**Article**

# The Arrow of Time Problem: Answering if Time Flow Initially Favouritizes One Direction Blatantly

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## Abstract

The following article begins an analysis of if by first principles there is a way to show time unidrectionality at the start of creation. Recently J. Vaccarro appealed to Kaon physics to show how time asymmetry could be violated after the big bang, as well as how and why time asymmetry vanishes later. We examine the nature of the evolution Hamiltonian after stepping through Vacarro's analysis and go to where we think it needs to be improved. This answers questions to if both the CPT theorem and related physics are manifestly violated, or not violated at the beginning of emergent space time, via judicious use of the Wheeler-DeWitt equation, a base of quantum cosmology and quantum gravity. We initially detail the limits of a derivation given the authors by Vaccaro which claims using of the Wheeler-DeWitt equation, and suggest ways to suppress the CPT violation, as suggested by Kaon physics after inflation at the onset of inflation, using refinements suggested by the authors. We propose to support the Kauffmann treatment of CPT violation as the useful one, as well, and will be part of long term solutions to this problem. Note that though, Vaccaro incorrectly identifies a Euclidian representation of the Hamiltonian (energy) function as equal to the Hamiltonian constraint, which leads to identifying the Schrodinger equation used for Kaon physics with the Wheeler-DeWitt equation, as $H|\psi\rangle = 0$ as written by Vaccaro is still usual non-relativistic time-dependent quantum mechanics and not necessarily linkable to the Wheeler-DeWitt equation, which in the result of primary quantization of the Hamiltonian constraint and has manifestly absent time, in the vicinity of a 'quantum bounce', or singularity.

**Key Words**: arrow of time, big bang, CPT violation, Kaon physics, Wheeler-DeWitt equation.

## 1. Introduction

Recently Dr. J.A. Vaccaro sent the authors her DICE 2010 discussion [1] as to direction of time flow, which concluded that, contrary to expectations, that time flow invariance, with respect to kaons could be violated for regimes of space time well after the big bang. For the regime of space time as to the big bang, Vaccarro asserts that forward and backward time evolution Hamiltonian contributions as to time evolution asymmetry would cancel out, leaving forward time evolution. This due to investigations of cancellations of forward and backward time evolution Hamiltonians, i.e. the hypothesis is that if the forward time Hamiltonian for time evolution is the same as for the backward time evolution Hamiltonian then there would be just a one direction evolution of time.

The approach proposed by Vaccaro, however, is factually studying the problem of time in non-relativistic quantum mechanics described by the time-dependent Schrodinger equation,

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and is manifestly the attempt to do a straightforward relation of these results to the Wheeler-DeWitt equation, the base of quantum cosmology and quantum gravity. However, the suggested relationship is incomplete and arguably flawed. The incorrectness follows from identification of the Hamiltonian with a Hamiltonian constraint, what is true if and only if there is a linear dependence on energy in the Hamiltonian constraint. Otherwise, like as what happens in both Einstein's Special Relativity and in quantum gravity based Wheeler-DeWitt equation, such a dependence and situation is manifestly absent. For such situations quantization scheme based constraints is actually used, what P.A.M. Dirac originally proposed [2]. Also relativistic quantum mechanics based the Dirac equation can be obtained in such a way, as an approximation of the Klein-Gordon equation. Strictly speaking, the phenomenon of a linear dependence on energy within the Hamiltonian constraint is factually true for non-relativistic quantum mechanics, i.e. in general for the Schrödinger wave equation. Other cases are usually approximations. Particularly, the energy linearity is not validated for the Wheeler-DeWitt equation, which is formally degenerated case. Also using of formal Feynman path integral in the case of Wheeler-DeWitt equation, like suggested by Hawking [3], is factually based on the analogy of quantum gravity with Schrödinger equation. However, the Hartle-Hawking wave function [4], following from such approach, was not factually computed in general. The problem is the only using of path integration in that concrete case arises from some kind of tautology based analogy with Schrödinger equation.

Otherwise, it is noticeable to also be aware that others have a similar fuzzy understanding of the role of classical and quantum physics, and the role of the Wheeler-DeWitt equation in general. Note that Mersini-Houghton writes [5]

*"Since our classical trajectory (a) goes through a turning point induced by the gravitational instabilities of back reaction in the De Sitter epoch= 0, it follows that after losing casual contact with the internal environment, the universe becomes quantum again as we approach the turning point in a time $a*$. If observers bound to the causal patch survived while the universe is transiting to a quantum state, they would perceive the arrow of time as being reversed near the turning point. However notice that very close to the turning point, the WKB approximation breaks down, therefore the concept of time and any statement about what can be observed becomes very fuzzy. A better estimate as to when effects related to 'recoherence' display significant effect in our universe is obtained from the density matrix below, Eq. (11)".*

This analysis is correct, up to a point, but its author is making factually the same logical error like Vaccaro.

Our comment is that the problem lies, as we identified it, as to the proper relationship between constraint equations and its solutions. As can be put succinctly. $H|\psi\rangle = 0$ is also a Schrödinger equation, but the only formally as the result of analogical quantization procedure of the Hamiltonian constraint $H=0$ that is not the Hamiltonian (energy), which is known as 'degenerate' but that $H|\psi\rangle = 0$ is Wheeler-DeWitt equation only if the Hamiltonian is a Hamiltonian constraint. The two things, a Hamiltonian, and a Hamiltonian constraint are very different things in general. The best example is Einstein Special Relativity, in which the

Hamiltonian constraint is $E^2 - p^2c^2 - m^2c^4 = 0$, i.e. energy is present in non-linear way, but the Hamiltonian (energy) is $E = \pm\sqrt{p^2c^2 + m^2c^4}$. In this particular case the Hamiltonian follows from the Hamiltonian constraint as the approximation. Vaccaro uses the Hamiltonian, but not the Hamiltonian constraint, so his presentation factually has **no** relation to quantum gravity given by Wheeler-DeWitt equation. To a certain degree, Mersini-Houghton [5] does the same thing. As defined by Kiefer, one can generalize a kinematic momentum $\prod_v$ via the constraint, for J being the Jacobi determinant of X with respect to x, where $X^u(x^u)$ is a set of space-like hypersurfaces, i.e. with fixed time coordinate, in the Minkowski space, whereas $x^0 = (c=1)\cdot t$ parameterizes the hypersurface $X^u(x^u)$, as given by Kiefer [6]

