Exploration

How Might the Hierarchy of Planck Constants Be Related to the Almost Vacuum Degeneracy for Twistor Lift of TGD?

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Abstract

The twistor lift of classical TGD is attractive physically but it is still unclear whether it satisfies all constraints. The basic implication of twistor lift would be the understanding of gravitational and cosmological constants. Cosmological constant removes the infinite vacuum degeneracy of Kähler action but because of the extreme smallness of cosmological constant Λ playing the role of inverse of gauge coupling strength, the situation for nearly vacuum extremals of Kähler action in the recent cosmology is non-perturbative. Cosmological constant and thus twistor lift make sense only in zero energy ontology (ZEO) involving causal diamonds (CDs) in an essential manner. One motivation for introducing the hierarchy of Planck constants was that the phase transition increasing Planck constant makes possible perturbation theory in strongly interacting system. Nature itself would take care about the converge of the perturbation theory by scaling Kähler coupling strength α_K to α_K/n, $n = h_{eff}/h$. This hierarchy might allow to construct gravitational perturbation theory as has been proposed already earlier. This would for gravitation to be quantum coherent in astrophysical and even cosmological scales. In the following this picture is studied in detail. Perhaps the most interesting observation is that a fundamental length scale of biology - size scale of neuron and axon - would correspond to the p-adic length scale assignable to vacuum energy density assignable to cosmological constant and be therefore a fundamental physics length scale.

Keywords: Planck Constant, hierarchy, vacuum degeneracy, twistor lift, TGD framework, cosmological constant.

1 Introduction

The twistor lift of classical TGD [19] is attractive physically but it is still unclear whether it satisfies all constraints. The basic implication of twistor lift would be the understanding of gravitational and cosmological constants. Cosmological constant removes the infinite vacuum degeneracy of Kähler action but because of the extreme smallness of cosmological constant Λ playing the role of inverse of gauge coupling strength, the situation for nearly vacuum extremals of Kähler action in the recent cosmology is non-perturbative.

One motivation for introducing the hierarchy of Planck constants [7, 16] was that the phase transition increasing Planck constant makes possible perturbation theory in strongly interacting system. Nature itself would take care about the converge of the perturbation theory by scaling Kähler coupling strength α_K to α_K/n, $n = h_{eff}/h$. This hierarchy might allow to construct gravitational perturbation theory as has been proposed already earlier. This would for gravitation to be quantum coherent in astrophysical and even cosmological scales.

In the following I describe briefly what is involved.

1. Either $L_\lambda = \sqrt{8\pi/\Lambda}$ or the length L characterizing vacuum energy density as $\rho_{vac} = \hbar/L^4$ or both can obey p-adic length scale hypothesis as analogs of coupling constant parameters. The third option makes sense if the ratio R/l_P of CP_2 radius and Planck length is power of two: it can be indeed chosen to be $R/l_P = 2^{12}$ within measurement uncertainties. $L(now)$ corresponds to the p-adic length scale $L(k) =\propto 2^{k/2}$ for $k = 175$, size scale of neuron and axon.

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2. A microscopic explanation for the vacuum energy realizing strong form of holography is in terms of vacuum energy for radial flux tubes emanating from the source of gravitational field. The independence of energy from the value of $h_{eff} = n$ implies analog of Uncertainty Principle: the product Nn for the number N of flux tubes and the value of n defining the number of sheets of the covering associated with $h_{eff} = n \times h$ is constant. This picture suggests that holography is realized in biology in terms of pixels whose size scale is characterized by L rather than Planck length.

3. Vacuum energy is explained both in terms of Kähler magnetic energy of flux tubes carrying dark matter and of the vacuum energy associated with cosmological constant. The two explanations could be understood as two limits of the theory in which either homological non-trivial and trivial flux tubes dominate. Assuming quantum criticality in the sense that these two phases can tranform to each other, one obtains a prediction for the cosmological constant, string tension, magnetic field, and thickness of the critical flux tubes.

4. An especially interesting result is that in the recent cosmology the size scale of a large neuron would be fundamental physical length scale determined by cosmological constant. This gives additional boost to the idea that biology and fundamental physics could relate closely to each other: the size scale of neuron would not be an accident but "determined in stars" and even beyond them!

2 Twistor lift of TGD, hierarchy of Planck constant, quantum criticality, and p-adic length scale hypothesis

Kähler action is characterized by enormous vacuum degeneracy: any four-surface, whose CP_2 projection is Lagrangian sub-manifold of CP_2 having therefore vanishing induced Kähker form, defines a vacuum extremal. The perturbation theory around canonically imbedded M^4 in $M^4 \times CP_2$ defined in terms of path integral fails completely as also canonical quantization. This led to the construction of quantum theory in "world of classical worlds" (WCW) and to identification of quantum theory as classical physics for the spinor fields of WCW: WCW spinors correspond to fermionic Fock states. The outcome is 4-D spin glass degeneracy realizing non-determinism at classical space-time level [8, 6, 13, 18].

The twistorial lift of TGD is based on unique properties of the twistor spaces of M^4 and CP_2. Note that M^4 allows two notions of twistor space. The first one involves conformal compactification allowing only conformal equivalence class of metrics. Second one is equal to Cartesian product $M^4 \times S^2$. CP_2 has flag manifold $SU(3)/U(1) \times U(1)$ as twistor space having interpretation as the space for the choices for quantization axis of color hypercharge and isospin. Both these spaces Kähler structure (strictly speaking E^4 and S^4 allow it but the notion generalizes to M^4) and there are no others. Therefore TGD is unique both from standard model symmetries and twistorial considerations.

