Article

On Dynamical Instability of Kantowski-Sachs Space-Time in *f*(*R*) Theory of Gravitation

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Abstract

In this paper, we have studied the instability regions of Kantowski-Sachs space-time in the framework of *f(R)* theory of gravitation.The dynamical equations are constructed with the help of contracted Bianchi identities. The perturbation approach is used for the stability analysis of the system under consideration. We have obtained the general collapse equation from the dynamical equations through perturbation scheme. This collapse equation with adiabatic index Γ_1 is used to study the instability regions in Newtonian (N) and Post-Newtonian (pN) regimes. We have concluded that the adiabatic index Γ_1 or stiffness parameter plays a key role to define the instability of the system which depends upon the physical parameters.

Keywords: Kantowski-Sachs, space-time, *f(R)* theory, perturbation, adiabatic index.

1.Introduction

The observational data such as Type Ia Supernovae, the cosmic microwave background indicate that our universe is expanding with an increasing rate [1-2]. It is believed that the dark energy is driving this cosmic acceleration. Modified theories of gravitation are used to explain the mysterious nature of dark energy. The modification of Einstein-Hilbert action may be the correct approach to explain the evolution of the universe. There exists a class of modified gravity theories such as *f(R)* gravity, *f(T)* gravity, *f(R,T)* gravity, *f(R,G)* gravity and *f(G)* gravity etc. Singh and Singh [3] have studied reconstruction of modified *f(R,T)* gravity with perfect fluid cosmological models. Reddy et al. [4] have investigated vacuum solutions of Bianchi Type-I and V models in *f(R)* gravity with a special form of deceleration parameter. Katore et al. [5] have discussed unified description of Bianchi type-I universe in *f(R)* gravity.

The stability analysis of cosmological models under perturbation has been the topic of interest among many researchers to understand the dynamical aspects. The importance of the stability analysis is evident because any stellar model is inconsequential if it is unstable against fluctuations, and different degrees of stability/ instability will give different patterns of evolution in the collapse of self-gravitating objects [6]. The pioneering and the classical one has been done by Chandrasekhar [7] which explores the study of the dynamical instability of gaseous masses approaching the Schwarzschild limit in general relativity under the most notable technique of

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perturbation. At the beginning of the black hole formation, the matter is just about to form a black hole and an infinitesimal small perturbation can either cause the matter to disperse to infinity or to form a black hole [8]. Fowler [9] has discussed stability regions of supermassive stars in terms of adiabatic index. Gingold and Monaghan [10] have studied instability of fast rotating non spherical stars with smooth particles hydrodynamic technique. This work is extended by Chan et al. [11] for heat flow and dynamical instability in spherical collapse. Sharif and Azam [12] have investigated the effects of electromagnetic field on the dynamical instability of expansion-free gravitational collapse. They have concluded that instability range depends upon pressure anisotropy, radial profile of energy density and electromagnetic field, but not on the adiabatic index Γ. Sharif and Bhatti [13] have explored the study of stability of restricted non static axial symmetry.

Recently many researchers have done the stability analysis in modified theories of gravitation. Sharif and Kausar [14] have investigated the Newtonian and Post Newtonian expansion-free fluid evolution in *f(R)* gravity. Sharif and Yousaf [15] have discussed the stability of cylindrically symmetric self-gravitating systems in $R+\in R^2$ gravity. They have also studied the instability of meridional axial system in $f(R)$ gravity [16]. Sharif and Rani [17] have discussed the dynamical instability of expansion free collapse in *f(T)* gravity. Abbas and Sarwar [18] have explored the dynamical stability of collapsing stars in Einstein Gauss-Bonnet gravity. Noureen and Zubair [19] have studied dynamical instability of spherical star in *f(R,T)* gravity.

Motivated by the above work, the present paper aims to study the dynamical instability of Kantowski-Sachs cosmological model in *f(R)* theory of gravitation. The paper has the following format. Sect. 2 deals with the field equations and dynamical equations. Sect. 3 is devoted to study the perturbation scheme on physical quantities and dynamical equations. In sect. 4, Newtonian and post Newtonian regimes are taken into account. Sect. 5 covers the conclusion.