$$H_v = \prod_v + J\frac{\partial x^0}{\partial X^u} T^u{}_v = 0 \qquad (1)$$

If one introduces orthogonal and parallel components of Eq. (1) parallel to $x^0 = (c=1)\cdot t =$ constant, with normal vectors $n_v$ to $x^0 = (c=1)\cdot t =$ constant and tangential vectors $X^v{}_{,a}$ obeying $n_v X^v{}_{,a} = 0$, then the identified Hamiltonian constraint and momentum (or diffeomorphism) constraint defined by

$$H_\perp = H_v n^v \approx 0 \qquad (2)$$

and

$$H_a = H_v X^v{}_{,a} \approx 0 \qquad (3)$$

Similar situation is present in the case the Hamiltonian approach to General Relativity proposed by Arnowitt, Deser, and Misner [7], which results in Wheeler-DeWitt equation. In Loop Quantum Gravity, based on the Ashtekar formulation of General Relativity, a Hamiltonian constraint is given by

$$\tilde{H}_\perp = H_E + \frac{\beta^2\sigma - 1}{\beta^2\sqrt{h}} tr\left(R_{ab}\left[E_a, E_b\right]\right) \qquad (4)$$

Here, $H_E$ is the so called Euclidean part of $\tilde{H}_\perp$, and frequently, $E^a = \tau^i E_i^a$. With $\tau^i = i\sigma^i/2$ and $\sigma^i$ the Pauli matrices, as well as a basis for the volume as $V = \int_\Sigma d^3x\sqrt{h} = \int_\Sigma d^3x\sqrt{\left|\det E_i^a\right|}$. From LQG point of view, the mistake which Vaccaro has been maked is equivalent to writing $\frac{\beta^2\sigma - 1}{\beta^2\sqrt{h}} tr\left(R_{ab}\left[E_a, E_b\right]\right) = 0$, since, even in the situation of the beginning of space time $V = \int_\Sigma d^3x\sqrt{h} = \int_\Sigma d^3x\sqrt{\left|\det E_i^a\right|} \sim l^3{}_{Planck} \neq 0$, there is no situation for the vanishing of the 2nd term in Eq. (4) above, one has to conclude that Vaccaro has not studied as to how to relate $H|\psi\rangle = 0$ to a de facto construction of the Wheeler-DeWitt equation.

Sawayama has a statement as to an appropriate Hamiltonian as given by a higher dimensional embedding. In his article, he states, namely [8]

*The Hamiltonian constraint is the generator of the time translation and the diffeomorphism constraint are the generator of space translations [8]. The theory of quantum gravity contains many unsolved problems which contain problem of the time and problem of the norm. However, most important problem is the difficulty of the constraint equations, i.e. Wheeler-DeWitt equation [9].*

*Our motivation is simple and that is to find at least one local solution of the Wheeler-DeWitt equation.*

To put it mildly, that is extremely difficult. I.e. and it involves issues as stated below, by Sawayama, as

$$R\left|\psi^5(g)\right\rangle = 0 \qquad (5)$$

Sawayama writes, as given below [8]

*Here ˆR is the operator, corresponding to the usual 4-dimensional Ricci scalar.*

Whereas the function $\left|\psi^5(g)\right\rangle$ would be part of the Wheeler-DeWitt equation in 5-dimenional quantum gravity, in spite that the Ricci scalar is actually the Hamiltonian constraint in absence of Matter fields.

Vaccaro does none of this, and up to the point that she gets close to Planck time scales in evolution, she is misapplying the situation as given by Eq. (4) above. Applying the Wheeler de Witt equation as to the problem of finding an appropriate embedding space has not been done by either Vaccaro [1], or Mersini-Houghton [5]. Note though, that away from the Planck time scale, and using quantum physics, that one can do Kaon physics as to the Schrödinger equation. Now what if one misinterpreted the Schrödinger equation solution of $H|\psi\rangle = 0$, as Wheeler-DeWitt, as done by Vaccaro [1]?. While this is a defensible analysis for Kaon physics, away from the big bang, and yet the analysis has severe problems in the vicinity of the big bang itself. Let us list what some of the problems are

1. The wave function of the universe interpretation of the Wheeler-DeWitt equation depends upon a WKB airy function, which has its argument dependent upon z. When z ~ $\left(\frac{3\pi \cdot \tilde{a}_0^2}{4G}\right)^{2/3} \cdot \left[1 - \left[\frac{\tilde{a}^2}{\tilde{a}_0^2}\right]\right] \xrightarrow[\tilde{a}\to 0]{} \left(\frac{3\pi \cdot \tilde{a}_0^2}{4G}\right)^{2/3}$ right at the start of the big bang, the wave function of the universe is a small positive value, as given by Kolb and Turner [9]. Having $\tilde{a} \to 0$ corresponds to a classically forbidden region, with a Schrödinger equation of the form (assuming a vacuum energy $\rho_{Vacuum} = [\Lambda/8\pi \cdot G]$ initially), with $\Lambda$ part of a closed FRW Friedman equation solution

$$a(t) = \frac{1}{\sqrt{\Lambda/3}} \cosh\left[\sqrt{\Lambda/3} \cdot t\right] \qquad (6)$$

to a flat space FRW equation of the form [9, 10]

$$\left[\frac{\dot{a}}{a}\right]^2 + \frac{1}{a^2} = \frac{\Lambda}{3} \qquad (7)$$

Which is so one forms a 1-dimensional Schrödinger equation to mimic the Wheeler-DeWitt equation [9, 10]

$$\left[\frac{\partial^2}{\partial \cdot \tilde{a}^2} - \frac{9\pi^2}{4G^2}\left[\tilde{a}^2 - \frac{\Lambda}{3}\tilde{a}^4\right]\right]\psi = 0 \qquad (8)$$

with $\tilde{a}_0$ a turning point to potential

$$U(a) = \frac{9\pi^2}{4G^2}\left[\tilde{a}^2 - \frac{\Lambda}{3}\tilde{a}^4\right]. \qquad (9)$$

Note that as $\tilde{a} \rightarrow 0$, the wave function in a classical sense would never leave a potential system defined by U(a) and that much more seriously, the definition of a vacuum energy, as set by the 1-dimensional Schrödinger equation is not defined, properly for a FRW classical Friedman equation. .The vacuum energy, is for $\rho_{Vacuum} = [\Lambda/8\pi \cdot G]$, for definition of the $\Lambda$ FRW metric, and is undefined for the regime $0 < \tilde{a} < 1/\sqrt{\Lambda/3}$. I.e. the classically undefined regions for evolution of Eq. (7) and Eq (8) are the same.

