Article

Magnetized Quark Attached to String Cloud in Spherical Symmetric Space-Time Admitting Conformal Motion

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Abstract

In this paper, we have examined string cloud with magnetized quark matter in spherical symmetric space- time admitting one-parameter group of conformal motions. For this purpose, the exact solutions of Einstein field equations have been obtained for spherical symmetric space-time in isotropic form via conformal motions. Also, we have discussed the properties of the obtained solutions.

Keywords: Spherical symmetric space-time, string cloud, magnetized quark, conformal motions.

1. Introduction

In general relativity, it is a subject of long- standing interest to look for the exact solutions of Einstein's field equations. To know the exact physical situation at early stage of the formation of our universe is still challenging subject of study. At the very early stages of evolution of the universe, it is generally assumed that during phase transition (as the universe passes through its critical temperature) the symmetry of the universe is broken spontaneously. It can give rise to topologically stable defects such as strings, domain walls and monopoles.

G. P. Singh & T. Singh [1] presented a new class of string cosmological models with and without magnetic field in the context of a space-time with G_3 symmetry. For this purpose the standard energy-momentum tensor for cosmic strings is modified by including an additional term for magnetic field. Bali and Upadhaya [2] investigated L. R. S. Bianchi type I string dust magnetized cosmological models with and without magnetic field following the techniques used by Letelier and Stachel. Sahoo and Mishra [3] studied plane symmetric space- time with quark matter attached to the string cloud and domain wall in the context of Rosen's biometric theory and observed that, in this theory, string cloud and domain walls do not exist and biometric relativity does not help to describe the early era of the universe. Sahoo and Mishra [4] also studied axially symmetric space- time with strange quark matter attached to the string cloud in Rosen's biometric theory and shown that there is no contribution from strange quark matter and hence vacuum model is presented. Deo [5] studied spherically symmetric Kantowski-Sachs space- time in the context of Rosen's biometric theory with the source matter cosmic strings and domain walls and observed that the space- time does not accommodate the cosmic strings as well as domain walls and it is observed that the resulting space- time represents Robertson-Walker flat space- time which expands according to the signature of the parameter uniformly along the space directions with time. Yilmaz [6] obtained Kaluza- Klien cosmological solutions for quark matter coupled to the string cloud and domain wall in the context of general relativity by using anisotropy feature of the universe. Rao and Neelima [7] studied the anisotropic Bianchi type-VI space-time with strange quark matter attached to string cloud in Barber's second self creation theory and general relativity and noticed that the presence of scalar field does not affect the geometry of the space- time but changes the matter distribution.

General relativity provides a rich arena to use symmetries in order to understand the natural relation between geometry and matter furnished by Einstein equations. Symmetries of geometrical/ Physical relevant quantities of this theory are known as collineations and the most useful collineation is conformal killing vector defined by

$$\pounds_{\xi} g_{ij} = \xi_{i;j} + \xi_{j;i} = \psi g_{ij} , \ \ \psi = \psi(x^i),$$

where $\pounds_{\xi}$ signifies the Lie derivative along ξ^i and $\psi = \psi(x^i)$ is the conformal factor. In particular, ξ is a special conformal killing vector, if $\psi_{;ij} = 0$ and $\psi_{,i} \neq 0$. Here (;) and (,) denote covariant and ordinary derivatives respectively (Aktas and Yilmaz [8]).

Conformal killing vectors provide a deeper insight into the space-time geometry and facilitate generation of exact solutions to the field equations. Aktas and Yilmaz [8] solved Einstein's field equations for spherical symmetric space- time via conformal motions and examined magnetized quark and strange quark matter in spherical symmetric space- time admitting one- parameter group of conformal motions. Kandalkar, Wasnik and Gawande [9] investigated spherically symmetric string cosmological model with magnetic field admitting conformal motion. Shobhane and Deo [10] examined the wet dark fluid matter in the spherical symmetric space-time admitting one parameter group of conformal motion.Shobhane and Deo [11] also studied string cloud with quark matter in plane symmetric space- time admitting conformal motion. Sharif [12] classified the static plane symmetric space- time according to their matter collineations.