The existence of Kähler structure is a unique hint for how to proceed in the twistorial formulation of classical TGD. One must lift Kähler action to that in the twistor space of space-time surface having also S^2 as a fiber and identify the preferred extremals of this 6-D Kähler action as those of dimensionally reduced Kähler action, which is 4-D Kähler action plus volume term identifiable in terms of cosmological constant. A more detailed construction is discussed in [19]. The most conservative assumption is that the twistorial approach is only an alternative for the space-time formulation: in this formulation preferred extremal property might reduce to twistor space property.

Kähler action gives as fundamental constants the radius $R \simeq 2^{12} l_P$ of CP_2 serving as the TGD counterpart of the unification scale of GUTs and Kähler coupling strength α_K in terms of which gauge coupling strengths can be expressed. Twistor lift gives 2 additional dimensional constants. The radius of S^2 fiber of M^4 twistor space $M^4 \times S^2$ is essentially Planck length $l_P = \sqrt{G/\hbar}$, and the cosmological constant $\Lambda = 8\pi G \rho_{vac}$ defining vacuum energy density is dynamical in the sense that it allows p-adic coupling constant evolution as does also α_K.

There are two options for p-adic coupling constant evolution of cosmological constant.

1. $\rho_{vac} = k_1 \times \hbar/L_p^4$, where $p \simeq 2^k$ characterizes a given level in the p-adic length scale hierarchy for space-time sheets. Here one can in principle allow $k_1 \neq 1$.

2. $\Lambda/8\pi = k_2/L_\Lambda^2 \propto \frac{1}{p_\Lambda^2}$. Also k_2 could differ from unity. Number theoretical universality suggests $k_1 = k_2 = 1$. The that here secondary p-adic length scale is assumed.

The first option seems more natural physically. During very early cosmology $\Lambda R^2/8\pi$ approaches l_P^2/R^2 for the first option, where $R \simeq 2^{12} l_P$ is the size scale of CP_2 so that one has $\Lambda R^2/8\pi \simeq 2^{-24} \simeq 6 \times 10^{-8}$ at this limit. Therefore perturbation theory would fail also in early cosmology near vacuum extremals. In the recent cosmology Λ is extremely small. Note that vacuum energy density would be always smaller than $\hbar/R^4$ and thus by a factor $(l_P/R)^4 \simeq 2^{-48} \simeq 3.6 \times 10^{-15}$ lower than in GRT based cosmology.

It must be made clear that the earlier identification of cosmological constant was in terms of the effective description for the magnetic energy density of the magnetic flux tubes. Magnetic tension would give effective negative pressure. This raises several questions.

1. Which of these identifications is the correct one?

2. Or are both contributions present separately?

3. Do the descriptions correspond to two different limits of the theory corresponding to the dominance of Kähler action (monopole flux tubes) and cosmological term dominating for almost vacuum extremals (no monopole fluxes) respectively.

It seems that second and third option could be correct.

2.1 Hierarchy of Planck constants

One motivation (besides motivations from bio-electromagnetism and Nottale's work [1]) for the hierarchy of Planck constants $h_{eff} = n \times h$ identified as gravitational Planck constants $\hbar_{gr} = GMm/v_0$ at the magnetic flux tubes mediating the gravitational interaction was that it effectively replaces the the large coupling parameter GMm with dimensionless coupling $v_0/c < 1$. This assumes quantum coherence in even astrophysical length and time scales. For gauge interaction corresponding to gauge coupling g one $\hbar_g = Q_1 Q_2 \alpha/v_0$. Also Kähler coupling strength α_K to α_K/n and makes perturbation theory converging for large enough value of n.

The geometric interpretation for $h_{eff} = n \times h$ emerges if one asks how to make the action large for very large value of coupling parameter to guarantee convergence of functional integral.

1. The answer is simple: space-time surfaces are replaced with n-fold coverings of a space-space giving n-fold action and effectively scaling h to $h_{eff} = n \times h$ so that coupling strength scale down by $1/n$. The coverings would be singular in the sense that at the 3-D ends of space-time surface at the boundaries of causal diamond (CD) the sheets co-incide.

2. The branches of the space-time surface would be related by discrete symmetries. The symmetry group could be Galois group in number theoretic vision about finite measurement resolution realized in terms of what I call monadic or adelic geometries [23] [21].

   On the other hand, the twistorial lift suggests that covering could be induced by the covering of the fiber $S^2(X^6)$ by the spheres $S^2(M^4 \times S^2)$ and the twistor space $S^2(SU(3)/U(1) \times U(1))$ defining fibers of twistor spaces of M^4 and CP_2. There would be gauge transformations transforming the light-like parton orbits to each other and the the discrete set would consists of gauge equivalence classes. These two identifications for the symmetries could be equivalent.

$h_{eff} = h_{gr} = n \times h$ would make perturbation theory possible for the space-time surfaces near vacuum extremals. For far from vacuum extremals Kähler action dominates and one would have $h_{eff} = h_{g_K} = n \times h$. This picture would conform with the idea that gravitational interactions are mediated by massless extremals (MEs) topologically condensed at magnetic flux tubes obtained as deformations of string like objects $X^2 \times S^2_I$, S^2_I a homologically trivial geodesic sphere of CP_2. The other interactions could be mediated in the similar manner. The flux tubes would be deformations of $X^2 \times S^2_{II}$, S^2_{II} a homologically non-trivial sphere so that the flux tubes would carry monopole flux.