The problem is this, having $\tilde{a} \rightarrow 0$ makes a statement about the existence, quantum mechanically about having a (semi classical) approximation for $\psi$, when in fact the key part of the solution for $\psi$, namely $\rho_{Vacuum} = [\Lambda/8\pi \cdot G]$ is not definable for Eq. (7) if $0 < \tilde{a} < 1/\sqrt{\Lambda/3}$, whereas the classically forbidden region for Eq. (7) depends upon $0 < \tilde{a} < \tilde{a}_0$ where $\tilde{a}_0$ is a turning point for Eq. (9) above. $\Lambda$ is undefined classically, and is a free parameter, of sorts especially in the regime $0 < \tilde{a} < 1/\sqrt{\Lambda/3}$. As $\tilde{a} \rightarrow 0$, unless $\Lambda \rightarrow 0$, there is no classical way to justify the WKB as $\tilde{a} \rightarrow 0$.

Vaccarro in general incorrectly assumes the following, namely that one can, in Eq. (4) assume that one can assume $\frac{\beta^2\sigma - 1}{\beta^2\sqrt{h}} tr\left(R_{ab}\left[E_a, E_b\right]\right) = 0$ so then if that were true the following would hold [1, 6].

2. The statement as to if there is a Wheeler-DeWitt treatment of time, itself depends upon[6] the identification $H \cdot \psi \rightarrow 0$, with $\psi$ the wave function of the universe, dependent upon a WKB analysis. If $\psi$ does not exist classically at all at z ~ $\left(\frac{3\pi \cdot \tilde{a}_0^2}{4G}\right)^{2/3}$ in the WKB approximation then the entire program has to be revisited and re done.
3. In addition, Vaccarro asserts that there would be the situation $H_B = H_F$ at the start of the big bang. If $H_B$ is for a Hamiltonian in the prior universe, and if there are a

different set of cosmological constant parameters, the statement $H_B = H_F$ cannot be defended. More likely would be, to keep into consideration making a statement to the effect that $H_B = H_F \bmod(some - space - time - parameter(s?))$. The idea would be that there would have to be some group theoretic extension of how to embed the initial Hamiltonian parameter as to make the cancellation as indicated by Vaccarro valid. Speculations as to what this extension could be will be included in the document.

Next, will be a statement as to J. Vaccarro's analysis as to the asymmetry problem.

## 2. Vaccarro's analysis of the time problem

The authors wish to thank J. Vaccarro for sharing the DICE 2010 presentation with them. We find that the starting point is a good one, and that what is being asked is what to put in for the very hard to analyze start of the universe initial conditions [1].

J. Vaccarro presented the time evolution operator, which is called $\breve{T}$ , in the context of a quantum Schrödinger equation, as in the following situation, where U is the unitary operator, and K is the anti conjugation operator, so that, if $\breve{T}$ is applied to the wave function $\psi$ there is a sign change in both final momentum and angular momentum of the object represented by wave function $\psi$

$$\breve{T} = UK . \qquad (10)$$

As to the direction of time as to normal operators, in the Schrödinger equation, one obtains $H\breve{T}\psi = E\breve{T}\psi \Rightarrow \breve{T}H\breve{T}^{-1} = H$. For kaon physics, however, away from the big bang $\breve{T}H\breve{T}^{-1} \neq H$. This inequality as to $\breve{T}H\breve{T}^{-1} \neq H$, the following results. Vaccarro indicates that usually people (the consensus) treat this as a no consequence datum. To her credit, Vaccarro re defines the problem in terms of a unitary time evolution operator hitting upon an initial wave function $\psi_0$ with $H_f$ being the "forward time evolution Hamiltonian"

$$U_F = \exp\left[-iH_f\tau\right] \Leftrightarrow \left|\psi_F(\tau)\right\rangle \equiv U_F\left|\psi_0\right\rangle \qquad (11)$$

Vaccarro also defends the backward time evolution operator via use of with $H_B$ being the "backward time evolution Hamiltonian" with $H_B = \breve{T}H_F\breve{T}^{-1}$, and

$$U_B = \exp\left[iH_B\tau\right] \Leftrightarrow \left|\psi_B(\tau)\right\rangle \equiv U_B\left|\psi_0\right\rangle \qquad (12)$$

As far as application to physics after the big bang, Vaccaro looked at what the wave function

$$\left|\psi(\tau)\right\rangle \propto \left[U_F + U_B\right]\cdot\left|\psi_0\right\rangle \qquad (13)$$

The claim made was as of cancellation between most of the $U_F$ and $U_B$ operations for N iterations of Eq. (12) above so that one would obtain, if $\tau = t/N \approx 10^{-44}$ seconds, i.e. a Planck time interval

$$\left|\psi(N\tau)\right\rangle \propto \frac{1}{2^N}\left[U_F(\tau) + U_B(\tau)\right]^N \cdot\left|\psi_0\right\rangle \equiv \left[\exp\left[-i\cdot\frac{(H_F - H_B)}{2}\tau\right] + \vartheta(\tau^2)\right]^N \left|\psi_0\right\rangle \qquad (14)$$

As $N \to \infty$, Eq (20) would be such that due to destructive interference, i.e. for large N limit, the above would be

$$\left|\psi(N\tau)\right\rangle \propto \frac{1}{2^N}\left[U_F(N\tau)+U_B(N\tau)\right] \cdot \left|\psi_0\right\rangle \quad (15)$$

According to Vaccarro, unidirectionality of time in kaon physics would lead to observing only one of the terms

$$\left|\psi(N\tau)\right\rangle \propto \frac{1}{2^N}\left[U_F(N\tau)\right]\cdot\left|\psi_0\right\rangle \text{ or } \left|\psi(N\tau)\right\rangle \propto \frac{1}{2^N}\left[U_B(N\tau)\right]\cdot\left|\psi_0\right\rangle \quad (16)$$

This analysis is for when the kaons are well away from the big bang itself. I.e. the authors find Vaccarro's analysis appropriate up to here.

