Article

Plane Symmetric Bulk Viscous Cosmological Model with Time Dependent Λ-Term in General Theory of Relativity

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Abstract

In this paper Einstein's field equations with time dependent cosmologicalterm Λ are consider in presence of bulk viscous fluid for plane symmetric space time .For the simplicity of $\xi(t)$, we assumeto be a simple power function of the energy density $\xi(t)=\xi_0\rho^n$, where ξ_0 and n are constants and an equation of state of the form $p=\gamma\rho$, where γ $(0\le\gamma\le 1)$ is a constant.Time dependent cosmological term Λ is found to be a decreasing function of time. Also some geometrical and physical aspects of the modelsare discussed.

Keywords: Bulk viscous fluid, time-dependent,lambda, plane-symmetric, spacetime.

1. Introduction

Einstein in 1917 introduced the cosmological term Λ to modify his own equations of general relativity. Now this Λ- term remains a focal point of interest in modern theories. In 1930's , distinguished cosmologists A.S. Eddington and Able George Lemaitre felt that introduction of Λ- term has attractive features in cosmology and models based on it should be discussed deeply.Inhomogeneous cosmological model plays an important role in understanding some essential features of the universe such as the formation of galaxies during the early stages of evolution and process of homogenization. The early attempt at the construction of such models hasdone by Tolman [1] and Bondi[2], who consider spherically symmetric models. Inhomogeneous plane symmetric model was first considered by Taub [3] and later by Tomimura [4], Szekeres [5], Collins and Szafron [6]. Pradhan et.al.[7]have studied plane symmetric inhomogeneous model in presence of perfect fluid.

The generalized Einstein's theory of gravitation with time dependent G and Λ have been proposed by Lau [8]. The possibility of variable G and Λ in Einstein's theory has also been studied by Dersarkissian [9].The cosmological model with variable G and Λ have been recently studied by several authors. Some of the recent discussions on the cosmological-constant "problem" and consequence on cosmology with a time varying cosmological constant have been

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discussed byRatra and Peebles [10], Sahni and Starobinsky [11], Peebles [12], Singhet.al.[13-14],Vermaet. al. [15-16] and Pradhan et al. [17-18].Weinberger [19], Heller and Klimek [20], Misner [21-22], Collins and Stewart [23] have studied the effect of viscosity on the evolution of cosmological models. Xing-Xiang Wang [24] discussed Kantowski-Sachs string cosmological model with bulk viscosity in general relativity.

Also several aspects of viscous fluid cosmological model in early universe have been extensively investigated by many authors Raj Bali and Dave [25], Adhav et.al. [26], Verma M.K. and Shri Ram [27], Mete et.al.[28-33].

The purpose of the present work is to study inhomogeneous bulk viscous fluid cosmological model with time dependent Λ term in general theory of relativity. Our paper is organize as follows, in section 2, we have discussed the metric and field equations in general theory of relativity. Section 3 contains solution of the field equations with different cases. The last section is the conclusion.

2. The metric and field equations

We consider the plane-symmetric space –time in the general form

$$ds^2 = D^2dt^2 - A^2dx^2 - B^2(dy^2 + dz^2), \tag{1}$$

where A, B and D are functions of x and t.

The Einstein field equations with the cosmological term Λg_{ij} are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -8\pi T_{ij}. \tag{2}$$

The energy momentum tensor in the presence of bulk stress has the form

$$T_{ij} = (\rho + \bar{p})u_i u_j - \bar{p}g_{ij} \tag{3}$$

and

$$\bar{p} = p - \xi u^i_{;i}. \tag{4}$$

Here $\rho, p, \bar{p}$ ξ and Λ are energy density, isotropic pressure, effective pressure, bulk viscous coefficient and cosmological constant respectively and u_i is the four velocity vector satisfying the relation

$$g^{ij}u_i u_j = 1 \tag{5}$$

Hereafter, the semicolon(;)denotes covariant differentiation.

Using commoving coordinates system, the set of field equations (2) for the metric (1) reduces to the following forms,

$$\frac{2}{BD^2}\left[B_{44}-\frac{DB_1D_1}{A^2}-\frac{B_4D_4}{D}\right]-\frac{1}{B^2}\left[\frac{B_1^2}{A^2}-\frac{B_4^2}{D^2}\right]-\Lambda=-8\pi\,\bar{p} \tag{6}$$

$$\frac{1}{BD^2}\left[B_{44}-\frac{DB_1D_1}{A^2}-\frac{B_4D_4}{D}\right]-\frac{1}{A^2B}\left[B_{11}-\frac{A_1B_1}{A}-\frac{AA_4B_4}{D^2}\right]+\frac{A}{A^2D^2}\left(AA_{44}-\frac{AA_4D_4}{D}-DD_{11}+\frac{DA_1D_1}{A}\right)-\Lambda=-8\pi\,\bar{p} \tag{7}$$