The enormously small value of cosmological constant would require large value of $h_{eff}/h = n$ explaining the huge value of h_{gr} whereas for other interactions the value of n would be much smaller. One can consider also variants of this working hypothesis. For instance, all long range interactions mediated by massless quanta might correspond to S^2_I. The reason is that very long homologically non-trivial magnetic flux tubes tend to have large energy unless the flux tube thickness is magnetic field is very large (the energy goes as $1/S$).

Quantum criticality would suggest that both phases homologically trivial and non-trivial phases are important. In TGD inspired quantum biology [9] I have considered the possibility that structures with size scaled by $h_{eff}/h = n$ can transform to structures with $n = 1$ but p-adic length scale scaled up by n. Here n would be power of two by p-adic length scale hypothesis.

This would have interpretation in terms of quantum criticality. Homologically non-trivial string like objects with given string tension determined by Kähler action would be transformed to homologically trivial string like objects with the same string tension but determined by the cosmological constant term. This would give a condition on the value of the cosmological constant and thickness of flux tubes to be discussed later.

2.2 Magnetic flux tubes as mediators of interactions

The gravitational Planck constant $\hbar_{gr} = GMm/v_0$ [12, 11, 17, 16] introduced originally by Nottale [1] depends on the large central mass M and small mass m. This makes sense only if h_{gr} characterizes a magnetic flux tube connecting the two masses. Similar conclusion holds true for h_g. This leads to a picture in which mass M has involves a collection of radial flux tubes emanating radially from it. This assumption makes sense in many-sheeted space-time since the fluxes can go to the another space-time sheets through wormhole contacts associated also with elementary particles. For single-sheeted space-time one should have genuine magnetic charges.

This picture encourages a strongly simplified vision about how holography is realized. From center mass flux tubes emanate and in given size scale of the space-time sheet from by the flux tubes having say spherical boundary, the boundary is decomposed of pixels representing finite number of qubits. Each pixel receives one flux tube.

The holographic picture leads to a picture about vacuum energy.

1. Vacuum energy can be expressed as a sum of energies assignable to the flux tubes. Same applies to Kähler interaction energy. The contribution of individual flux tube is proportional to its length given by radius r of the large sphere considered. The total vacuum energy must be proportional to r^3 so that the number of flux tubes must be proportional to r^2. This implies that single flux tube correponds to constant area ΔS of the boundary sphere for given value of cosmological constant. The natural guess is that ΔS is of the same order of magnitude as the area defined by the length scale defined L by the vacuum energy density $\rho_{vac} = \Lambda/8\pi G$ allowing parameterization $\rho_{vac} = k_1\hbar/L^4$.

2. In the recent cosmology one has $\hbar/L(now) \simeq .029$ eV, which equals roughly to $M/10$, where $M = \sum m(\nu_i) \simeq .032 \pm 0.081$ eV is the sum of the three neutrino masses. L is given as a geometric mean

$$L = \sqrt{L_\Lambda l_P} \simeq .42 \times 10^{-4}$$

meters of length scales $l_P = \sqrt{G/\hbar}$ and $L_\Lambda = (8\pi/\Lambda)^{1/2}$. $L(now)$ corresponds to the size scale of large neuron. This is perhaps not an accident.

The area of pixel must be of order $L^2(now)$ suggesting strongly a p-adic length scale assignable with neuron: maybe neuronal system wold realize holography. $L(151) = 10$ nm (cell length scale thickness) and $L(k) \propto \sqrt{p} \simeq 2^{k/2}$ gives the estimate $p \simeq 2^k$, $k = 175$: the p-adic length scale is 4 per cent smaller than $L(now)$.

3. The pixel area would be by a factor $L^2(now)/l_P^2$ larger than Planck length squared usually assumed to define the pixel size but would conform with the p-adic variant of Hawking-Bekenstein law in which p-adic length scale replaces Planck length [10].

The value of the vacuum energy density for a given flux tube is proportional to the value of $h_{eff}/h = n$ by the multi-sheeted covering property. Vacuum energy cannot however depend on n. There are two manners to achieve this: local and global.

1. For the local option the energy of each flux tube would remain invariant under $h \to n \times h$ as would also the number N of flux tubes. This requires that the cross section S of the radial gravitational flux tube to which energy is proportional, scales down as S/n. This looks strange.

2. For the global option flux tubes are not changed but the number N of the radial flux tubes scales down as $N \propto 1/n$: one has $Nn = constant$. In the situation in which Kähler magnetic energy dominant local option demands $S \propto n$ and global option $N \propto 1/n$. Nn constant conditions brings in mind something analogous to Uncertainty Principle. The resolutions characterized by N and n are associated with complementary variables.

The global option applies to both homologically trivial and non-trivial options and is more promising.

2.3 Quantum criticality condition

Strong form of holography in the bosonic sector suggests that the twistorial lift of the action can be written as stringy action for string world sheets assignable to the space-time surface. The string would accompany magnetic flux tube. This implies that the action can be written as

$$Action = TA \ , \tag{2.1}$$

where T is string tension and A is the total area of string world sheets. As a special case this gives the action assignable to single flux tube.