The paper is outlined as follows: In Sec.2, we have obtained Einstein field equations for string cloud with magnetized quark in spherical symmetric space- time. In Sec.3, the solutions of the field equations are obtained for string cloud with magnetized quark in spherical symmetric space- time admitting one parameter group of conformal motions and some particular cases are discussed. In Sec.4, concluding remarks are given.

2. Field Equations

The most general static spherically symmetric line element in isotropic form (Banerjee & Santos [13] and Hajj-Boutros & Sfeila [14]) is given by

$$ds^2 = e^{\nu(r)} dt^2 - e^{\omega(r)} (dr^2 + r^2 d\Omega^2), \tag{1}$$

where

$$d\Omega^2 = d\theta^2 + sin^2\theta\ d\phi^2, x^{1,2,3,4} = r, \theta, \phi, t.$$

The energy momentum tensor for string cloud in the presence of magnetic field may be defined as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j + E_{ij}, \tag{2}$$

where E_{ij} is the electromagnetic field given by (Aktas & Yilmaz[8])

$$E_{ij} = \left(u_i u_j + \frac{1}{2} g_{ij}\right) h^2 - h_i h_j, \tag{3}$$

$u^i = \delta_4^i e^{-\nu/2}$ is the four-velocity, x^i is unit space-like vector in the direction of string, h^i is the magnetic field in radial direction with

$$u^i u_i = 1 = -x^i x_i,\ u^i x_i = 0,\ h_i u^i = 0,\ h^2 = h^i h_i,$$

ρ is the rest energy density for string cloud with particles attached to them and λ is string tension density satisfying the relation

$$\rho = \rho_p + \lambda\ or\ \rho_p = \rho - \lambda\ . \tag{4}$$

Here, ρ_p is the particle energy density. The string is free to vibrate and different vibrational modes are seen as different masses or spins. Therefore, we will consider quarks instead of particle in the string cloud. In this case, we get

$$\rho = \rho_q + B_c + \lambda\ or\ \rho_q + B_c = \rho - \lambda\ , \tag{5}$$

where ρ_q is the quark density and B_c is the bag constant.

Einstein field equations can be expressed as

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi G T_{ij}. \tag{6}$$

Here, we shall use geometrized units so that $8\pi G = c = 1$.

Then using (1) and (2), we obtain

$$e^{-\omega}\left(\frac{\omega' + \upsilon'}{r} + \frac{\omega'\upsilon'}{2} + \frac{\omega'^2}{4}\right) = -\lambda + \frac{h^2}{2}, \tag{7}$$

$$e^{-\omega}\left(\frac{\omega''+v''}{2}+\frac{\omega'+v'}{2r}+\frac{v'^2}{4}\right)=\frac{h^2}{2}, \tag{8}$$

and

$$e^{-\omega}\left(\omega''+\frac{2\omega'}{r}+\frac{\omega'^2}{4}\right)=-\rho-\frac{h^2}{2}, \tag{9}$$

where primes denote differentiation w.r.t. r.

Using (7) and (8), we get

$$\lambda=e^{-\omega}\left[\left(\frac{\omega''+v''}{2}\right)-\left(\frac{\omega'+v'}{2r}\right)+\left(\frac{v'^2-\omega'^2}{4}\right)-\frac{\omega'v'}{2}\right] \tag{10}$$

and using (8) and (9), we get

$$\rho=-e^{-\omega}\left[\left(\frac{3\omega''+v''}{2}\right)+\left(\frac{5\omega'+v'}{2r}\right)+\left(\frac{\omega'^2+v'^2}{4}\right)\right] \tag{11}$$

3. Solutions of Field Equations

Now we shall assume that space- time admits a one- parameter group of conformal motions (Aktas and Yilmaz[8]) i.e.

$$£_\xi\, g_{ik}=\xi_{i;k}+\xi_{k;i}=\psi g_{ik}, \tag{12}$$

where ψ is an arbitrary function of r.

Using (1) and (12) by virtue of spherical symmetry, we get the following expressions:

$$\xi^1 v'=\psi, \tag{13}$$

$$\xi^1=\frac{\psi r}{r\omega'+2}, \tag{14}$$

$$\omega'\xi^1+2\,\xi^1_{,1}=\psi, \tag{15}$$

and

$$\xi^2=\xi^3=0,\ \xi^4=\alpha=constant, \tag{16}$$

where a comma denotes partial derivatives.