## 2.1 . **When at the source of the big bang, can be** $H_B = H_F$ **?**

We assert that the analysis given by $H_B = H_F$ has to be revisited. This will be brought up in detail later, but to see what we find questionable, we will present Vaccarro's concluding remarks if $H_B = H_F$ at the start of the big bang. Assuming $H_B = H_F = H$ and no interference

$$\left|\psi(N\tau)\right\rangle \propto \frac{1}{2^N}\left[U_F(\tau)+U_B(\tau)\right]^N \cdot \left|\psi_0\right\rangle \equiv \left[\cos(H\tau)\right]^N \left|\psi_0\right\rangle \equiv \left|\psi_0\right\rangle \quad (17)$$

The problem in this, is two fold. First of all, Vaccarro asserts that[1]

$$H\left|\psi(N\tau)\right\rangle = 0 \quad (18)$$

Such an analysis is defensible, if one assumes that the Wheeler-DeWitt equation holds with the Hartle-Hawking "no boundary" condition for which one can write via a WKB style

$$\psi_0 \approx Ai(z) = \frac{1}{\pi}\int_0^\infty \left[\cos\left(\frac{t^3}{3}+zt\right)\right]dt \cong$$

$$\left[N_0/2\right]\cdot\left|9\pi c^5/2\hbar G\Lambda\right|^{-1/2} \exp\left[\left(3\pi c^5/2\hbar G\Lambda\right)\cdot\left(\left[\frac{\tilde{a}}{\tilde{a}_0}\right]^2 - 1\right)\right] \xrightarrow{\tilde{a}\to 0} \left[N_0/2\right]\cdot\left|9\pi c^5/2\hbar G\Lambda\right|^{-1/2} \quad (19)$$

For $a_0 \sim 0^+$ for red shift at the start of the big bang, eq. (19) in the right hand has an undefined by classical range of $0 < \tilde{a} < 1/\sqrt{\Lambda/3}$. As $\tilde{a} \to 0$, unless $\Lambda \to 0$, there is no classical way to justify the WKB as $\tilde{a} \to 0$. Note that the expression of probability for a particle to move from the point $\tilde{a} \to 0$, to escaping Eq. (3) above is given by Kolb and Turner as a less than 100% value of. Having $\Lambda \to 0$ would mean $P_{Transition} \to 0$! But the existence of setting $\Lambda \neq 0$ violates the semi classical nature of Eq. (18) above, as $\tilde{a} \to 0$.

$$P_{Transition} \sim \exp\left[-3\pi/G\Lambda\right] \quad (20)$$

Secondly, we assert that $H_B = H_F$ can only hold if there is 100% certain that the laws of physics are identical in the prior to present universe. LQG asserts that there are identical values for physical parameters before and after the LQG "big bounce" and 'super inflation'. Note that Vaccarro is not assuming LQG, and is instead working with a WDW framework! As stated by A. Ashtekar [11] to Beckwith, at inaugural opening of the Penn state cosmology center, in 2007 asserted that the universe definitely preserves its " memory " and sets identical to virtually identical values for the fine structure constant, G, and more of the same from a prior to a present universe.

The authors raise two questions. First of all, is there any proof as to $H_B = H_F$**,** at the start of the expansion of the universe, and secondly, what to make of the analysis of Eq. (18) if $\psi_0 \neq 0$ is formed for values of the $\Lambda \neq 0$ regime if the wave function has no over lap with a vacuum energy which would be congruent with Eq. (2) above ?

## 3. Alternatives to a single universe bounce calculation. How could this affect the analysis of if $H\psi|(\tau N)\rangle = 0$ is brought into question with $\psi|(\tau N)\rangle \propto \psi_0$ if $\psi_0$ no longer is WKB compliant?

Vaccarro's main point is the collapse of $H\psi|(\tau N)\rangle = 0$ happens, then the initial wave function of the universe argument forces unidirectionality of time itself. We find Vaccarro's argument to be interesting but incomplete on essential details. $\psi|(\tau N)\rangle \propto \psi_0$ no longer WKB compliant means that the arrangement assuming $H\psi|(\tau N)\rangle = 0$ **has to be revisited [1].**

**Note that having** $\Lambda \neq 0$ is necessary for a non zero transition probability, as given by Eq. (19). But that $\Lambda \neq 0$, and $\tilde{a} \to 0$ has that $\Lambda \neq 0 \Rightarrow \psi|(\tau N)\rangle \propto \psi_0$ not necessarily compliant with the WKB analysis, and semi classicality for when $\psi|(\tau N)\rangle \propto \psi_0$ being violated draws into question $H\psi|(\tau N)\rangle = 0$**.** I.e. having a quantum solution for Eq. (8) above, and a classically forbidden solution for Eq. (7) above leads to trouble. Last but not least , $H_B = H_F$ no longer necessarily true would mean that the formation of $H\psi|(\tau N)\rangle = 0$ as given by Vaccarro would be questionable. Note that in making this assumption we are ignoring the problem as stated in the beginning about forming a Hamiltonian constraint equation, which involves, once again the issue of $\frac{\beta^2\sigma - 1}{\beta^2\sqrt{h}} tr\left(R_{ab}\left[E_a, E_b\right]\right) = 0$, even when $V = \int_\Sigma d^3x\sqrt{h} = \int_\Sigma d^3x\sqrt{\left|\det E_i^a\right|} \sim l^3{}_{Planck} \neq 0$ [6]

Another way to investigate this problem would be employing the Vilenkin solution; it involves zero values of the wave function if $\tilde{a} \to 0$. As stated by Kolb and Turner [9], $\psi|(\tau N)\rangle \propto \psi_0$ as set equal to zero, would lead to no analysis of $H\psi|(\tau N)\rangle = 0$ **which makes sense. More seriously, the** $\psi_0$ values have imaginary components

**The core difficulty of both approaches lies in the fixed nature of how to look at** $\Lambda \neq 0$. One way to remove some of the pathologies as to what to expect for a suitable wave function obeying $H\psi|(\tau N)\rangle = 0$**, as well as** $\psi|(\tau N)\rangle \propto \psi_0$ is to set $\Lambda \neq 0$ as becoming proportional to a scalar field nucleation field. I.e. $\psi\left|(\tau N)\right\rangle \propto \psi_0 \Leftrightarrow \Lambda \sim V(\phi) \sim m^2\phi^2/2$ for a varying $\phi$ scalar field. As of 2007, both Huang and Weng [12] did a take off on setting up $\Lambda \sim V(\phi) \sim \phi^2$ so as to avoid the pathologies inherent in setting $\Lambda \neq 0$ as a fixed parameter. Let us review shortly what they have done.