2. Field Equations and Dynamical Equations

The $f(R)$ gravity is obtained by the Einstein-Hilbert action as

$$S_{f(R)}=\frac{1}{2k}\int d^4x\sqrt{-g}\,f(R)+S_M\,, \tag{1}$$

where $f(R), k, S_M$ are nonlinear Ricci Scalar function, coupling constant and matter action, respectively. The corresponding metric field equations are

$$f_R\,R_{\alpha\beta}-\frac{1}{2}f_R\,g_{\alpha\beta}-\nabla_\alpha\nabla_\beta f_R+g_{\alpha\beta}\nabla_\gamma\nabla^\gamma f_R=k\,T_{\alpha\beta}\,. \tag{2}$$

We have a formulation in the form of Einstein field equations as

$$G_{\alpha\beta}=\frac{k}{f_R}\left(\overset{(D)}{T}_{\alpha\beta}+T_{\alpha\beta}\right), \tag{3}$$

where $\overset{(D)}{T}_{\alpha\beta}=\frac{1}{k}\left\{\frac{f-Rf_R}{2}g_{\alpha\beta}+\nabla_\alpha\nabla_\beta f_R-\nabla_\gamma\nabla^\gamma f_R g_{\alpha\beta}\right\}$ is the effective stress energy momentum tensor.

The trace of equation (1) gives

$$Rf_R + 3\nabla_\gamma \nabla^\gamma f_R - 2f = kT, \tag{4}$$

which under constant curvature condition yields

$$Rf_R - 2f = 0. \tag{5}$$

This represents de-Sitter point, a vacuum solution, i.e., $T=0$.
We have Kantowski-Sachs metric in the form

$$ds^2 = dt^2 - A^2 dr^2 - B^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{6}$$

where A and B are the functions of r and t only.

The Kantowski-Sachs space-time describes spatially homogeneous but anisotropic cosmologies [20]. This is the only spatially homogeneous universe which is not included in the Bianchi classification. It is categorized outside this classification of models with three-dimensional isometry groups.The field equations of this space-time describe three different anisotropic3+1 dimensional space-times. Those with zero and negative curvature are just axi-symmetric Bianchi type I and III universes. The positive curvature models constitute the Kantowski-Sachs models. In the special case where they become isotropic, they reduce to the closed Friedmann-Robertson-Walker universes [21]. This provides the realistic picture of the universe immediately after the big-bang. Collins [22] has studied global structure of the Kantowski- Sachs cosmological models. Reddy and Naidu [23] have discussed Kantowski-Sachs cosmological models in biometric theory of gravitation. Rao and Neelima [24] have investigated Kantowski-Sachs string cosmological models with bulk viscosity in general scalar tensor theory of gravitation.

The energy momentum tensor for a locally anisotropic fluid is [14,19]

$$T_{\alpha\beta} = (\rho + p_\perp) u_\alpha u_\beta - p_\perp g_{\alpha\beta} + (p_r - p_\perp)\chi_\alpha \chi_\beta, \tag{7}$$

where $\rho, p_r, p_\perp, u_\alpha$, and χ_α are the energy density, anisotropic stresses, four velocity and four vector. Four vectors in co-moving co-ordinates are $\chi^\alpha = A^{-1}\delta_1^\alpha$ and $u_\alpha = \delta_4^\alpha$ satisfying $u_\alpha u^\alpha = 1$, $\chi_\alpha \chi^\alpha = -1$ and $u_\alpha \chi^\alpha = 0$.

The Ricci scalar for equation (6) is given by

$$R = 2\left[\frac{\ddot{A}}{A} + \frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} - \frac{2B''}{A^2 B} + \frac{2A'B'}{A^3 B} - \frac{B'^2}{B^2 A^2}\right]. \tag{8}$$

The $f(R)$ field equations for the metric (6), using equations (3) and (7) are as follows

$$-\frac{2\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} + \frac{B'^2}{B^2 A^2} - \frac{2}{B^2} = \frac{-k\,p_r}{f_R} + \frac{1}{f_R}\left[\frac{f - Rf_R}{2} - \ddot{f} - \frac{2\dot{f}\,\dot{B}}{B} + \frac{2f'B'}{A^2 B}\right], \tag{9}$$