Using equations (13) and (14), we obtain

$$\omega'=v'-\frac{2}{r}\ . \tag{17}$$

Using equations (14) and (15) , we get

$$\xi^1 = kr \ , \tag{18}$$

where k *(> 0)* is an arbitrary constant. Therefore using equations (13), (17) and (18), we get

$$v' = \frac{\psi}{kr} \ , \tag{19}$$

and

$$\omega' = \frac{\psi}{kr} - \frac{2}{r} \ . \tag{20}$$

Further, on integration equation (17) gives

$$e^{\omega} = \frac{e^{v}}{c^2 r^2}, \tag{21}$$

where c *(> 0)* is an arbitrary constant and $r > 0$.

Particular cases

Case 1: Let

$$\psi = \sqrt{2}\, k \left[\frac{1 + (c_1 r)^{\sqrt{2}}}{1 - (c_1 r)^{\sqrt{2}}}\right], \tag{22}$$

where c_1 (> 0) is an arbitrary constant and $r \neq 1/c_1$.

Then using equation (19) - (21), we get

$$e^{v} = \frac{(c_2 r)^{\sqrt{2}}}{\left[1 - (c_1 r)^{\sqrt{2}}\right]^2} \tag{23}$$

and

$$e^{\omega} = \frac{(c_2 r)^{\sqrt{2}}}{c^2 r^2 \left[1 - (c_1 r)^{\sqrt{2}}\right]^2}\, , \tag{24}$$

where c_2 *(> 0)* is an arbitrary constant.

Using equations (4), (10) and (11), we get

$$\lambda = 0 \tag{25}$$

and

$$\rho = \rho_p = \rho_q + B_c = -\frac{12 c^2 (c_1)^{\sqrt{2}}}{(c_2)^{\sqrt{2}}} < 0. \tag{26}$$

Using equation (8), we get

$$h^2 = \frac{c^2\left[1 + 10(c_1 r)^{\sqrt{2}} + (c_1 r)^{2\sqrt{2}}\right]}{(c_2 r)^{\sqrt{2}}}. \quad (27)$$

Case 2: Let

$$\psi = \sqrt{2}k\left[\frac{(c_3 r)^{1/\sqrt{2}} - 1}{(c_3 r)^{1/\sqrt{2}} + 1}\right], \quad (28)$$

where $c_3\ (> 0)$ is an arbitrary constant.

Then using equation (19) - (21), we get

$$e^{\nu} = \frac{\left[(c_3 r)^{1/\sqrt{2}} + 1\right]^4}{(c_4 r)^{\sqrt{2}}} \quad (29)$$

and

$$e^{\omega} = \frac{\left[(c_3 r)^{1/\sqrt{2}} + 1\right]^4}{c^2 r^2 (c_4 r)^{\sqrt{2}}}, \quad (30)$$

where $c_4\ (> 0)$ is an arbitrary constant.

Using equations (4), (10) and (11), we get

$$\lambda = \frac{6c^2 (c_3 r)^{1/\sqrt{2}}\,(c_4\, r)^{\sqrt{2}}}{\left[(c_3 r)^{1/\sqrt{2}} + 1\right]^6}, \quad (31)$$

$$\rho = 0 \quad (32)$$

and

$$\rho_p = -\lambda = \rho_q + B_c = -\frac{6c^2 (c_3 r)^{1/\sqrt{2}}\,(c_4 r)^{\sqrt{2}}}{\left[(c_3 r)^{1/\sqrt{2}} + 1\right]^6} < 0. \quad (33)$$

Using equation (8), we get

$$h^2 = \frac{c^2\,(c_4 r)^{\sqrt{2}}}{\left[(c_3 r)^{1/\sqrt{2}} + 1\right]^4}. \quad (34)$$

Case 3: Let

$$\psi = c_5 = constant. \quad (35)$$

Then using equation (19) - (21), we get

$$e^{\nu} = (c_6 r)^{\beta} \quad (36)$$

and

$$e^{\omega} = \frac{(c_6 r)^{\beta}}{c^2 r^2} \quad (37)$$

where $\beta = c_5/k$ and $c_6\ (>0)$ are arbitrary constants.