Essentially what they did was to re define the Wheeler-DeWitt equation with $\Lambda \sim V(\phi) \sim \frac{1}{2}m^2\phi^2$ so that Eq. (7) becomes, instead

$$\left[\frac{\partial^2}{\partial\tilde{a}^2} - \left[\frac{144\pi^2}{k^4}\right]\cdot\left[\tilde{a}^2 - \frac{k^2\tilde{a}^4 V(\phi)}{3}\right]\right]\psi = 0 \Leftrightarrow \rho_{transition} = c_1 \exp\left[-\frac{24\pi^2}{k^4 V(\phi)}\right] \qquad (21)$$

As the decay of the inflaton commences, with $\Lambda \sim V(\phi) \sim m^2\phi^2/2$ gets smaller, the same phenomenon as reported for Eq. (20) commences. However, the value of $\Lambda \sim V(\phi) \sim m^2\phi^2/2$ is not dependent upon scale factor, $\tilde{a}$ so one can avoid the phenomenon of the probability function for transmission through the potential barrier being dependent upon a cosmological parameter, fixed, which has no classical analog. In addition Huang and Weng, using $\frac{\partial\psi}{\partial\tilde{a}} = 0$ obtain for the chaotic inflaton potential contribution $\Lambda \sim V(\phi) \sim m^2\phi^2/2$, if $\tilde{a} = \alpha_0/\phi^2$ as Huang and Weng assert, then [12]

$$\frac{\partial^2\psi}{\partial\phi^2} = \frac{576\pi^2}{k^4}\cdot\left(\frac{\left[k_c\alpha_0{}^2\phi^{-4} - \frac{k^2}{3}\cdot\alpha_0^4\lambda_0\phi^{-6}\right]}{\left[1 - \frac{24\phi^{-2}}{k^2}\right]}\right)\cdot\alpha_0^2\phi^{-6}\psi = 0 \qquad (22)$$

We assert that Eq. (24) could for a wide range of $\tilde{a} = \alpha_0/\phi^2$ values be solved for $\psi$ dependent upon the nucleation of an inflaton field, $\phi$, while avoiding the pathologies as of Eq. (7) and Eq. (8) above, while, if $\Lambda \sim V(\phi) \sim m^2\phi^2/2$ holds, for scale dependent values of $\tilde{a} = \alpha_0/\phi^2$ obtain sensible values for the $\rho_{transition} = c_1 \exp\left[-\frac{24\pi^2}{k^4 V(\phi)}\right]$ transition.

What if one wishes to make a path integral treatment of Eq. (24) above? Huang and Weng state, near the end of their article, a general treatment of the wave functional, as given by [12]

$$\psi \equiv c_2 \exp\left[\sqrt{\frac{\pi^4\alpha_0^6\lambda_0}{6\phi^8}}\right] \qquad (23)$$

As in the Vilenkin case, having $\tilde{a} \to 0$ will lead to Eq. (23) going to zero. I.e. then making an analysis of what to do with $H\left|\psi(N\tau)\right\rangle = 0$ becomes messy again. At least though, there is no requirement as in the Vilenkin case that the wave function has to be imaginary, or complex valued! What the authors propose, instead to do, is to describe, for Eq. (22) a path integral way to parameterize the evolution of the $\tilde{a} = \alpha_0/\phi^2$ relationship as put into Eq. (23) to talk about different histories for the evolution of the $\phi$ field. This Feynman Kernel treatment for Eq. (23) will then be followed up with a future works interpretation of what Steven Kenneth Kauffmann proposed for a modified path integral treatment to obtain an answer as to the suppression of CPT violation, and a resumption of having a common history for a range of $\phi$ values.

*3.1.1. Putting in the traditional way to obtain a Kernel evolution for the wave functional for Eq.(22)*

We shall recast Eq. (22) in terms of a force equation. Doing so leads to

$$\frac{1}{2}\cdot\left[\frac{k^4}{288\pi^2}\right]\cdot\frac{\partial^2\psi}{\partial\phi^2}-\left[\frac{\left[k_c{\alpha_0}^2\phi^{-4}-\frac{k^2}{3}\cdot\alpha_0^4\lambda_0\phi^{-6}\right]}{\left[1-\frac{24\phi^{-2}}{k^2}\right]}\cdot\alpha_0^2\phi^{-6}\right]\cdot\psi=0 \qquad (24)$$

The effective mass of this problem is, if $\psi$ is an effective 'distance' written as:

$$\frac{k^4}{288\pi^2}=M_{effective} \qquad (25)$$

The emergent time is in terms of the variable $\phi$, whereas the effective force is

$$\text{Force}=\left[\frac{\left[k_c{\alpha_0}^2\phi^{-4}-\frac{k^2}{3}\cdot\alpha_0^4\lambda_0\phi^{-6}\right]}{\left[1-\frac{24\phi^{-2}}{k^2}\right]}\cdot\alpha_0^2\phi^{-6}\right]\cdot\psi \qquad (26)$$

The effective frequency, is then

$$\omega_{effective}=\sqrt{\left[\frac{\left[k_c{\alpha_0}^2\phi^{-4}-\frac{k^2}{3}\cdot\alpha_0^4\lambda_0\phi^{-6}\right]}{\left[1-\frac{24\phi^{-2}}{k^2}\right]\cdot\frac{k^4}{288\pi^2}}\cdot\alpha_0^2\phi^{-6}\right]} \qquad (27)$$

Using the mnemonic of , if the prime denotes $\frac{\partial}{\partial\phi}$, of

$$\frac{1}{2}M_{eff}\psi''-\omega^2_{effective}\psi=0 \qquad (28)$$

If we go to the Feynman semi classical kernel, as given by Grosche and Steiner [13] then start with the decomposition as given in Eq. (29) below. The variable $R[x(t)]$, with $x=\psi, t=\phi$ as defined below will be expanded upon, and we will then write out what this Kernel $K(x'',x',T)$ is, as a semi classical operator to be constructed. Note, that the substitution $\wp\cdot x[t]$ represents the paths taken over all available trajectories. Note that in the representation of the kernel below, the following are used:

$x[T]=x''=\psi[\phi[Max]]=\psi''$, $x[0]=x'=\psi[\phi[Min]]$, $\wp\cdot x[t]=\wp\cdot\psi[\phi]$

Then, the Kernel below is, starting off as:

$$K(x'',x',T) = \int_{x[0]=x'}^{x[T]=x''} \wp \cdot x[t] \exp\left\{ \frac{i}{\hbar} R[x[t]] \right\} \qquad (29)$$

In our problem, involving the suppression of CPT invariance, at the onset of the inflationary era,

$$R[x[t]] = R[\psi[\phi]] = \int_{\phi(\min)}^{\phi(\max)} \left[ \frac{M_{eff}}{2} \cdot \left( \frac{\partial \psi}{\partial \phi} \right)^2 - V[\psi] \right] \cdot d\phi \qquad (30)$$

Note that in this case, the potential term would then be equivalent to a term roughly proportional to Eq. (26) above, squared, times $\psi$ .

