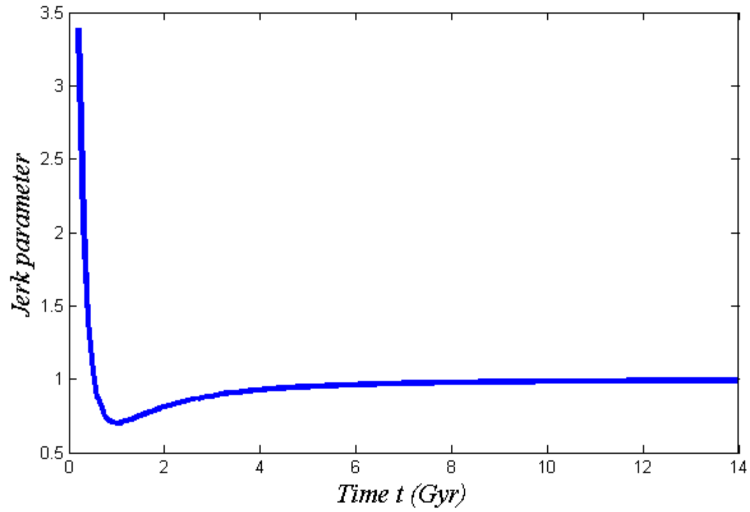

Figure 3: Plot of jerk parameter versus cosmic time t for $\alpha_1 = 0.3$ and $\alpha_2 = 0.7$.

These parameters are useful to find the distance of a given dark energy model from ΛCDM limit and they characterize the properties of dark energy in a model-independent manner. Also, this pair can be used to differentiate the dark energy regions as follows:

(i) {r, s} = (1,0) indicates ΛCDM limit
(ii) {r, s} = (1,1) shows CDM limit
and (iii) $s > 0$, $r < 1$ represent ,respectively, phantom and quintessence dark energy eras.

These parameters can also be expressed in terms of Hubble parameter and its derivatives with respect to cosmic time as

$$r = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3}, \quad s = \frac{-2(3H\dot{H} + \ddot{H})}{3H(3H^2 + 2\dot{H})} \tag{40}$$

Hence from Eqs.(26) and (40) we have

$$r = 1 - \frac{3\alpha_1}{(\alpha_1 + \alpha_2 t)^2} + \frac{2\alpha_1}{(\alpha_1 + \alpha_2 t)^3}, \quad s = \frac{2\alpha_1[3(\alpha_1 + \alpha_2 t) - 2]}{3(\alpha_1 + \alpha_2 t)\ [3(\alpha_1 + \alpha_2 t)^2 - 2\alpha_1]} \tag{41}$$

The above results are useful to discuss the physical behavior of the model (24). The spatial volume of the universe increases with time. This shows the spatial expansion of the universe .It can be observed that the physical and kinematical parameters of the model, $H, \theta, \sigma, \rho_\Lambda, \rho_M\ \delta$, all diverge at t=0 and they all become constant as $t \to \infty$. Fig-1 shows the variation of EoS parameter ω_Λ with time. It can be seen that the dark energy model lies in the quintessence region and attains a constant value. Fig-2 gives the plot of deceleration parameter which shows that there is a smooth transition of the model from decelerated phase to the accelerated phase of the universe. Fig-3 depicts the behavior of cosmic jerk parameter with time t. It may be observed that it is positive throughout the history of the universe and becomes ΛCDM model at late times. Also, when $\alpha_1 = 0$ we have the diagnostic pair as $\{r, s\} = (0,1)$ which gives us ΛCDM limit. It can be observed that the anisotropy parameter vanishes when n=1 which implies that the model becomes isotropic and shear free at late times. The displacement vector field in the model which arises naturally from the geometry and the average density parameter show an exponential increase with time.

5. Conclusions

The recent scenario of accelerated expansion of the universe is attracting many researchers to investigate dark energy models in modified theories of gravitation which are supposed to explain this late time acceleration of the universe. The work presented in this paper is one such attempt to investigate Bianchi type-II modified holographic Ricci dark energy model in the modified theory formulated in Lyra manifold. The model presented is expanding and evolves to attain a constant value in the quintessence region. It may be noted that anisotropy and skewness parameters of the universe approach at late times. Also, the model exhibits a smooth transition from decelerated phase to accelerated phase with time. The results discussed here are in good agreement with the observations modern cosmology.

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