$$-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}-\frac{\dot{A}\dot{B}}{AB}-\frac{2B''}{A^2B}-\frac{A'B'}{A^3B}$$
$$=\frac{-k\,p_\perp}{f_R}+\frac{1}{f_R}\left[\frac{f-Rf_R}{2}-\ddot{f}-\frac{\dot{f}\,\dot{A}}{A}-\frac{\dot{f}\,\dot{B}}{B}+\frac{f''}{A^2}+\frac{f'\,A'}{A^3}+\frac{f'\,B'}{A^2B}\right], \tag{10}$$

$$-\frac{2\dot{A}\dot{B}}{AB}-\frac{\dot{B}^2}{B^2}-\frac{1}{B^2}+\frac{2B''}{A^2B}-\frac{2A'B'}{A^3B}+\frac{B'^2}{B^2A^2}$$
$$=\frac{k}{f_R}\left[\rho+\frac{1}{k}\left\{\frac{f-Rf_R}{2}-\frac{\dot{f}\,\dot{A}}{A}-\frac{2\dot{f}\,\dot{B}}{B}+\frac{f''}{A^2}-\frac{f'\,A'}{A^3}+\frac{2f'\,B'}{A^2B}\right\}\right], \tag{11}$$

$$-\frac{\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}+\frac{A''}{A}-\frac{A'^2}{A^2}-\frac{2\dot{A}B'}{AB}+\frac{\dot{B}B'}{B^2}=\frac{1}{k}\left\{\dot{f}'-\frac{\dot{A}f'}{A}\right\}, \tag{12}$$

where prime and dot denote the derivatives with respect to *r* and *t* respectively.

The dynamical equations are found from the non-trivial contracted components of the Bianchi identities. The dynamical equations explore the evolutionary behaviour of fluid parameters with the passage of time during gravitational collapse. Therefore the equations $(T^{\alpha\beta}+\overset{(D)}{T}{}^{\alpha\beta})_{;\beta}u_\alpha=0$ and $(T^{\alpha\beta}+\overset{(D)}{T}{}^{\alpha\beta})_{;\beta}\chi_\alpha=0$ have the following independent components

$$\dot{\rho}+(\rho+p_r)\frac{\dot{A}}{A}+2(\rho+p_\perp)\frac{\dot{B}}{B}+\frac{D_0}{k}=0\ , \tag{13}$$

$$p_r'+2(p_r-p_\perp)\frac{B'}{B}+\frac{D_1}{k}=0\ , \tag{14}$$

where D_0 and D_1 are given in appendix.

3. Perturbation Scheme

We apply the perturbation to the metric coefficients and matter variables upto first order in λ, where $0<\lambda<<1$ is known as perturbation parameter. It is assumed that the system is in static equilibrium initially. We further assume that the metric functions have same time dependence in their perturbation. Therefore, the perturbation scheme is defined as follows [15, 25]

$$A(r,t)=A_0(r)+\lambda T(t)a(r), \tag{15}$$
$$B(r,t)=B_0(r)+\lambda T(t)b(r), \tag{16}$$
$$\rho(r,t)=\rho_0(r)+\lambda\bar{\rho}(r,t), \tag{17}$$
$$p_r(r,t)=p_{r0}(r)+\lambda\,\bar{p}_r(r,t), \tag{18}$$
$$p_\perp(r,t)=p_{\perp0}(r)+\lambda\,\bar{p}_\perp(r,t). \tag{19}$$

The Ricci scalar in $f(R)$ model leads to

$$R(r,t)=R_0(r)+\lambda T(t)e(r), \tag{20}$$

$$f = R_0(1+\in R_0) + \lambda T(t)e(r)(1+2\in R_0), \tag{21}$$

$$f_R = (1+2\in R_0) + 2\lambda \in T(t)\, e(r). \tag{22}$$

Applying perturbations given in equations (15) - (22), the static part of the Ricci scalar is given by

$$R_0 = 2\left[\frac{1}{B_0^2} - \frac{2B_0''}{B_0 A_0^2} - \frac{B_0'^2}{A_0^2 B_0^2} + \frac{2A_0' B_0'}{A_0^3 B_0}\right]. \tag{23}$$

The perturbed part of Ricci Scalar is obtained as

$$Te = 2\ddot{T}\left(\frac{a}{A_0} + \frac{b}{B_0}\right) - \frac{4T}{A_0 B_0}\left[a'B_0' + b'A_0' + \frac{b''}{A_0} - \frac{aB_0''}{A_0^2} - \frac{aA_0'B_0'}{A_0} - \frac{b'B_0'}{A_0 B_0} + \frac{b'B_0'}{A_0^2 B_0} - \frac{bB_0'^2}{4B_0} + \frac{b}{2}R_0\right]. \tag{24}$$