For the CPT theorem to no longer apply, the authors assert that the range of integration of Eq. (30) would be almost closed, so we could use a simple Riemann integral summation, with few terms to approximate Eq. (30). We are examining this, and claim it has merit.

Still another is to investigate what can be done by using a different path integral approach to the problem of CPT violation and its suppression at the start of the universe. As implied by Kauffman's treatment of a variant of the path integral approach.[10] and his subsequent research work, he obtains results which lead him to conclude that

*The negative-energy "free particles" of entrenched relativistic quantum theory are well-known features of the Klein-Gordon and Dirac equations, which are shown to have many other unphysical features as well. The correspondence principle for relativistic particles is incompatible with these two equations, produces no unphysical features and implies only positive energies for free particles, which eliminates the very basis of the entrenched notion of antiparticles, as well as of the CPT theorem.*

The authors state, that if the CPT theorem is eliminated, that this removes Vaccaro's [1] entire premise; i.e. it has to be re done. Also that Eq. (30) above may be approximated by a simple Riemannian summation, may also make the CPT theorem not applicable in early universe conditions. Both suppositions are being investigated. We now then refer to what entropy and an arrow of time configuration of time flow would be right after the big bang.

**4.0 Linking a treatment of the arrow of time , after a Planck time, and Planck distance from the "start" of the universe. Set up to have a comparison with initial entropy.**

T. Padmanabhan at DICE 2010 introduced the theme of Post Planckian physics evolution of this document [15]. I.e. to reverse engineer GR emergent structure into initial component space time "atoms", to permit a "Gibbs" style treatment of the thermodynamics of space time physics [16, 17, 18] . We are using his idea, in part, as a way to understand 'atoms" of space time as a component of entropy past the start point. This section summarizes the post Planck time and Planck length in evolution state of entropy, taking into consideration that entropy is part of the arrow of time problem. We hope to eventually match this discussion of entropy, and a resultant arrow of time.

This part of the paper after a Planck time, and Planck length in evolution from the initial start of inflation answers the question as to what would be optimal conditions for initial entropy production initially . To begin this inquiry we start with examining candidates for the initial configuration of the normalized energy density. The normalized energy density of gravitational waves, as given by Maggiore [19] is

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{\nu=0}^{\nu=\infty} d(\log \nu) \cdot \Omega_{gw}(\nu) \Rightarrow h_0^2 \Omega_{gw}(\nu) \cong 3.6 \cdot \left[\frac{n_\nu}{10^{37}}\right] \cdot \left(\frac{\nu}{1kHz}\right)^4 \quad (31)$$

where $n_\nu$ is a frequency-based count of gravitons per unit cell of phase space. Eq. (31) leads to, as given to Fig. 1. candidates as to early universe models to be investigated experimentally. The author, Beckwith, wishes to determine inputs into $n_\nu$ above, in terms of frequency, and also initial temperature. What is in the brackets of the exponential is a way of counting the number of space time $e^+ e^-$ charges nucleated in a space time volume $V \cdot t$. Beckwith used a very similar constructions with Density wave physics [20] and also has extended this idea to use in graviton physics, in a kink- anti kink construction [21] The idea, after one knows how to obtain a counting algorithm, with additional refinements will be to use what can be understood by the above analogy, assuming a minimal mass $m \cong m_{eff}$ for $m_{eff}$ , as Beckwith brought up [21] as will be discussed as inputs into the models represented by Fig 1 below

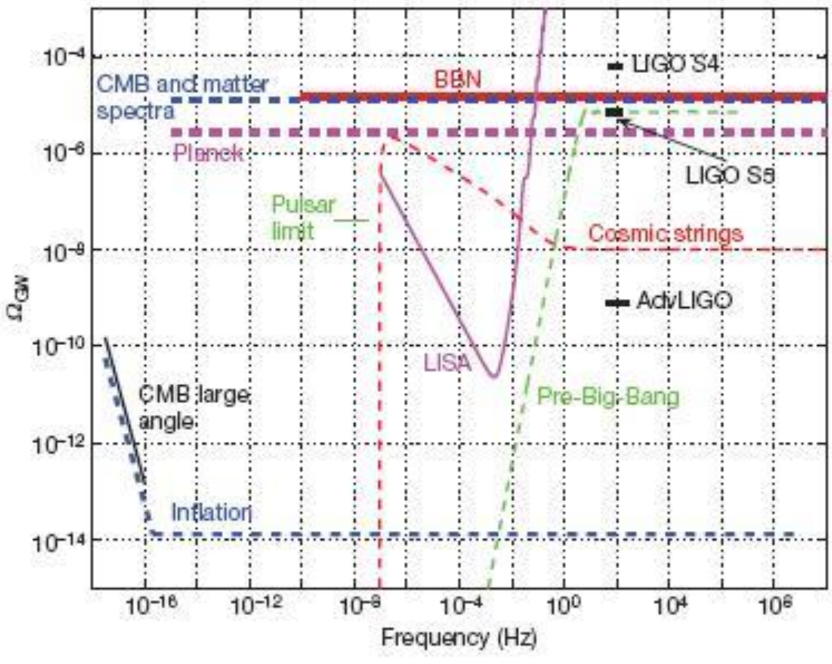


**Figure 1**. From. Abbott et al. **[18]** shows the relation between $\Omega_g$ and frequency.

What one of us, A.W.B., intends to do is to use the emergent structure set by the cosmological Schwinger method, as outlined by J. Martin [23] to obtain the number of emergent particles in initial phase space counting, and tie in the phase space numerical counting , and entropy with different candidates for the inflaton potential. Beckwith, via dimensional analysis [21] made the following identifications. I.e. Force = qE = $[T\Delta S / dist] = \hbar \cdot [[\omega_{Final} - \omega_{initial}] / dist]$

which in terms of inflaton physics leads to (if $V' = dV/d\phi$, where V is an inflaton potential, and dist = distance of Planck length, or more)

$$[T\Delta S / dist] = [\hbar/dist]\cdot\left[2k^2 - \frac{1}{\eta^2}\left[M_{Planck}^2\cdot\left[\frac{6}{16\pi}-\frac{3}{4\pi}\right]\cdot\left[\frac{V'}{V}\right]^2 - \frac{3}{4\pi}\cdot M_{Planck}^2\cdot\left[\frac{V''}{V}\right]\right]\right]^{1/2} \quad (32)$$

