Article

A Potential for the Lanczos Spinor in Arbitrary Spacetimes of Petrov Types III, N, and O

V. Barrera-Figueroa[1], J. López-Bonilla[*2] & R. López-Vázquez[2]

[1]SEPI-UPIITA, Instituto Politécnico Nacional (IPN), Posgrado en Tecnología Avanzada, Av. IPN 2580, Col. Barrio la Laguna 07340, CDMX, México
[2]ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 5, 1er. Piso, Lindavista 07738, CDMX, México

Abstract

Andersson-Edgar proved that the Lanczos spinor can be generated via $L_{ABC\dot{D}} = \nabla^{E}{}_{\dot{D}}\, T_{ABCE}$. Here we determine the potential T_{ABCE} for any R_4 of Petrov types O, N, and III.

Keywords: Lanczos spinor, spin coefficients.

1. Introduction

In [1] was obtained the Lanczos generator [2, 3] for an arbitrary spacetime of Petrov type N, with the following Newman-Penrose components:

$$\Omega_0 = \frac{\kappa}{2}, \qquad \Omega_3 = -\frac{\lambda}{2}, \qquad \Omega_4 = \frac{\sigma}{2}, \qquad \Omega_7 = -\frac{\nu}{2},$$
$$\Omega_1 = \frac{\rho}{6}, \qquad \Omega_2 = -\frac{\pi}{6}, \qquad \Omega_5 = \frac{\tau}{6}, \qquad \Omega_6 = -\frac{\mu}{6}, \tag{1}$$

in terms of the spin coefficients [4, 5] associated to the canonical null tetrad [6]:

$$l^{\mu} \leftrightarrow o^A o^{\dot{B}}, \qquad n^{\mu} \leftrightarrow \iota^A \iota^{\dot{B}}, \qquad m^{\mu} \leftrightarrow o^A \iota^{\dot{B}}, \qquad \overline{m}^{\mu} \leftrightarrow \iota^A o^{\dot{B}}; \tag{2}$$

hence for the type N the Lanczos spinor [7, 8] is given by:

$$L_{ABC\dot{D}} = \frac{1}{6}[3\, o_A o_B o_C(-\nu\, o_{\dot{D}} + \lambda\, \iota_{\dot{D}}) + (o_A o_B \iota_C + (o_A * \iota_B) o_C)(\mu\, o_{\dot{D}} - \pi\, \iota_{\dot{D}}) +$$
$$+ (\iota_A \iota_B o_C + (o_A * \iota_B)\iota_C)(\tau\, o_{\dot{D}} - \rho\, \iota_{\dot{D}}) + 3\, \iota_A \iota_B \iota_C\, (-\sigma\, o_{\dot{D}} + \kappa\, \iota_{\dot{D}})]. \tag{3}$$

* Correspondence: J. López-Bonilla, ESIME-Zacatenco-IPN, Edif. 5, Col. Lindavista CP 07738, CDMX, México
E-mail: jlopezb@ipn.mx

On the other hand, Andersson-Edgar [9-11] proved that any Lanczos spinor can be generated via the relation:

$$L_{ABC\dot{D}} = \nabla^{E}{}_{\dot{D}}\, T_{ABCE}\,, \qquad\qquad T_{ABCE} = T_{(ABC)E}\,. \tag{4}$$

In Sec. 2 we give an explicit expression for the potential T_{ABCE} in terms of the dyad (o^F, ι^G), for the Lanczos spinor (3).

Let's remember that [1]:

$$(\Omega_r)_{\,Type\ III} = 2\,(\Omega_r)_{\,Type\ N}\,, \qquad r = 0, \dots, 7, \tag{5}$$

in the canonical tetrad for the type III, therefore:

$$(T_{ABCE})_{\,Type\ III} = 2\,(T_{ABCE})_{\,Type\ N}\,. \tag{6}$$

2. Potential for the Lanczos spinor

In the spin coefficients formalism [4] are very known the following relations:

$$\begin{gathered}
\nabla^{A}{}_{\dot{B}}\, o_A = (\tau-\beta)o_{\dot{B}} + (\varepsilon-\rho)\iota_{\dot{B}}, \quad \nabla^{A}{}_{\dot{B}}\, \iota_A = (\gamma-\mu)o_{\dot{B}} + (\pi-\alpha)\iota_{\dot{B}}, \\
o_A\nabla^{A}{}_{\dot{B}}\, o_C = (-\beta\, o_{\dot{B}} + \varepsilon\, \iota_{\dot{B}})o_C + (\sigma\, o_{\dot{B}} - \kappa\, \iota_{\dot{B}})\iota_C, \\
o_A\nabla^{A}{}_{\dot{B}}\iota_C = (-\mu o_{\dot{B}} + \pi\iota_{\dot{B}})o_C + (\beta o_{\dot{B}} - \varepsilon\iota_{\dot{B}})\iota_C, \\
\iota_A\nabla^{A}{}_{\dot{B}}\, o_C = (-\gamma\, o_{\dot{B}} + \alpha\, \iota_{\dot{B}})o_C + (\tau\, o_{\dot{B}} - \rho\, \iota_{\dot{B}})\iota_C, \\
\iota_A\nabla^{A}{}_{\dot{B}}\, \iota_C = (-\nu\, o_{\dot{B}} + \lambda\, \iota_{\dot{B}})o_C + (\gamma\, o_{\dot{B}} - \alpha\, \iota_{\dot{B}})\iota_C,
\end{gathered} \tag{7}$$

hence:

$$\begin{gathered}
\nabla^{E}{}_{\dot{D}}[(o_A o_B \iota_C + (o_A * \iota_B)o_C)\iota_E] = 3o_A o_B o_C(-\nu\, o_{\dot{D}} + \lambda\, \iota_{\dot{D}}) + \\
+(o_A o_B \iota_C + (o_A * \iota_B)o_C)(-\mu\, o_{\dot{D}} + \pi\, \iota_{\dot{D}}) + 2(\iota_A \iota_B o_C + (o_A * \iota_B)\iota_C)(\tau\, o_{\dot{D}} - \rho\, \iota_{\dot{D}}), \\
\nabla^{E}{}_{\dot{D}}[(\iota_A \iota_B o_C + (o_A * \iota_B)\iota_C)o_E] = 3\iota_A \iota_B \iota_C(\sigma\, o_{\dot{D}} - \kappa\, \iota_{\dot{D}}) + \\
+(\iota_A \iota_B o_C + (o_A * \iota_B)\iota_C)(\tau\, o_{\dot{D}} - \rho\, \iota_{\dot{D}}) + 2(o_A o_B \iota_C + (o_A * \iota_B)o_C)(-\mu\, o_{\dot{D}} + \pi\, \iota_{\dot{D}}),
\end{gathered} \tag{8}$$

then it is immediate that (4) implies (3) for:

$$T_{ABCE} = \frac{1}{6}[(o_A o_B \iota_C + (o_A * \iota_B) o_C)\iota_E - (\iota_A \iota_B o_C + (o_A * \iota_B)\iota_C) o_E], \tag{9}$$

where it is evident the symmetry (4) in ABC.

The result (9) is applicable to the type III in according with (6). The type O accepts as Lanczos spinor to any multiple of (3) [1], thus (9) also is valid for this type.

Received November 23, 2016; Accepted December 25, 2016

References

1. G. Ares de Parga, O. Chavoya, J. López-Bonilla, *Lanczos potential*, J. Math. Phys. **30**, No. 6 (1989) 1294-1295
2. C. Lanczos, *The splitting of the Riemann tensor*, Rev. Mod. Phys. **34**, No. 3 (1962) 379-389
3. H. Takeno, *On the spintensor of Lanczos*, Tensor N. S. **15** (1964) 103-119
4. J. López-Bonilla, R. López-Vázquez, J. Morales, G. Ovando, *Spin coefficients formalism*, Prespacetime Journal **6**, No. 8 (2015) 697-709
5. V. Barrera-Figueroa, J. López-Bonilla, R. López-Vázquez, *Spin coefficients and Lanczos spinor*, Prespacetime Journal **7**, No. 15 (2016) 1908-1911
6. J. López-Bonilla, R. López-Vázquez, J. Morales, G. Ovando, *Petrov types and their canonical tetrads*, Prespacetime Journal **7**, No. 8 (2016) 1176-1186
7. J. D. Zund, *The theory of the Lanczos spinor*, Ann. di Mat. Pura Appl. **109**, No. 1 (1975) 239-268
8. J. López-Bonilla, R. López-Vázquez, J. Morales, G. Ovando, *Maxwell, Lanczos, and Weyl spinors*, Prespacetime Journal **6**, No. 6 (2015) 509-520
9. F. Andersson, S. B. Edgar, *Local existence of spinor-and tensor potentials*, arXiv: gr-qc/9902080v2 13 Jan 2000
10. F. Andersson, S. B. Edgar, *Existence of Lanczos potentials and superpotentials for the Weyl spinor / tensor*, Class. Quantum Grav. **18**, No. 12 (2001) 2297-2304
11. F. Andersson, S. B. Edgar, *Local existence of symmetric spinor potentials for symmetric (3, 1) – spinors in Einstein space-times*, J. Geom. Phys. **37**, No. 4 (2001) 273-290