It is found that equation (13) and (14) have only one static part

$$p_{r_0}' + 2(p_{r0} - p_{\perp 0})\frac{B_0'}{B_0} + \frac{D_2}{k} = 0. \tag{25}$$

The perturbed parts of (13) and (14) are

$$\dot{\bar{\rho}} + (\rho_0 + p_{r0})\frac{\dot{T}a}{A_0} + 2(\rho_0 + p_{\perp 0})\frac{\dot{T}b}{B_0} + \frac{D_3\dot{T}}{k} = 0, \tag{26}$$

$$\bar{p}_r' + 2(p_{r0} - p_{\perp 0})\left(\frac{b}{B_0'}\right)' T + 2(\bar{p}_r - \bar{p}_\perp)\frac{B_0'}{B_0} + \frac{P(x,t)}{k} = 0. \tag{27}$$

Integrating equation (26), we get

$$\bar{\rho} = -(\rho_0 + p_{x0})\frac{Ta}{A_0} - 2(\rho_0 + p_{\perp 0})\frac{Tb}{B_0} - \frac{D_2}{k}T, \tag{28}$$

where D_2, D_3, P are given in appendix.

From equation (24), we obtain

$$2\ddot{T}\left(\frac{a}{A_0} + \frac{b}{B_0}\right) - \frac{4T}{A_0 B_0}\left[a'B_0' + b'A_0' + \frac{b''}{A_0} - \frac{aB_0''}{A_0^2} - \frac{aA_0'B_0'}{A_0} - \frac{b'B_0'}{A_0 B_0} + \frac{b'B_0'}{A_0^2 B_0} - \frac{bB_0'^2}{4B_0} + \frac{b}{2}R_0 - \frac{A_0 B_0 e}{4}\right] = 0 \; . \tag{29}$$

Equation (29) can be rewritten as

$$\ddot{T}(t) - \mu^2(r)T(t) = 0, \tag{30}$$

where $\mu^2(r)$ is given in appendix.

Our aim is to find the instability region, hence we assume that all the functions involved in the above equations are such that μ is positive, hence the solution of equation (30) becomes

$$T(t) = -e^{\mu t}\,. \tag{31}$$

Here we assume that the system goes to the collapsing state with $T(-\infty) = 0$ at $t = -\infty$, keeping it in static position.

The equation of state is given by [26]

$$\bar{p}_i = \Gamma_1 \frac{p_{i0}}{\rho_0 + p_{i0}} \bar{\rho}, \tag{32}$$

where Γ_1 is the adiabatic index which is taken as constant identity.
Equations (28) and (32) follow

$$\bar{p}_r = -\Gamma_1 \frac{p_{r0}}{\rho_0 + p_{r0}} \left[(\rho_0 + p_{r0}) \frac{a}{A_0} + 2(\rho_0 + p_{\perp 0}) \frac{b}{B_0} + \frac{D_2}{k} \right] T, \tag{33}$$

$$\bar{p}_\perp = -\Gamma_1 \frac{p_{\perp 0}}{\rho_0 + p_{\perp 0}} \left[(\rho_0 + p_{r0}) \frac{a}{A_0} + 2(\rho_0 + p_{\perp 0}) \frac{b}{B_0} + \frac{D_2}{k} \right] T. \tag{34}$$

Substituting equations (33) and (34) in equation (27), we obtain

$$\begin{aligned} &-\Gamma_1 T \left[\frac{p_{r0}}{\rho_0 + p_{r0}} \left\{ (\rho_0 + p_{r0}) \frac{a}{A_0} + 2(\rho_0 + p_{\perp 0}) \frac{b}{B_0} + \frac{D_2}{k} \right\} \right]' + (p_{r0} - p_{\perp 0}) \left(\frac{b}{B_0} \right)' T \\ &- 2T\Gamma_1 \left[\begin{array}{l} \left\{ \frac{p_{r0}}{\rho_0 + p_{r0}} \left((\rho_0 + p_{r0}) \frac{a}{A_0} + 2(\rho_0 + p_{\perp 0}) \frac{b}{B_0} + \frac{D_2}{k} \right) \right\} \\ - \left\{ \frac{p_{\perp 0}}{\rho_0 + p_{\perp 0}} \left((\rho_0 + p_{r0}) \frac{a}{A_0} + 2(\rho_0 + p_{\perp 0}) \frac{b}{B_0} + \frac{D_2}{k} \right) \right\} \end{array} \right] \frac{B_0'}{B_0} + \frac{P}{k} = 0. \end{aligned} \tag{35}$$

Equation (35) is the general collapse equation.