Eq. (32) divided by a charge, q, gives a relic electric field. While the existence of a charge, q, as an independent entity, at the onset of inflation is open to question, what the author, Beckwith [20] is paying attention to is using inputs into free energy, as can be specified by the following: If one identified the evolution of temperature, with energy, and made the following identification, $\breve{T}$ for time, and $\Omega_0$ for a special frequency range, as inputs into [21]

$$E_{thermal} \approx \frac{1}{2}k_B T_{temperature} \propto \left[\Omega_0 \breve{T}\right] \sim \tilde{\beta} \quad (33)$$

Here, the thermal energy, as given by temperature ranging as $T_{temperature}\varepsilon(0^+, 10^{19}GeV)$, up to the Planck interval of time $t_P \sim 10^{-44}\sec$, so that one is looking at $\tilde{\beta} \approx |F| \equiv \frac{5}{2}k_B T\cdot\overline{N}$, as a free energy. For this parameter, if $\ddot{N}$, as an initial entropy , arrow of time configuration, were fixed, then the change in temperature would lead to change in 'free energy' , so that work, is here, change in energy, and dE = TdS – p dV. In basic physics, this would lead to force being work (change in energy) divided by distance. Then, $\Delta\tilde{\beta} \cong (5k_B \Delta T_{temp}/2)\cdot\overline{N} \sim$ Force times $dist$ =distance. We assume that there would be an initial fixed entropy arising, with $\overline{N}$ a nucleated structure arising in a short time interval as a temperature $T_{temperature}\varepsilon(0^+, 10^{19}GeV)$ arrives. So [21], leads to a force value

$$\frac{\Delta\tilde{\beta}}{dist} \cong (5k_B \Delta T_{temp}/2)\cdot\frac{\overline{N}}{dist} \sim qE_{net-electric-field} \sim [T\Delta S/dist] \quad (34)$$

Next, will be the identification of inflation physics, as dimensionally argued in Eq. (34) to choices in the inflaton potential. To see that, consider the following, as given by Eq. (35) below [21]. Candidates as to the inflaton potential would be in powers of the inflaton, i.e. in terms of $\phi^N$, with perhaps N=2 an admissible candidate (chaotic inflation). For N = 2, one gets [21]

$$[\Delta S] = [\hbar/T]\cdot\left[2k^2 - \frac{1}{\eta^2}\left[M_{Planck}^2\cdot\left[\left[\frac{6}{4\pi}-\frac{12}{4\pi}\right]\cdot\left[\frac{1}{\phi}\right]^2 - \frac{6}{4\pi}\cdot\left[\frac{1}{\phi^2}\right]\right]\right]\right]^{1/2} \sim n_{Particle-Count} \quad (35)$$

Making a comparison with a weighted average of $\Delta S \sim 10^5$ and varying values of a scalar field of $0 < \phi < 2\pi$, when one has $\eta\varepsilon(-10^{-44}\sec, 0)$, and $0 < T \le T_{Planck} \sim 10^{19}GeV$ leads to a rich phenomenology, where one could see variations as of a time parameter, and how the wave length, k , evolved, especially if $\Delta S \sim 10^5$ remains constant . I.e. why did the value of wavelength, k, vary so much, in a short period of time, i.e. less than Planck time? As mentioned before in [21] this question asks how the initial wave vector, k, forms and to what degree variation in the inflaton $0 < \phi < 2\pi$ occurs. I.e. it gives a way to vary the inflaton, and understand relic entropy generation.

### 4.1 First principle evaluation of initial bits of information, as opposed to numerical counting, and entropy

A consequence of Verlinde's [24] generalization of entropy as also discussed by Beckwith [21] , and the number of 'bits' yields the following consideration, which will be put here for startling effect. Namely, if a net acceleration is such that $a_{accel} = 2\pi k_B cT/\hbar$ as mentioned by Verlinde [24] as an Unruh result, and that the number of 'bits' is

$$n_{Bit} = \frac{\Delta S}{\Delta x} \cdot \frac{c^2}{\pi \cdot k_B^2 T} \approx \frac{3 \cdot (1.66)^2 g^*}{\left[\Delta x \cong l_p\right]} \cdot \frac{c^2 \cdot T^2}{\pi \cdot k_B^2} \tag{36}$$

Eq. (6) has a $T^2$ temperature dependence for information bits , as opposed to [21]

$$S \sim 3 \cdot \left[1.66 \cdot \sqrt{\tilde{g}_*}\right]^2 T^3 \sim n_f \tag{37}$$

Should the $\Delta x \cong l_p$ order of magnitude minimum grid size hold, then when T ~ $10^{19}$ GeV [21]

$$n_{Bit} \approx \frac{3 \cdot (1.66)^2 g^*}{\left[\Delta x \cong l_p\right]} \cdot \frac{c^2 \cdot T^2}{\pi \cdot k_B^2} \sim 3 \cdot \left[1.66 \cdot \sqrt{\tilde{g}_*}\right]^2 T^3 \tag{38}$$

The situation for which one has [17], $\Delta x \cong l^{1/3} l_{Planck}^{2/3}$ with $l \sim l_{Planck}$ corresponds to $n_{Bit} \propto T^3$ whereas $n_{Bit} \propto T^2$ if $\Delta x \cong l^{1/3} l_{Planck}^{2/3} >> l_{Planck}$. This issue of either $n_{Bit} \propto T^3$ or $n_{Bit} \propto T^2$ will be analyzed in future publications. If the bits of information can be related to a numerical count, the next step will be to make a linkage between thermal heat flux, due to the initial start of inflation, with degrees of freedom rising from a point, almost zero to over 1000 in a Planck time interval. Furthermore, we also have evidence of at least a spontaneous creation of 'particles' which may reflect upon the arrow of time well after the Planck time, and Planck spatial evolution after the source of the big bang. As Beckwith wrote up [17], including in additional energy due to an increase of $\tilde{\beta}$ due to increasing temperature $T$ would have striking similarities to the following Observe the following argument as given by Mukhanov and Winitzki [25], as to additional particles being 'created' due to what is an infusion of energy in an oscillator, obeying the following equations of motion [16, 25]

$$\ddot{q}(t) + \omega_0^2 q(t) = 0, \text{ for } t < 0 \text{ and } t > \breve{T}; \qquad \ddot{q}(t) - \Omega_0^2 q(t) = 0, \text{ for } 0 < t < \breve{T} \tag{39}$$

given $\Omega_0 \breve{T} >> 1$ , with a starting solution of $q(t) \equiv q_1 \sin(\omega_0 t)$ if t <0, Mukhanov state that for [21, 25] $t > \breve{T}$;

$$q_2 \approx \frac{1}{2}\sqrt{1 + \frac{\omega_0^2}{\Omega_0^2}} \cdot \exp\left[\Omega_0 \breve{T}\right] \tag{40}$$