$$\frac{2}{A^2B}\left[B_{11}-\frac{A_1B_1}{A}-\frac{AA_4B_4}{D^2}\right]+\frac{1}{B^2}\left[\frac{B_1^2}{A^2}-\frac{B_4^2}{D^2}\right]+\Lambda=-8\pi\rho \tag{8}$$

$$\frac{2}{B}\left[B_{14}-\frac{B_1A_4}{A}-\frac{D_1B_4}{D}\right]=0\cdot \tag{9}$$

Here, the suffixes 1 and 4 at the symbols A, B and D denotes partial differentiation with respect to x and t respectively.

3. Solution of the field equations

The solutions of the field equations (6)-(9) are highly nonlinear, therefore we consider the explicit solutions of the field equations, A, B and D are taken in the following form (Sahu et.al. [34])

$$A=t^l(1+x^2)^a,\ B=t^m(1+x^2)^b \text{ and } D=(1+x^2)^d, \tag{10}$$

where l, m, a, b, d are real constant $(l\neq 0, m\neq 0)$

Using the values of A, B and D from equation(10), equations (6)-(9) leads to

$$\frac{3m^2-2m}{t^2(1+x^2)^{2d}}-\frac{4b(2d+b)x^2}{t^{2l}(1+x^2)^{2l+2}}=\Lambda-8\pi\,\bar{p} \tag{11}$$

$$\frac{l^2+m^2-l-m+lm}{t^2(1+x^2)^{2d}}+\frac{(4ad-4bd+2d-4d^2-4b^2+2b+4ab)x^2-2(d+b)}{t^{2l}(1+x^2)^{2l+2}}=\Lambda-8\pi\,\bar{p} \tag{12}$$

$$\frac{4b\{(3b-2a-1)x^2+1\}}{t^{2l}(1+x^2)^{2l+2}}-\frac{m(2l+m)}{t^2(1+x^2)^{2d}}=-\Lambda-8\pi\rho \tag{13}$$

and

$$m(d-b)+lb=0 \tag{14}$$

From equations (11) and (12), we obtain

$$b+d=0, \quad 3m^2-2m=l^2+m^2-l-m \tag{15}$$

and

$$4ad-4bd+2d-4d^2-4b^2+2b+4ab=-4b(2d+b) \tag{16}$$

Now thecorresponding to the equations (15) and (16), we have two sets of solutions,

i.e., (i) $b=d=0$ and $l=m$
(ii) $b=d=0$ and $l=-2m+1$

Case-I

When

$$b=0, d=0, l=m=k \text{ (say)} \tag{17}$$

In this case, the geometryof the space time(1) takes the form

$$ds^2=dt^2-t^{2k}[(1+x^2)^{2a}dx^2+dy^2+dz^2] \tag{18}$$

Using these values from equation (17), equations (11) - (14) leads to

$$8\pi\bar{p}=\frac{2k-3k^2}{t^2}+\Lambda \tag{19}$$

and

$$8\pi\rho=\frac{3k}{t^2}-\Lambda \tag{20}$$

For the simplification of $\xi(t)$, we assume that the fluid obeys an equation of state of the form

$$p=\gamma\rho, \tag{21}$$

where $\gamma(0\le\gamma\le1)$ is a constant.

Thus, given $\xi(t)$ we can solve the cosmological parameters. In most of the investigations involving bulk viscosity is assumed to be a simple power function of the energy density (Pavon[35],Maartens[36], Zimdahl [37])

$$\xi(t)=\xi_0\rho^n, \tag{22}$$

where ξ_0 and n are constants.

If $n=1$,Equation (22) may correspond to a relative fluid (Winberg[38]). However, more realistic models (Santos,[39]) are based on n lying in the regime $0\le n\le\frac{1}{2}$.