4. Instability Regions for Newtonian (N) and post-Newtonian (pN) Approximation

Now we proceed to calculate constraints at which the system under consideration undergo instability at both Newtonian and post-Newtonian eras in *f(R)* theory. We also investigate the role of adiabatic index in this scenario.

Newtonian Approximation

We take $A_0 = B_0 = 1$, for the investigation of instability ranges in the N era such that the static part of energy density is much greater than the static part of principle stresses, i.e., $\rho_0 >> p_{r0}, \rho_0 >> p_{\perp 0}$. Using these limits and $t \to -\infty$ the collapse equation, (35) turns out to be

$$\Rightarrow \Gamma_1 < \frac{2(p_{r0} - p_{\perp 0})b' + \Omega_1}{\left[p_{r0} \{ (\rho_0 + p_{r0})a + 2(\rho_0 + p_{\perp 0})b \} \right]' + \Omega_2}, \tag{36}$$

where $\Omega_1 = \frac{D_{4N_0}}{k}$ and $\Omega_2 = \left[p_{r0} \left\{ \frac{D_2}{k} \right\} \right]'$, P_{N_0} represent the P in Newtonian limit.

The system would be unstable if it satisfies equation (36).

Post Newtonian Approximation

For the dynamical ranges of instability in the pN limit, we assume

$$A_0 = 1 - \frac{m_0}{r}, \qquad B_0 = 1 + \frac{m_0}{r}, \tag{37}$$

and take effects upto $O\left(\frac{m_0}{r}\right)$. Using these quantities in the collapse equation (35), we get the instability range as

$$\Rightarrow \Gamma_1 < \frac{2(p_{r0} - p_{\perp 0})\left(1 + \frac{m_0}{r}\right) + \Omega_3 + \Omega_4}{\left[\frac{p_{r0}}{\rho_0 + p_{r0}}\left\{(\rho_0 + p_{r0})\frac{a}{\left(1 - \frac{m_0}{r}\right)} + 2(\rho_0 + p_{\perp 0})\frac{b}{\left(1 + \frac{m_0}{r}\right)}\right\}\right]' + \Omega_5}, \tag{38}$$

where

$$\Omega_3 = 2\left[\begin{array}{l}\left\{\frac{p_{r0}}{\rho_0 + p_{r0}}\left((\rho_0 + p_{r0})\frac{a}{\left(1 - \frac{m_0}{r}\right)} + 2(\rho_0 + p_{\perp 0})\frac{b}{\left(1 + \frac{m_0}{r}\right)}\right)\right\} \\ -\left\{\frac{p_{\perp 0}}{\rho_0 + p_{\perp 0}}\left((\rho_0 + p_{r0})\frac{a}{\left(1 - \frac{m_0}{r}\right)} + 2(\rho_0 + p_{\perp 0})\frac{b}{\left(1 + \frac{m_0}{r}\right)}\right)\right\}\end{array}\right]\left(1 + \frac{m_0}{r}\right)^{-1},$$

$$\Omega_4 = \frac{p_{r0}}{\rho_0 + p_{r0}}\frac{D_2}{k} - \frac{p_{\perp 0}}{\rho_0 + p_{\perp 0}}\frac{D_2}{k} + \frac{P_{pN_0}}{k},$$

$\Omega_5 = \left[\frac{p_{r0}}{\rho_0 + p_{r0}}\left\{\frac{D_2}{k}\right\}\right]'$ and P_{pN_0} represent the D_4 in Post-Newtonian (pN) limit.

Equation (38) corresponds to the instability range of system in pNlimit.

