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Perfect Fluid Cosmological Model in the Frame Work Lyra's Manifold

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Abstract

In this paper we have obtained Perfect Fluid cosmological model in the frame work Lyra's Manifold in Ruban's back ground. To get the deterministic solutions assume the relation between metric potential. Some physical and kinematical properties of the models are also discussed.

Keywords: Ruban's metric, perfect fluid, Lyra's geometry.

1. Introduction

Einstein proposed his general theory of relativity, in which gravitation is described in terms of geometry and it motivated the geometrization of other physical fields. One of the attempts in this direction was made by Weyl [16] who proposed a more general theory in which gravitation and electromagnetism is also described geometrically. But this theory was not accepted as it was based on non-integrability of length transfer. Later Lyra [3] introduced a gauge function i.e. displacement vector in Riemannian space-time which removes the non-integrability condition of a vector under parallel transport. This modified geometry was named as Lyra's geometry.

Lyra [3] proposed a modification of Riemannian geometry by introducing gauge function into structure less manifold, as a result of which the cosmological constant arises naturally from the geometry. In subsequent investigation Sen [14] and Sen and Dunn [15] formulated a new scalar-tensor theory of gravitation and constructed an anologue of the Einstein field equations based on Lyra's geometry. Reddy and Innaih [10], Reddy and venkateswarlu [11] are some of the authors who have investigated various aspects of the four dimensional cosmological models in Lyra's manifold. Bhowmik and Rajput [1] obtained anisotropic Bianchi type cosmological models on the basis of Lyra's geometry. Reddy DRK [12] examined plane symmetric cosmic strings in Lyra's manifold

Mohanty *et.al*. [4] has studied non-Existence of five dimensional perfect fluid cosmological model in Lyra's manifold.

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Samanta *et.al*.[13] discussed five dimensional Bianchi-I string cosmological model in Lyra manifold. Nimkar *et. al.* [6] have been studied wet dark fluid Cosmological Model in Lyra's Manifold.

Also,recently Nimkar *et.al*.[7], Pund *et.al*.[9], Nimkar [8], and Mete *et.al*.[5] have studied different cosmological model in Ruban's background.

The purpose of the present work is to obtain Perfect fluid Ruban's cosmological model in the frame work of Lyra's Manifold . Our paper is organized as follows. In section 2, we derive the field equation in presence Perfect fluid with the aid of Ruban's space time. Section 3, contain solution of the field Equations, Section 4 is mainly concerned with the physical and kinematical properties of the model. The last section contains some conclusion.

2. The Metric and field equations

We consider the space- time of Ruban's [2] in the form

$$ds^2 = dt^2 - Q^2(x,t)dx^2 - R^2(t)(dy^2 + h^2 dz^2) \tag{1}$$

where $h(y) = \frac{\sin\sqrt{k}y}{\sqrt{k}} = \begin{matrix} \sin y & if & k=1 \\ y & if & k=0 \\ \sinh y & if & k=-1 \end{matrix}$

and k is the curvature parameter of the homogeneous 2-spaces t and x constants. The functions Q and R are free and will be determined.

The relativistic field equations in normal gauge in Lyra's manifold are as

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -T_{ij} \tag{2}$$

where ϕ_i is a displacement vector field and the other symbols have their usual meaning as in Riemmanian geometry. We now assume the vector displacement field β to be the time like constant vector.

$$\phi_i = (0,0,0,\beta) \tag{3}$$

where $\beta = \beta(t)$ is a function of time alone.

The energy momentum tensor for Perfect fluid is given by

$$T_{ij} = (p+\rho)\ u_i u_j - pg_{ij}\ , \tag{4}$$

where p and ρ are the energy density and pressure of the fluid respectively together with co-moving coordinates $u^i u_i = 1$ where

$$u_i = (0,0,0,1)$$

Comoving co-ordinates for perfect fluid is given by

$$T_1^1 = T_2^2 = T_3^3 = -p\,, \quad T_4^4 = \rho \ \text{ and } \ T_j^i = 0 \ \text{ for } i \neq j\,. \tag{5}$$

The field equations (2) for the metric (1) with the help of equation (3)-(5) reduce to

$$2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} + \frac{3}{4}\beta^2 = -p \tag{6}$$

$$\frac{\dot{R}\dot{Q}}{RQ} + \frac{\ddot{R}}{R} + \frac{\ddot{Q}}{Q} + \frac{3}{4}\beta^2 = -p \tag{7}$$

$$2\frac{\dot{R}\dot{Q}}{RQ} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} - \frac{3}{4}\beta^2 = \rho \tag{8}$$

Here over head dot represent partial differentiation with respect to *t*.

3. Solution and the model

Here we have three independent field equations (6)-(8) connecting five unknown R, Q, p, ρ and β. Therefore in order to obtain exact solutions, we must need one more relation connecting the unknown quantities. We assume the equation of state $p = \gamma\rho$,where $0 \leq \gamma \leq 1$.Since the field equations are highly nonlinear, we also assume the relation between metric coefficients

$$Q = x^n R^n\,.$$

Using this relation, the field equations (6)-(8) admit the exact solution.

$$R = M(c_1 t + c_2)^{\frac{1}{n+2}} \tag{9}$$

$$Q = x^n N(c_1 t + c_2)^{\frac{n}{n+2}} \tag{10}$$

Using equations (9) and (10) the Metric (1) becomes

$$ds^2 = dt^2 - N^2 x^{2n}(c_1 t + c_2)^{\frac{2n}{n+2}} dx^2 - M^2 (c_1 t + c_2)^{\frac{2}{n+2}} (dy^2 + h^2 dz^2) \tag{11}$$

This metric can be transformed through a proper choice of coordinates to the form

$$ds^2 = dT^2 - N^2 x^{2n} T^{\frac{2n}{n+2}} dx^2 - M^2 T^{\frac{2}{n+2}} (dy^2 + h^2 dz^2) \tag{12}$$

where $T = c_1 t + c_2$

4. The Physical and Kinematical Properties

The energy density ρ and pressure p for the model (12) are given by

$$p = \rho = c_5 \frac{1}{(T)^{\frac{2}{n+2}}} \tag{13}$$

$$\frac{3}{2}\beta^2 = (2n+1)\left(\frac{c_1}{n+2}\right)^2 \frac{1}{T^2} - \frac{k}{M^2 T^{\frac{2}{n+1}}} \tag{14}$$

Also.

The spatial volume , $V = x^n NM^2 T$ (15)

The Scalar expansion, $\theta = \frac{\dot{Q}}{Q} + 2\frac{\dot{R}}{R} = \frac{c_1}{T}$ (16)

Shear scalar, $\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{c_3}{T^2}$ (17)

Hubble parameter, $H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\theta}{3} = \frac{c_1}{3T}$ (18)

The model (12) has no initial singularities while the energy density and pressure given by (13) and β given by (14) possess initial singularities. However as T increases these singularities vanish. Spatial volume of the model given by (15) shows anisotropic expansion of the universe (12) with time. For the model the Scalar expansion θ and shear scalar σ tend to zero as $T \to \infty$

5. Conclusion

In this paper, we have obtained Ruban's cosmological model in presence of perfect fluid in the frame work of Lyra Manifold. For finding the exact solution the relation between the metric potential and the equation of state of are used. The model is free from initial singularities and they are expanding, shearing and non- rotating in the standard way. Our model throws some light on the understanding of structure formation of the universe in Lyra's manifold.

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