Model-I: Solution for $\xi = \xi_0$

When $n = 0$ equation (22) reduces to $\xi = \xi_0$ =constant.
Hence in this case equation (19) with the use of (21) become

$$8\pi\rho = \frac{1}{1+\gamma}\left[\frac{24\pi\xi_0 k}{t} + \frac{2k}{t^2}\right]. \tag{23}$$

Eliminating $\rho(t)$ between (20) and (23), we have

$$\Lambda = \frac{3k^2}{t^2} - \frac{1}{1+\gamma}\left[\frac{24\pi\xi_0}{t} + \frac{2k}{t^2}\right] \tag{24}$$

Model-II: Solution for $\xi = \xi_0 \rho$

When $n = 1$, Equation (22) reduces to $\xi = \xi_0 \rho$. Hence in this case equation (19) with the help of (21) become

$$8\pi\rho = \frac{2k}{\left[1+\gamma - \frac{3\xi_0 k}{t}\right]t^2} \tag{25}$$

Eliminating ρ between (20) and (25), we obtain

$$\Lambda = \frac{3k^2}{t^2} - \frac{2k}{\left[1+\gamma - \frac{3\xi_0 k}{t}\right]t^2} \tag{26}$$

From Equations (24) and (26), we observed that cosmological constantis a decreasingfunction of time and approaches a small value in the present epoch.

3.1 The physical parameters of the model

The physical parameters such as spatial volume (V), Hubble parameter (H), expansion scalar (θ) and shear scalar (σ) of the model (18) are given by

$$V = R^3 = \sqrt{-g} = t^{3k}(1+x^2)^a \tag{27}$$

$$H = \frac{k}{t} \tag{28}$$

$$\theta = \frac{A_4}{AD} + \frac{2B_4}{BD} + \frac{D_1}{A^2} = \frac{3k}{t} \tag{29}$$

and

$$\sigma^2 = \frac{1}{2\sigma_{ij}\sigma^{ij}} = \frac{1}{12}\left[\left(\frac{(g_{11})_4}{g_{11}} - \frac{(g_{22})_4}{g_{22}}\right)^2 + \left(\frac{(g_{22})_4}{g_{22}} - \frac{(g_{33})_4}{g_{33}}\right)^2 + \left(\frac{(g_{33})_4}{g_{33}} - \frac{(g_{11})_4}{g_{11}}\right)^2\right] = 0 \tag{30}$$

Case-II:

When $b = 0, d = 0, l = -2m+1$ (31)

In this case,the geometry of the space time(1) takes the form

$$ds^2 = dt^2 - t^{2(1-2m)}(1+x^2)^{2a}dx^2) - t^{2m}(dy^2 + dz^2) \tag{32}$$

Using these values from equation (31), equations (11) - (14) reduces to

$$8\pi\,\bar{p} = \frac{2m - 3m^2}{t^2} + \Lambda \tag{33}$$

and

$$8\pi\rho = \frac{2m - 3m^2}{t^2} - \Lambda \tag{34}$$

Model-I: Solution for $\xi = \xi_0$

When $n = 0$ equation (22) reduces to $\xi = \xi_0$ =constant. Hence in this case equation (33) with the use of (21) become

$$8\pi\rho = \frac{1}{1+\gamma}\left[\frac{2(2m - 3m^2)}{t^2} + \frac{8\pi\xi_0}{t}\right] \tag{35}$$

Eliminating $\rho(t)$ between (34) and (35), we have

$$\Lambda = \frac{(2m - 3m^2)}{t^2} - \frac{1}{1+\gamma}\left[\frac{2(2m - 3m^2)}{t^2} + \frac{8\pi\xi_0}{t}\right] \tag{36}$$

Model-II: Solution for $\xi = \xi_0\rho$

When $n = 1$, Equation (22) reduces to $\xi = \xi_0\rho$. Hence in this case equation (33) with the help of (21) becomes

$$8\pi\rho = \frac{2(2m-3m^2)}{\left[1+\gamma-\frac{8\pi\xi_0}{t}\right]t^2} \tag{37}$$

Eliminating ρ between (34) and (37), we obtain

$$\Lambda = \frac{(2m-3m^2)}{t^2} - \frac{2(2m-3m^2)}{\left[1+\gamma-\frac{8\pi\xi_0}{t}\right]t^2} \tag{38}$$

From Equations (36) and (38), we observed that cosmological constant is a decreasing function of time and approaches a small value in the present epoch.

3.2 The physical parameters of the model

The physical parameters expansion scalar (θ) and shear scalar (σ) of the model (32) are given by

$$\theta = \frac{1}{t} \tag{39}$$

and

$$\sigma^2 = \frac{2}{3}\left(\frac{3m-1}{t}\right)^2 \tag{40}$$

4. Conclusions

In this paper, we have investigated inhomogeneous cosmological model with bulk viscous fluid and time dependent cosmological term-$\Lambda$by using a simple power function of energy density $\xi(t)=\xi_0\rho^n$, where ξ_0 and n are constants. The values of the parameters can be obtained by using explicit solutions of the field equations. In both the cases, cosmological term Λ is a decreasing function of time and this approaches a small value as time increases. It is observed that at $t=0$, the involved parameters in both physical and kinematical properties of the models diverges while the parameters remain finite and well behaved for $t>0$.

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