5.Conclusions

This paper deals with the dynamical instability of Kantowski-Sachs Space-time in the context of *f(R)* theory of gravitation. We have considered the locally anisotropic fluid distribution. The instability range of self-gravitating source is defined by adiabatic index Γ_1, which is also known as the stiffness parameter of the fluid under consideration. We have formulated the collapse equation by using perturbation scheme. For the collapsing phenomenon, we have assumed the complete hydrostatic equilibrium at large past time, i.e., $T(-\infty) = 0$. We have, then, explored the dynamical instability at N (Newtonian) and pN (Post-Newtonian) regimes. The system under investigation would be unstable if it satisfies equations (36) and (38) at N (Newtonian) and pN (Post-Newtonian) regimes respectively. However, on violation of these inequalities the system will move towards stability.

Here, we have studied the instability ranges for Kantowski-Sachs space-time and found that it depends upon the adiabatic index which is well consistent with [16, 19]. It is interesting to mention that equation (30) gives both oscillating and non-oscillating configuration of the collapsing system. But only non-oscillating constraint has been taken into account to examine the stability of the system and the role of adiabatic index and it is found that the adiabatic index Γ_1 is fully involved in the instability analysis and it plays a pivotal role in our investigation.

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Appendix:

$$D_0 = -\frac{1}{AB^2}\left[\left\{\frac{B^2}{A}\left(\dot{f}_R' - \frac{\dot{A}f_R'}{A}\right)\right\}_{,r} + \left\{AB^2\left(\frac{f - Rf_R}{2} + \frac{f_R''}{A^2} - \frac{f_R'A'}{A^3} - \frac{2f_R'B'}{A^2B} - \frac{2\dot{f}_R\dot{B}}{B}\right)\right\}_{,t}\right]$$
$$-\frac{\dot{A}}{A}\left\{\frac{f - Rf_R}{2} + \frac{\dot{f}_R\dot{A}}{A} + \frac{2f_R'B'}{A^2B} - \frac{2\dot{f}_R\dot{B}}{B} - \ddot{f}_R\right\} - \frac{2\dot{B}}{B}\left\{\frac{f - Rf_R}{2} - \frac{\dot{f}_R\dot{B}}{B} + \frac{f_R'B'}{A^2B} + \frac{f_R''}{A^2} - \frac{f_R'A'}{A^3} - \ddot{f}_R\right\}$$

$$D_1 = \frac{1}{AB^2}\left[\left\{\frac{B^2}{A}\left(\frac{f - Rf_R}{2} + \frac{\dot{f}_R\dot{A}}{A} + \frac{2f_R'B'}{A^2B} - \frac{2\dot{f}_R\dot{B}}{B} - \ddot{f}_R\right)\right\}_{,r} - \left\{\frac{B^2}{Ak}\left(\dot{f}_R' - \frac{\dot{A}f_R'}{A}\right)\right\}_{,t}\right]$$
$$-\frac{A'}{A^3}\left\{\frac{f - Rf_R}{2} + \frac{\dot{f}_R\dot{A}}{A} + \frac{2f_R'B'}{A^2B} - \frac{2\dot{f}_R\dot{B}}{B} - \ddot{f}_R\right\}$$
$$+\frac{2B'}{BA^2}\left\{\frac{f - Rf_R}{2} - \frac{\dot{f}_R\dot{B}}{B} + \frac{f_R'B'}{A^2B} + \frac{f_R''}{A^2} - \frac{f_R'A'}{A^3} - \ddot{f}_R\right\}$$

$$D_2 = -2\left[\begin{array}{l}\frac{1}{A_0B_0^2}\left\{\frac{B_0^2}{A_0}\left(e' - \frac{\in aR_0'}{A_0}\right)\right\}_{,r} + \frac{\in}{2A_0B_0^2}\left\{2\left(A_0b + aB_0^2\right)\left(\frac{-R_0^2}{2} + \frac{2R_0''}{A_0^2} - \frac{R_0'A_0'}{A_0^3} - \frac{4R_0'B_0'}{A_0B_0}\right)\right\} \\ -\frac{eR_0}{2} + \frac{\in}{A_0^2}\left\{e'' - e' - \frac{2aR_0''}{A_0} - \frac{aR_0'}{A_0} + \frac{3aR_0'A_0'}{A_0^2} - \frac{2}{B_0}\left(bR_0' + e'B_0 - \frac{2aR_0'B_0}{A_0} - bR_0'B_0'\right)\right\} \\ +\in\left\{-\left(\frac{a}{A_0} + \frac{2b}{B_0}\right)\frac{R_0^2}{4} + 2\left(\frac{a}{A_0} - \frac{b}{B_0}\right)\frac{R_0'B_0'}{A_0B_0} + \frac{2bR_0''}{A_0^2B_0}\right\}\end{array}\right]$$