As already argued, quantum criticality suggests that the actions associated with the flux tubes corresponding to the two geodesic spheres S^2_I and S^2_{II} are same if the two surfaces are obtained by performing same deformation for S^2_I meaning that it produces the same induced metric apart from scaling factor. The sphere S^2_i, $i = I, II$, defines transversal coordinates for the cross section of the string like object. Quantum criticality demands that the string tensions obtained by integrating the entire action over the S^2_i are same. A possible interpretation of this symmetry is that at quantum criticality gravitational and gauge interactions in the extreme situations are dual to each other.

1. One expects that a reasonable approximation for the action approximated as the cosmological term for almost vacuum extremals of Kähler action can be written as $Action = T_1 A$, A the area of string world sheet and

$$T_1 = \rho_{vac} S_1 \ , \tag{2.2}$$

where S is the area of the deformation of S^2_I and larger than that of S^2_I if the M^4 projection is topologically and also geometrically in good approximation a sphere. A good guess is that the area is determined by a p-adic length scale, most naturally that associated with $L = \hbar/\rho_{vac}^{1/4}$. As found, in the recent cosmology this would correspond to a size of large neuron and to p-adic length scale $L(175)$ with 4 per cent error.

2. One can estimate also the action for the flux tubes of type II for which Kähler action dominates. From flux conservation the magnetic field for which flux is quantized is in reasonable approximation $B_K = \Phi/S$, where Φ is the quantized Kähler flux for S^2_{II}. Energy density would be $(1/8\pi\alpha_K)B_K^2$. This would give for the string tension

$$T_2 = \frac{k_1}{8\pi\alpha_K}\frac{\Phi_K^2}{S_2} + \rho_{vac}S_2 \quad . \tag{2.3}$$

Here k_1 is a numerical constant, the value of which is near to unity. k_1 might be interpreted as reflecting the effects of the deformation.

Φ_K is flux quantum of Kähler flux normalized to contain the factor g_K. For Cooper pairs flux quantum is $\Phi_0 = \pi\hbar$: in this case the area of flux is area πR^2 of a disk and the charge is $2e$ assignable to Cooper pair. Now the area is the area $4\pi R^2$ of a geodesic sphere and $2e$ is replaced by g_K. This suggest that one has

$$\Phi_K = 8\Phi_0 = 8\pi\hbar \quad . \tag{2.4}$$

holds true.

Induced Kähler form includes Kähler coupling g_K as a scaling factor as do all induced classical gauge fields and Kähler magnetic flux is a multiple of $\Phi_0 = 2\pi$ is the elementary flux quantum. The condition $\Phi = B_K S = n\Phi_0$ guarantees $exp(i\Phi) = 1$. If the cross section were a disk as usually, this would give $B = \Phi/S = \Phi/\pi R^2$. For spherical cross section one has $B = \Phi/4\pi R^2$.

3. The condition

$$S_1 = xS_2 \tag{2.5}$$

is a natural guess. Since $T_2 > T_1$ for $k = 1$, $k > 1$ must hold true. At the limit when S_2 becomes the area $S_2 = 4\pi R^2$ of CP_2 geodesic sphere, one has from the condition

$$T_1 = T_2 \tag{2.6}$$

the result

$$x = \frac{S_1}{S_2} = 1 + \frac{k_1}{8\pi\alpha_K} \times \frac{\Phi_0^2}{16\pi^2} \times (\frac{L_{min}}{R})^4 \quad . \tag{2.7}$$

Here L parameterizes the value vacuum energy density as $\rho_{vac} = \hbar/L^4$ and L_{min} corresponds to the minimum value of L. A natural estimate is $L_{min} \sim R$. p-Adic length scale hypothesis $\Lambda = R^2 p_\Lambda$ and the relationship $G/R^2 = 2^{-12}$ to be discussed later together with $L_{min}/R \geq 1$ give $p_\Lambda > 2^{15}\pi$ giving $p_\Lambda \geq 2^{17}$. A tempting working hypothesis is that the ratio S_1/S_2 obeys this formula quite generally.

4. The condition $T_1 = T_2$ assuming $L_{min} = R$ would give for the area S_2, string tension, and Kähler magnetic field the expressions

$$\begin{aligned} S_2 &= k \times \Phi\sqrt{\frac{\hbar}{\rho_{vac}}} = k\Phi_0 L^2 \ , \\ T_1 &= T_2 = T = \frac{k}{L^2} \ , \\ B_K &= \frac{1}{kL^2} \ , \\ k &= \frac{1}{\sqrt{x-1}} \times \sqrt{\frac{k_1}{8\pi\alpha_K}} = \frac{4\pi}{\Phi_0} \ . \end{aligned} \tag{2.8}$$

What looks amazing that the dependences on Kähler coupling strength α_K and parameter k_1 disappear. The formulas reduce to

$$\begin{aligned} S_2 &= 4\pi L^2 \ , \\ T_1 &= T_2 = T = \frac{k}{L^2} = \frac{4\pi}{\Phi_0 L^2} \ , \\ B_K &= \frac{\Phi_0}{4\pi L^2} \ . \end{aligned} \tag{2.9}$$

The area of critical flux tubes would be completely determined in terms of the vacuum energy density. For this flux tube thickness the two kinds of flux tubes could transform to each other without change in energy. In the recent cosmology the flux tube thickness would be of the order of $L(175)$ and Kähler magnetic field would correspond to magnetic field of order Earth's magnetic field.

