Article

Two-Fluid Cosmological Models in Scalar-Tensor Theory of Gravitation

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Abstract

In this paper, we have investigated two-fluid Kantowski-Sachs cosmological models with matter and radiating source in the scalar-tensor theory of gravitation proposed by Saez and Ballester. Two-fluid model in Saez and Ballester theory of gravitation, one fluid represents the matter content of the universe and another fluid is the CMB radiation. To get the deterministic model, we have assumed that a relation between metric potentials $A = B^n$, where A and B are metric potentials and n is constant. We have also investigated the behaviors of some physical parameters.

Keywords: Two-fluid, Kantowski-Sachs, Saez, Ballester, gravitation.

1. Introduction

The cosmic evolution based on two-fluid big-bang model appears better than two sequences of a single fluid model. In these models, we have assumed that both the fluids are present throughout the cosmic evolution with one fluid dominating the other. Such type of models can be used to describe, the transition between a radiation dominated phase to a matter dominated phase as the universe evolves. Many researchers have formulated several aspects of two fluid cosmological models. Cosmological models with two fluids studied by McIntosh [1]. Coley and Dunn [2] have evaluated Bianchi type VI_0 model with two fluid sources. Pant and Oli [3] have investigated two-fluid Bianchi type II, cosmological models. Oli [4] has constructed Bianchi type-I two fluid cosmological models with a variable G and $\wedge$.

Qualitative analysis of two fluids FRW cosmological models has formulated by Verma [5]. Sandin [6] has examined Tilted two fluid Bianchi type-I models. Adhav et al. [7] have derived two fluid cosmological models in Bianchi type V space-time. Bianchi type IX two fluids cosmological models in General Relativity are developed by Pawar and Dagwal [8]. Kaluza-Klein mesonic cosmological model with the two-fluid source is investigated studied by Venkateswarlu [9]. Two-fluid cosmological model of Bianchi type-V with negative constant

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deceleration parameter studied by Singh et al.[10]. Samanta [11] have evaluated two fluid cosmological models in Kaluza-Klein space-time. Two fluid scenarios for dark energy model in a scalar-tensor theory of gravitation obtained by Reddy and Kumar [12]. Two fluids tilted cosmological model in General Relativity constructed by Pawar and Dagwal [13]. Axially Bianchi type-I Mesonic cosmological models with two fluid sources in Lyra Geometry presented by Pawar et al. [14]. Pawar and Dagwal [15, 16] have investigated Conformally flat tilted cosmological models and tilted Kantowski-Sachs cosmological models with disordered radiation in the scalar-tensor theory of gravitation proposed by Saez and Ballester. Recently Two fluid Axially Symmetric Cosmological Models in *f*(R, T) Theory of Gravitation, tilted Cosmological Models in *f*(R, T) Theory of Gravitation and tilted Kasner-Type Cosmological Models in Brans-Dicke Theory of Gravity formulated by Pawar and Dagwal [17-19].

2. Field Equation

We consider metric in the form –

$$ds^2 = -dt^2 + A^2\,dr^2 + B^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right), \tag{1}$$

where A and B are functions of t alone.

The Einstein's field equation is given by Saez and Ballester for the combined scalar and tensor field are

$$G_i^j - wV^h\left(V_{,i}V^{,j} - \frac{1}{2}g_i^jV_{,a}V^{,a}\right) = T_i^j, \tag{2}$$

and the scalar field satisfies the equation

$$2V^hV_{;j}^{\ i} + hV^{h-1}V_{,a}V^{,a} = 0\ , \tag{3}$$

where $G_i^j = R_i^j - \frac{1}{2}g_i^jR$ is Einstein tensor, h an arbitrary exponent and w a dimensionless coupling constant.

The energy-momentum tensor for the two fluid sources is given by

$$T_i^j = \overset{(m)}{T_i^j} + \overset{(r)}{T_i^j}\ . \tag{4}$$

Where $\overset{(m)}{T_i^j}$ is the energy-momentum tensor for matter field with the energy density ρ_m and pressure p_m and $\overset{(r)}{T_i^j}$ is the energy-momentum tensor for radiation field having energy density ρ_r and pressure $p_r = \left(1/3\right)\rho_r$, which are respectively given by

$$T_{ij}^{(m)} = \left(p_m + \rho_m\right)u_i^{(m)}u_j^{(m)} + p_m g_{ij}, \tag{5}$$

$$T_{ij}^{(r)} = \frac{4}{3}\rho_r u_i^{(r)} u_j^{(r)} + \frac{1}{3}\rho_r g_{ij}. \tag{6}$$

The four velocity vectors are given by $u_i^{(m)} = \left(0,0,0,1\right)$ and $u_i^{(r)} = \left(0,0,0,1\right)$ with
$g^{ij} u_i^{(m)}u_j^{(m)} = -1$ *and* $g^{ij} u_i^{(r)}u_j^{(r)} = -1$.

The field equation (2) for metric (1) reduce to

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} + \frac{1}{B^2} - \frac{wV^h}{2}V_4^2 = p_m + \frac{\rho_r}{3}, \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{wV^h}{2}V_4^2 = p_m + \frac{\rho_r}{3}, \tag{8}$$

$$\frac{2A_4B_4}{AB} + \frac{B_4^2}{B^2} + \frac{1}{B^2} + \frac{wV^h}{2}V_4^2 = -\left(\rho_m + \rho_r\right), \tag{9}$$

$$V_{44} + \left(\frac{A_4}{A} + 2\frac{B_4}{B}\right)V_4 + \frac{h}{2}\frac{V_4^2}{V} = 0 . \tag{10}$$

Here the index 4 after a field variable denotes the differentiation with respect to time *t*.