$$D_3 = \frac{1}{A_0{B_0}^2}\left[\left\{\frac{B_0^2}{A_0}\left(-\frac{\in R_0^2}{2} + \frac{4\in R_0'B_0'}{A_0^2B_0}\right)\right\}_{,r} + \left(\frac{B_0^2}{A_0}\right)_{,r}\left(-\frac{\in R_0^2}{2} + \frac{4\in R_0'B_0'}{A_0^2B_0}\right)\right]$$
$$-\frac{A_0'}{{A_0}^3}\left(-\frac{\in R_0^2}{2} + \frac{4\in R_0'B_0'}{A_0^2B_0}\right) - \frac{2B_0'}{B_0{A_0}^2}\left\{-\frac{\in R_0^2}{2} + \frac{2\in R_0'B_0'}{A_0^2B_0} + \frac{2\in R_0''}{{A_0}^2} - \frac{2\in R_0'A_0'}{{A_0}^3}\right\}$$

$$P=\frac{1}{A_0^3}\left\{-eR_0+\frac{4\in}{A_0^2B_0}\left(bR_0'+e'B_0-\frac{2aR_0'B_0}{A_0}-bR_0'B_0'\right)\right\}_{,r}-\frac{1}{A_0B_0^2}\left(\frac{a}{A_0^3}+\frac{2b}{A_0^2B_0}\right)\left\{\frac{-\in R_0^2}{2}+2\left(\frac{\in R_0'B_0'}{A_0B_0}\right)_{,r}\right\}$$
$$-\frac{1}{A_0B_0^2}\left(\frac{a}{A_0^4}+\frac{2b}{A_0^3B_0}\right)\left\{e'-\frac{\in aR_0'}{A_0}\right\}-\frac{A_0'}{A_0^3}\left\{\frac{-2eR_0}{2}+\frac{4\in}{A_0^2B_0}\left(bR_0'+e'B_0-\frac{2aR_0'B_0}{A_0}-bR_0'B_0'\right)\right\}+\left(\frac{3aA_0'}{A_0^4}-\frac{a'}{A_0^3}\right)$$
$$\times\left(-\frac{\in R_0^2}{2}+\frac{4\in R_0'B_0'}{A_0B_0}\right)+\frac{2B_0'}{A_0^2B_0}\left\{-eR_o+\frac{4\in}{A_0^2B_0}\left(bR_0'+e'B_0-\frac{2aR_0'B_0}{A_0}-bR_0'B_0'\right)\right\}$$
$$+\frac{4\in B_0'}{A_0^4B_0}\left(-e'+e''-\frac{2aR_0''}{A_0}-\frac{aR_0}{A_0}+\frac{3aR_0'A_0'}{A_0^2}\right)-\frac{4\in aB_0'}{B_0A_0^3}\left\{\frac{-R_0^2}{2}+\frac{2R_0'B_0'}{A_0B_0}+\frac{2R_0''}{A_0^2}-\frac{2R_0'A_0'}{A_0^3}\right\}$$
$$-\left\{\left(\frac{2e}{A_0^2}\right)_{,r}-\left(\frac{B_0^2}{A_0}\right)_{,r}e+\frac{A_0'}{A_0^3}e+2\frac{B_0'}{B_0A_0^2}\right\}\in\ddot{T}+\left(\frac{B_0^2}{A_0}\right)_{,r}\frac{2B_0^2\dot{T}}{A_0^2}\left(e'-\frac{\in aR_0'}{A_0}\right)$$

$$\mu^2(r)=\frac{2T}{A_0B_0}\left(\frac{a}{A_0}+\frac{2b}{B_0}\right)^{-1}\left[a'B_0'+b'A_0'+\frac{b''}{A_0}-\frac{aB_0''}{A_0^2}-\frac{aA_0'B_0'}{A_0}-\frac{b'B_0'}{A_0B_0}+\frac{b'B_0'}{A_0^2B_0}-\frac{bB_0'^2}{4B_0}+\frac{b}{2}R_0-\frac{A_0B_0e}{4}\right]$$