5. In recent cosmology the value of B_K (more precisely, $g_K B_K$ using ordinary conventions) at criticality would be

$$B_K = \frac{\Phi_0}{4\pi}\frac{1}{L^2(175)} \ .$$

B_K corresponds to the U(1) magnetic field in standard model and is therefore as such not the ordinary magnetic field. For S^2_{II} Kähler magnetic field is non-vanishing. If Z^0 field vanishes, classical em field (with e included as normalization factor) equals to $\gamma = 3J$, where J is K"ahler induced Kähler form (see [**?**]append. One has

$$B_K = \frac{eB_{em}}{3} \ . \tag{2.10}$$

6. An interesting question is whether one could identify physically the ordinary magnetic field assignable to the critical Kähler magnetic field.

Earth's magnetic field $B_E = .5$ Gauss corresponds to magnetic length $L_B = \sqrt{\hbar e B} = 5\mu\text{m}$. Endogenous magnetic field $B_{end} \simeq 2B_E/5$ explaining the findings of Blackman [**?**]Blackman about the effects of ELF em fields on vertebrate brain in terms of cyclotron transitions corresponds to $L_B = 12.5\ \mu$m to be compared with the p-adic length scale $L(175) = 40\ \mu$m. Also these findings served as inspiration of $h_{eff} = n \times h$ hypothesis [**?**]lianPB,lianPN.

I have assigned large Planck constant phases with the flux tubes of B_{end}, which have however remained somewhat mysterious entity. Could B_{end} correspond to quantum critical value of B_K and therefore relate directly to cosmology?

One can check whether $B_K = eB_{end}/3$ holds true. The hypothesis would give

$$eB_{end} = \frac{1}{L_B^2} = 3 \times \frac{\Phi_0}{4\pi\hbar}\frac{1}{L^2(175)} \quad .$$

implying

$$r = \frac{L^2(175)}{L_B^2} = \frac{3\Phi_0}{4\pi\hbar} \quad .$$

The left hand side gives $r = 10.24$. For $\Phi_0 = 8\pi\hbar$ the right hand side gives $r = 6$. $B_E = .34$ Gauss left and right hand sides of the formula are identical.

7. One can wonder the proposed formulas might be exact for preferred extremals satisfying extremely powerful conditions to guarantee strong form of holography. This would require in both cases bundle structure with transversal cross section action as fiber. In the case of extremals of Kähler this would require that induce Kähler magnetic field is covariantly constant.

2.4 Two variants for p-adic length scale hypothesis for cosmological constant

There are two options for the dependence string tension T and and area S of the cross section of the flux tube on p-adic length scale: either $L_\Lambda = \sqrt{8\pi/\Lambda}$ or $L = (\hbar/\rho_{vac})^{1/4}$ satisfies p-adic length scale hypothesis. The "boundary condition" is that the radius of flux tubes would be of the order of neutron size scale in recent cosmology.

1. $L(now) = L_p$ scaling gives

$$S = S(now)\frac{p(now)}{p}$$

with $p_{now} \simeq 2^{175}$ by p-adic length scale hypothesis. $L(175)$ is by about 4 per cent smaller than the Compton length assignable to $\hbar/L(now) = .029$ eV.

If one wants $L(now) = L(175)$ exactly, one must increase R by 4 per cent, which is allowed by p-adic mass calculations fixing the value of R only with 10 per cent accuracy. Indeed, the second order contribution in p-adic mass calculations is uncertain and the ratio of maximal and minimal values of R is $R_{max}/R_{min} = \sqrt{6/5} \simeq 1.1$.

As already noticed, $L(now)$ corresponds to neutron size scale, which conforms with p-adic mass calculations since the radius of flux tubes would correspond to p-adic length scale. This option looks more natural and suggest a profound connection with biology and fundamental physics.

2. $L_\lambda \equiv \sqrt{8\pi/\Lambda}$ could be proportional to secondary p-adic length scale $L(2, p_\Lambda) \equiv \sqrt{p_\Lambda} L_{p_\Lambda}$. The scaling law

$$L_\Lambda \propto \frac{p_\Lambda(now)}{p_\Lambda}$$

gives

$$L_\Lambda^2(now) = \frac{8\pi}{\Lambda(now)} = (\frac{p}{p(now)})^2 \times \frac{L^4(now)}{l_P^2} \quad .$$

$L_\Lambda(now) \sim 50$ Gly (roughly the age of the Universe) holds true. Note that one has $S \propto \sqrt{p_{now}/p}S(now)$ and $T = T_{now}\sqrt{p/p_{now}}$.

$1/p$-dependence for the string tension T looks more natural in light of p-adic mass calculations. One must however notice that the $L = L(175)$ is 4 per cent small than $L(now)$.

The density of dark energy is uncertain by few per cent at least and one can ask whether $L(now) = L(175)$ could fix it. The change induced to ρ_{vac} by that of $L(now)$ is

$$\frac{\Delta\rho_{vac}}{\rho_{vac}} = -4\frac{\Delta L(now)}{L(now)}$$

and the reduction L by 4 per cent would reduce vacuum density by 16 per cent, which looks rather large change. The value of R can be determined by 10 per cent accuracy and the increase of R by four per cent is another manner to achieve $L(now) = L(175)$.