Using equations (4), (10) and (11), we get

$$\lambda = \rho = \frac{c^2(2-\beta^2)}{2(c_6 r)^\beta} \tag{38}$$

and

$$\rho_p = \rho_q + B_c = 0. \tag{39}$$

Using equation (8), we get

$$h^2 = \frac{c^2\beta^2}{2(c_6 r)^\beta}. \tag{40}$$

Case 4: Let

$$\psi = \frac{4k}{ln(c_7 r)}, \tag{41}$$

where $c_7\ (>0)$ is an arbitrary constant and $r > 0\ \ (\neq 1/c_7)$.

Then using equation (19) - (21), we get

$$e^\upsilon = (c_8)^4\,[ln(c_7 r)]^4 \tag{42}$$

and

$$e^\omega = \frac{(c_8)^4\,[ln(c_7 r)]^4}{c^2 r^2}, \tag{43}$$

where $c_8\ (>0)$ is an arbitrary constant.

Using equations (4), (10) and (11), we get

$$\lambda = \frac{c^2}{(c_8)^4[ln(c_7 r)]^4}\left\{1 - \frac{12}{[ln(c_7 r)]^2}\right\}, \tag{44}$$

$$\rho = \rho_p + \lambda = \frac{c^2}{(c_8)^4[ln(c_7 r)]^4}, \tag{45}$$

$$\rho_p = \rho - \lambda = \frac{12}{[ln(c_7 r)]^2} \tag{46}$$

and

$$\rho_q = \frac{12}{[ln(c_7 r)]^2} - B_c. \tag{47}$$

Using equation (8), we get

$$h^2 = 0. \tag{48}$$

Using (1), the space-time geometry i.e. the line element in each of the above cases is given by

$$ds^2 = e^{\upsilon(r)}\left[dt^2 + \frac{1}{c^2r^2}(dr^2 + r^2 d\Omega^2)\right], \quad (49)$$

where e^{υ} are respectively given by (23), (29), (36) and (42).

4. Conclusion

In this paper, we have studied string cloud with magnetized quark in spherical symmetric space-time admitting one- parameter group of conformal motions. Following properties are observed:
In case (1), our cosmological model has singularity at $r = 1/c_1$. e^{υ} and e^{ϖ} are positive and continuous, except at $r = 1/c_1$. Further, the energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are not satisfied.
In case (2), e^{υ} and e^{ϖ} are positive, continuous and non-singular, but the energy condition $\rho_p \geq 0$ is not satisfied.

In case (3), e^{υ} and e^{ϖ} are positive, continuous and non- singular. Further, if we set $-\sqrt{2} < \beta < \sqrt{2}$, the energy conditions are satisfied. In this case, matter disappears and we get geometric string solution i.e. $\rho_p = 0$. We observed here that for $0 < \beta < \sqrt{2}$, $\lambda, \rho, h^2 \to \infty$ as $r \to 0$ and for $-\sqrt{2} < \beta < 0$, $\lambda, \rho, h^2 \to 0$ as $r \to 0$ and at $\beta = 0$, $\lambda, \rho = c^2$ and $h^2 = 0$. The quark pressure and magnetic pressure are respectively given by (Aktas & Yilmaz [8])

$$p_q = \frac{\rho_q}{3} = \frac{-B_c}{3} \text{ and } p_{mag} = \frac{h^2}{2} = \frac{c^2\beta^2}{4(c_6 r)^{\beta}}.$$

In case (4), e^{υ} and e^{ϖ} are positive, continuous and non- singular. Further, the energy conditions are satisfied. The quark pressure is given by

$$p_q = \frac{4}{[ln\,(c_7 r)]^2} - \frac{B_c}{3}.$$

For $r < e^{2\sqrt{3}}/c_7$, $\lambda < 0$, as pointed by Letelier [15], [16], the string tension density λ may be positive or negative. Further, in this case, we obtained spherically symmetric cosmological solution for string cloud with quark matter and without electromagnetic field.

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