We assume an analytic relation between the metric coefficients as

$$A = B^n \tag{11}$$

where n is constant, and secondly we consider the equation of state, given by

$$p_m = \left(\gamma - 1\right)\rho_m . \tag{12}$$

Equation (7) and (8) leads to

$$B_{44}+\frac{(1+n)B_4^2+1}{B}=0. \tag{13}$$

$$2ff'+2\frac{\left[(1+n)f^2+1\right]}{B}=0 \tag{14}$$

Where

$$B_4=f(B) \quad , \qquad f'=\frac{df}{dB} \tag{15}$$

Integrating we get

$$f^2=(B_4)^2=\frac{c}{B^{2(n+1)}}-\frac{1}{n+1}, \tag{16}$$

where c is integration constant.

Hence the line element (1) reduced to

$$ds^2=-\frac{dB^2}{f^2}+B^{2n}dr^2+B^2\left(d\theta^2+\sin^2\theta d\phi^2\right). \tag{17}$$

Using co-ordinate transformation

$$ds^2=-\left[\frac{c}{T^{2(n+1)}}-\frac{1}{n+1}\right]^{-1}dT^2+T^{2n}dr^2+T^2\left(d\theta^2+\sin^2\theta d\phi^2\right), \tag{18}$$

where B = T .

3. Some Physical and Geometrical Property

Equation (10) , (11) and (16) leads to

$$V=\left[\frac{a_1}{T^{n+1}}\right]^{2/h+2} , \tag{19}$$

Where $a_1=-\frac{c_1}{(n+1)T^{n+1}}$

The energy density ρ_m, pressure p_m for matter field and the energy density ρ_r for radiation field as

$$\rho_m=\frac{1}{(3\gamma-4)}\left[\frac{-6(n+1)}{T^{2(n+2)}}\left(\frac{c}{T^{2(n+1)}}-\frac{1}{n+1}\right)^{-1/2}+\frac{2(2+n)}{T^2}\left(\frac{c}{T^{2(n+1)}}-\frac{1}{n+1}\right)+\frac{4}{T^2}-\frac{wc_1^2}{T^{2(n+2)}}\left(\frac{c_1}{(1+n)T^{(n+1)}}\right)^{2/h+1}\right], \tag{20}$$

$$\rho_m=\frac{(\gamma-1)}{(3\gamma-4)}\left[\frac{-6(n+1)}{T^{2(n+2)}}\left(\frac{c}{T^{2(n+1)}}-\frac{1}{n+1}\right)^{-1/2}+\frac{2(2+n)}{T^2}\left(\frac{c}{T^{2(n+1)}}-\frac{1}{n+1}\right)+\frac{4}{T^2}-\frac{wc_1^2}{T^{2(n+2)}}\left(\frac{c_1}{(1+n)T^{(n+1)}}\right)^{2/h+1}\right], \tag{21}$$

and

$$\rho_r=\frac{6(n+1)}{(3\gamma-4)T^{2(n+2)}}\left(\frac{c}{T^{2(n+1)}}-\frac{1}{n+1}\right)^{-1/2}-\left[\frac{2(2+n)}{(3\gamma-4)}+(2n+1)\right]\frac{1}{T^2}\left(\frac{c}{T^{2(n+1)}}-\frac{1}{n+1}\right)-\left[\frac{4}{(3\gamma-4)}+1\right]\frac{1}{T^2}+\left[\frac{1}{(3\gamma-4)}-\frac{1}{2}\right]\frac{wc_1^2}{T^{2(n+2)}}\left(\frac{c_1}{(1+n)T^{(n+1)}}\right)^{2/h+1} . \tag{22}$$

The rate of expansion H_i in the direction of *x*, *y*, *z*-axis are given by

$$H_1=\frac{2n}{T}\left[\frac{c}{T^{2(n+1)}}-\frac{1}{n+1}\right],$$

$$H_2 = H_3 = \frac{2}{T}\left[\frac{c}{T^{2(n+1)}} - \frac{1}{n+1}\right] \tag{23}$$

The scalar of expansion and shear scalar for the model are respectively given by

$$\theta = \frac{n+2}{T}\left[\frac{c}{T^{2(n+1)}} - \frac{1}{n+1}\right] \quad , \tag{24}$$

$$\sigma^2 = \frac{2}{3}\frac{(n-1)^2}{T^2}\left[\frac{c}{T^{2(n+1)}} - \frac{1}{n+1}\right]^2 \quad . \tag{25}$$

4. Conclusion

We have obtained two fluids Kantowski-Sachs cosmological models in the scalar-tensor theory of gravitation proposed by Saez and Ballester. Two fluids Kantowski-Sachs cosmological models are expanding and shearing universe but tilted Kantowski-Sachs cosmological models is only expanding and the non-shearing universe. When $T \rightarrow 0$, the scalar field V, the scalar of expansion and shear scalar, are undetermined but the large value of T the scalar field V, the scalar of expansion and shear scalar vanish. The scalar of expansion and shear scalar are vanished for $n = -2$ & $n = 1$. When $T = 0$, the rate of expansion H_i in the direction of *x*, *y*, *z*-axis is undetermined and the rate of expansion H_i in the direction of *x*, *y*, *z*-axis is vanished at $T = \infty$. At $\gamma = \frac{4}{3}$, the energy density ρ_m, pressure p_m for matter field are infinite. The pressure p_m for matter field vanishes at $\gamma = 1$. Since $\lim_{T \rightarrow \infty}\left(\frac{\sigma}{\theta}\right) \neq 0$ the models does not approach isotropy.

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