One can of course ask, whether both variants of p-adic length scale hypotheis could be correct. The reader night protest that this leads to the murky waters of p-adic numerology.

1. Could L_Λ be proportional to the secondary p-adic length scale $L(p,2) = \sqrt{p}L_p = 2^{k/2} \times L(k)$ associated with p characterizing L such that the proportionality constant is power of $\sqrt{2}$. The application of the condition defining L in terms of $L_\Lambda^2 = 8\pi/\Lambda$ gives

$$L_\Lambda^2 = \frac{L^4}{l_P^2} \quad .$$

Using $L_\Lambda = \sqrt{p_\Lambda}R$ and taking square roots, this gives

$$\sqrt{p_\Lambda} = pk^2 \quad , \quad k = \frac{R_{CP_2}}{l_P} \quad .$$

This conforms with the p-adic length scales hypothesis in its simplest form if k is power of $\sqrt{2}$.

2. The estimate from p-adic mass calculations for $r \equiv R(CP_2)/l_P$ is $r = 4.167 \times 10^3$ and is 2 per cent larger than 2^{12}. Could the $R(CP_2)/l_P = 2^{12}$ for the radii of CP_2 and M^4 twistorial sphere be an exact formula between fundamental length scales? As noticed, the second order contribution in p-adic mass calculations is uncertain by 10 per cent. This would allow the reduction of $R(CP_2)$ by 2 percent.

This looks an attractive option. The bad news is that the *increase* of $R(CP_2)$ by about 4 per cent to achieve $L(now) = L(175)$ is in conflict with its *reduction* by 2 per cent to achieve $R(CP_2)/l_P = 2^{12}$: this would reduce $L(175)$ by 2 per cent and increase ρ_{vac} by about 8 per cent. ρ_{vac} is however an experimental parameter depending on theoretical assumption and it value could allow this tuning. Therefore

$$\begin{aligned} \frac{R_{CP_2}}{l_P} &= 2^{12} \quad , \\ p_\Lambda &= 2^{48} \times p^2 \quad . \end{aligned} \tag{2.10}$$

is an attractive option fixing completely the value of $R(CP_2)/l_P$ and predicting relation between cosmological scale L_Λ and a fundamental scale in recent biology, which could be assigned to magnetic flux tubes assignable to axons. Note that for $k_{now} = 175$ the value of $k_\Lambda = k_{now} + 48$ is $k_\Lambda = 175 + 48 = 223$ which corresponds to p-adic length scale of 64 m.

3. Needless to say that one must be take these estimates with a big grain of salt. Number theoretical universality suggests that one might apply number theoretical constraints to fundamental constants like R, l_P, and Λ but one should be very critical concerning the values of empirical parameters such as ρ_{vac} depending on theoretical assumptions. Furthermore, p-adic length scale hypothesis is applied at the level of imbedding space metric and one can ask whether it actually applies for the induced metric (Robertson-Walker metric now).

3 ZEO, cosmological constant, and magnetic flux tubes

As I started to work with TGD around 1977, I adopted path integral and canonical quantization as the first approaches. One of the first guesses for the action principle was 4-volume in induced metric giving minimal surfaces as preferred extremals. The field equations are generalization of massless field equation and at least in the case of string models Hamiltonian formalism and second quantization is possible. The reason why for giving up this option was that for space-time surfaces of infinite duration the volume is infinite. This is not pleasant news concerning quantization since subtraction of exponent of infinite volume factor looked really ugly thing to do. At that time I did of course have no idea about ZEO and CDs.

For Kähler action there is however infinite vacuum degeneracy. All space-time surfaces with CP_2 projection, which is Lagrangian manifold (at most 2-dimensional) are vacuum extremals and canonical quantization fails completely. This implies classical non-determinism also for non-vacuum extremals obtained as small deformations of vacuum extremals. This feature seems to have nice implications such as 4-D spin glass degeneracy. It would however make WCW metric singular for nearly vacuum extremals.

3.1 Twistor lift brings volume term back

Concerning volume term the situation changed as I introduced twistor lift of TGD. One could say that twistor lift forces cosmological constant.

1. The twistorial lift of Kähler action is 6-D Kähler action for the twistor space $T(X^4)$ of space-time surface X^4. The analog of twistor structure would be induced from the product $T(M^4) \times T(CP_2)$, of twistor spaces $T(M^4) = M^4 \times S^2$ of M^4 and $T(CP_2) = SU(3)/U(1) \times U(1)$ of CP_2 having Kähler structure so that the induction of Kähler structure to $T(X^4)$ makes sense. Besides M^4 and CP_2 only the spaces E^4 and the S^4, which are variants of M^4 have twistor space with Kähler structure or analog of it. The induction conditions would imply dimensional reduction so that the 6-D Kähler action for the twistorial lift would reduce to 4-D Kähler action plus volume term identifiable in terms of cosmological constant Λ.

2. 4-D Kähler action has Kähler coupling strength α_K as coupling parameter and volume term has coefficient $1/L^4$ identifiable in terms of cosmological constant
$$\frac{1}{L^4} \equiv \frac{\Lambda}{8\pi l_P^2} \ .$$
$l_P = \sqrt{G/\hbar}$ would correspond to the radius of twistor sphere for M^4 and thus becomes fundamental length scale of twistorially lifted TGD besides radius of CP_2. Note that the radius of twistor sphere of CP_2 is naturally CP_2 radius.