The Mukhanov et al argument [21, 25] leads to an exercise which Mukhanov claims is solution to the exercise yields an increase in number count, as can be given by setting the oscillator in the ground state with $q_1 = \omega_0^{-1/2}$, with the number of particles linked to amplitude by $\breve{n} = [1/2] \cdot (q_0^2 \omega_0 - 1)$, leading to [21, 25]

$$\breve{n} = [1/2] \cdot \left(1 + \left[\omega_0^2 / \Omega_0^2\right]\right) \cdot \sinh^2\left[\Omega_0 \breve{T}\right] \tag{41}$$

I.e. for non zero $\left\lfloor \Omega_0 \breve{T} \right\rfloor$, Eq (40) leads to exponential expansion of the numerical state. For sufficiently large $\left\lfloor \Omega_0 \breve{T} \right\rfloor$, Eq. (37) and Eq. (38) are equivalent to placing of energy into a system, leading to vacuum nucleation. A further step in this direction is given by Mukhanov on page 82 of his book leading to a Bogoliubov particle number density of becoming exponentially large [21, 25]

$$\breve{\breve{n}} \sim \cdot \sinh^2 [m_0 \eta_1] \qquad (42)$$

The *equality*, i.e. not proportion like in Mukhanov et al analysis, have been established also by Glinka and Pervushin [26] in their approach to unification of gravity and particles based hybrid Hamiltonian formulation mixing both the Dirac and the Arnowitt-Deser-Misner methods. This approach seems to be sufficient to explain tremendous number of observational astrophysical data, and possibly will be analyzing and developing by the authors. Eq. (40) to Eq. (41) are , for sufficiently large $\left\lfloor \Omega_0 \breve{T} \right\rfloor$ a way to quantify what happens if initial thermal energy are placed in a harmonic system, leading to vacuum particle ' creation' Eq. (42) is the formal Bogoliubov coefficient limit of particle creation. Note that $\ddot{q}(t) - \Omega_0^2 q(t) = 0,$ for $0 < t < \breve{T}$ corresponds to a thermal flux of energy into a time interval 0 to T. Then $[T \Delta S / dist] = \hbar \cdot [[\omega_{Final} - \omega_{initial}] / dist]$ to obey, in the limits that k → 0

### *4.1.1 Effective "electric field" as proportional to temperature, to the first power. Its interpretation. This from inputs into the given frequencies as stated in Eq. (42)*

$$n_f = [1/4] \cdot \left[ \sqrt{\frac{v(a_{initial})}{v(a)}} - \sqrt{\frac{v(a)}{v(a_{final})}} \right] \qquad (43)$$

Eq. (43) above as given by Glinka as one of diverse fruitful results of his approach to quantum cosmology and quantum gravity [27, 28], could be investigated as being part of the bridge between phenomenology of what inflaton potentials should be used, i.e. there exist a number of permissible inputs into the inflaton potential which should be looked at . I.e. the values of the inflaton field which are acceptable, i.e. for $\phi \varepsilon (0, 2\pi)$. Furthermore, what Glinka put up above, may be a way to link the flow of the arrow of time, from initial configurations as to the evolution of entropy as given by the Schwinger cosmological effect. We intend to look at this in future publications. Furthermore, it may be a way to obtain a way to obtain a bridge between time flow at the onset of inflation, as may only be understood by a falsification of the CPT theorem, for reasons as given by Kauffmann, and later time/ entropy/ arrow of time dynamics.

## 5. Conclusion: Analyzing the problem of graviton "counting", atoms of space time, GW, and Khrennikov's signal theory treatment of QM with regards to the representation of gravitons and inflaton fields

According to Khrennikov, [29] the classical and quantum probabilities can be delineated via, in CM by

$$\langle f \rangle_\mu = \int_M f(\phi) \cdot d\mu(\phi) \qquad (44)$$

Here, M is the state space, and f is a functional in classical probability to be estimated, while $\mu$ is the measure. Eq (43) should be contrasted with a QM presentation of the probability to be estimated with

$$\left\langle \hat{A} \right\rangle_{\rho} = Tr\rho \cdot \hat{A} \qquad (45)$$

Khrennikov's main claim [29] is that randomness is the same in classical mechanics as in QM, and furthermore delineates a way to make a linkage between Eq. (44) and Eq. (45) via use of, if t is the time scale of fluctuation, and T is the time of measurement that one can write, to order $\vartheta(t/T)$

$$\int f_A(\phi) d\mu(\phi) = \left\langle \hat{A} \right\rangle_{\rho} = Tr\rho \cdot \hat{A} + \vartheta(t/T) \qquad (46)$$

How that is built up will enable a fuller creation of "space time atoms" as stated by T. Padmanabhan [15], [18] for a thermodynamic treatment of the evolution of the inflaton, in terms of gravitons and GW physics as mentioned by the author [21]. The hope is to establish the details of the embedding of QM as part of a larger non linear theory. Then, afterwards, would be matching that entropy value, with initial entropy, in Planckian space time, as was unsuccessfully done by Vaccaro [1]

Should this be done, then the issue of the Hamiltonian constraint, as to obtaining a proper fit to the Wheeler de Witt equation, may be understandable, and that a program of analysis which avoids the mistake Vaccaro [1] can be utilized. But the first step toward understanding the actual role of the Hamiltonian constraint equation, as given by Kiefer's abstract rendering of [5] the inter relationships given in Eq. (4) above may need suitable application of Glinka's [27, 28] numerical count of entropy, as a function of frequency, in local phase space at the start of the big bang. This will mean hard work ahead, as well as understanding the significance of Eq. (46) which the authors view as part of the process of understanding how to obtain a suitable Hamiltonian constraint to the Wheeler-DeWitt equation.

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