L is in the role of coupling constant and expected to obey discrete p-adic coupling constant evolution $L \propto \sqrt{p}$, prime or prime near power of two if p-adic length scale hypothesis is accepted. In the recent cosmology L could correspond to the p-adic length scale $L(175) \simeq 40\ \mu$m, the size of large neuron.

L $\simeq 40\mu$m corresponds to the energy scale $E = 1/L \simeq .031$ eV, which is thermal energy at temperature of 310 K (40 C) - the physiological temperature. A deep connection with quantum biology is suggestive. Also the energy scale defined by cell membrane potential is in this energy scale. This energy scale about 10 times smaller than the mass scale of neutrinos.

Also $L_\Lambda = \sqrt{8\pi/\Lambda}$ would satisfy p-adic coupling constant evolution as already discussed. Now the p-adic length scale would be secondary p-adic length scale $L_\Lambda = L(2,p) = \sqrt{p} \times (R/l_P)$, l_P Planck length. p-Adic length scale hypothesis demands that R/l_P - the ratio for the radii of CP_2 and twistor sphere is power of 2. p-Adic mass calculations indeed allow this ratio can be indeed chosen to be equal to $R/l_P = 2^{12}$.

3.2 What happens to extremals of Kähler action?

What happens to the extremals of Kähler action when volume term is introduced?

1. The known non-vacuum extremals [3, 4] such as massless extremals (topological light rays) and cosmic strings are minimal surfaces so that they remain extremals and only the classical Noether charges receive an additional volume term. In particular, string tension is modified by the volume term. Homologically non-trivial cosmic strings are of form $X^2 \times Y^2$, where $X^2 \subset M^4$ is minimal surface and $Y^2 \subset CP_2$ is complex 2-surface and therefore also minimal surface.

2. Vacuum degeneracy is in general lifted and only those vacuum extremals, which are minimal surfaces survive as extremals.

For CP_2 type vacuum extremals [3, 4] the roles of M^4 and CP_2 are changed. M^4 projection is light-like curve, and can be expressed as $m^k = f^k(s)$ with light-likeness conditions reducing to Virasoro conditions. These surfaces are isometric to CP_2 and have same Kähler and symplectic structures as CP_2 itself. What is new as compared to GRT is that the induced metric has Euclidian signature. The interpretation is as lines of generalized scattering diagrams. The addition of the volume term forces the random light-like curve to be light-like geodesic and the action becomes the volume of CP_2 in the normalization provided by cosmological constant. What looks strange is that the volume of any CP_2 type vacuum extremals equals to CP_2 volume but only the extremal with light-like geodesic as M^4 projection is extremal of volume term.

Consider next vacuum extremals, which have vanishing induced Kähler form and are thus have CP_2 projection belonging to at most 2-D Lagrangian manifold of CP_2 [3, 4].

1. Vacuum extremals with 2-D projections to CP_2 and M^4 are possible and are of form $X^2 \times Y^2$, X^2 arbitrary 2-surface and Y^2 a Lagrangian manifold. Volume term forces X^2 to be a minimal surface and Y^2 is Lagrangian minimal surface unless the minimal surface property destroys the Lagrangian character.

   If the Lagrangian sub-manifold is homologically trivial geodesic sphere, one obtains string like objects with string tension determined by the cosmological constant alone.

   Do more general 2-D Lagrangian minimal surfaces than geodesic sphere exist? For general Kähler manifold there are obstructions but for Kähler-Einstein manifolds such as CP_2, these obstructions vanish (see `http://tinyurl.com/gtkpya6`). The case of CP_2 is also discussed in the slides "On Lagrangian minimal surfaces on the complex projective plane" (see `http://tinyurl.com/jrhl6gy`). The discussion is very technical and demonstrates that Lagrangian minimal surfaces with all genera exist. In some cases these surfaces can be also lifted to twistor space of CP_2.

2. More general vacuum extremals have 4-D M^4 projection. Could the minimal surface condition for 4-D M^4 projection force a deformation spoiling the Lagrangian property? The physically motivated expectation is that string like objects give as deformations magnetic flux tubes for which string is thicknened so that it has a 2-D cross section. This would suggest that the deformations of string like objects $X^2 \times Y^2$, where Y^2 is Lagrangian minimal surface, give rise to homologically trivial magnetic flux tubes. In this case Kähler magnetic field would vanish but the spinor connection of CP_2 would give rise to induced magnetic field reducing to some U(1) subgroup of U(2). In particular, electromagnetic magnetic field could be present.

Cosmological constant is expected to obey p-adic evolution and in very early cosmology the volume term becomes large. What are the implications for the vacuum extremals representing Robertson-Walker metrics having arbitrary 1-D CP_2 projection? [3, 4, 5].

1. The TGD inspired cosmology involves primordial phase during a gas of cosmic strings in M^4 with 2-D M^4 projection dominates. The value of cosmological constant at that period could be fixed from the condition that homologically trivial and non-trivial cosmic strings have the same value of string tension. After this period follows the analog of inflationary period when cosmic strings condense are the emerging 4-D space-time surfaces with 4-D M^4 projection and the M^4 projections of cosmic strings are thickened. A fractal structure with cosmic strings topologically condensed at thicker cosmic strings suggests itself.

2. GRT cosmology is obtained as an approximation of the many-sheeted cosmology as the sheets of the many-sheeted space-time are replaced with region of M^4, whose metric is replaced with Minkowski metric plus the sum of deformations from Minkowski metric for the sheet. The vacuum extremals with 4-D M^4 projection and arbitrary 1-D projection could serve as an approximation for this GRT cosmology. Note however that this representability is not required by basic principles.

3. For cosmological solutions with 1-D CP_2 projection minimal surface property forces the CP_2 projection to belong to a geodesic circle S^1. Denote the angle coordinate of S^1 by Φ and its radius by R. For the future directed light-cone M^4_+ use the Robertson-Walker coordinates $(a = \sqrt{m_0^2 - r_M^2}, r = ar_M, \theta, \phi)$, where (m^0, r_M, θ, ϕ) are spherical Minkowski coordinates. The metric of M^4_+ is that of empty cosmology and given by $ds^2 = da^2 - a^2 d\Omega^2$, where Ω^2 denotes the line element of hyperbolic 3-space identifiable as the surface $a = constant$.

   One can can write the ansatz as a map from M^4_+ to S^1 given by $\Phi = f(a)$. One has $g_{aa} = 1 \rightarrow g_{aa} = 1 - R^2(df/da)^2$. The field equations are minimal surface equations and the only non-trivial equation is associated with Φ and reads $d^2f/da^2 = 0$ giving $\Phi = \omega a$, where ω is analogous to angular velocity. The metric corresponds to a cosmology for which mass density goes as $1/a^2$ and the gravitational mass of comoving volume (in GRT sense) behaves is proportional to a and vanishes at the limit of Big Bang smoothed to "Silent whisper amplified to rather big bang" for the critical cosmology for which the 3-curvature vanishes. This cosmology is proposed to results at the limit when the cosmic temperature approaches Hagedorn temperature [5].

4. The TGD counterpart for inflationary cosmology corresponds to a cosmology for which CP_2 projection is homologically trivial geodesic sphere S^2 (presumably also more general Lagrangian (minimal) manifolds are allowed). This cosmology is vacuum extremal of Kähler action. The metric is unique apart from a parameter defining the duration of this period serving as the TGD counterpart for inflationary period during which the gas of string like objects condensed at space-time surfaces with 4-D M^4 projection. This cosmology could serve as an approximate representation for the corresponding GRT cosmology.

   The form of this solution is completely fixed from the condition that the induced metric of $a = constant$ section is transformed from hyperbolic metric to Euclidian metric. It should be easy to check whether this condition is consistent with the minimal surface property.

3.3 ZEO makes volume term possible

Therefore the volume term, which I gave up 38 years ago, has creeped back to the theory! Should one forget twistor lift because of its infinite value for space-time surfaces of infinite duration? No! ZEO saves the situation.

In ZEO given CD defines a sub-WCW consisting of space-time surfaces inside CD. This implies that the volumes for the M^4 projections of allowed space-time surfaces are smaller than CD volume having the order of magnitude $L^4(CD)$, $L(CD)$ is the temporal distance between the tips of CD (one has $c = 1$). I have also proposed that $L(CD)$ is quantized in multiples of integers, primes or primes near power of two so that the identification might make sense. $L(CD) = L$ is not possible due to the small value 40 μm of L but $L(CD) = L_\Lambda$ could make sense.

The preferred extremal property realizing SH poses extremely strong constraints on the value of total action and it should force the phase defined by action to be stationary so that interference effects would be practically absent. The question is how this could be realized.

1. The most general possibility is that the phase of the vacuum functional can be large but is localized around very narrow range of values. The imaginary part of the action S_{Im} for preferred extremals should be around values $S_{Im} = A_0+n2\pi$. Standard Bohr orbitology indeed assumes the quantization of action in this manner. One could also argue that just the absence of destructive interference demands Bohr quantization of the action in the vacuum functional. Whether preferred extremal property indeed gives rise to this kind of Bohr quantization, is an open problem. The real exponent of the vacuum functional should in turn be large enough and positive values are favored. They are however bounded in ZEO because of the finite size of CDs.

2. To proceed further one must say something about the value spectrum of α_K. In the most general situation α_K is complex number: the proposal of [20] is that the discrete p-adic coupling constant evolution for $1/\alpha_K$ corresponds to a complex zero $s = 1/2 + iy$ of Riemann zeta: also the trivial real zeros can be considered. For large values of y the imaginary part of y would determine $1/\alpha_K$ and $Re(s) = 1/2$ would be responsible for complex value of α_K. This makes sense since quantum TGD can be regarded formally as a complex square root of thermodynamics.

3. Denote by $S = S_{Re} + iS_{Im}$ the exponent of vacuum functional. For complex values of $1/\alpha_K$ S_{Im} receives a contribution also from the Euclidian regions and S_{Re} a contribution also from the Minkowskian regions. For S_{Im} the contributions should obey the condition

$$S_{Im} = S_{Im}(M) + S_{Im}(E) \simeq A_0 + n2\pi \tag{3.1}$$

to achieve constructive interference.

For real parts the condition $S_{Re} = S_{Re}(M)+S_{Re}(E)$ must be small if negative. Large positive values of S_{Re} are favored. S_{Re} automatically selects the configurations which contribute most and among these configurations the phase $exp(iS_{Im})$ must be stationary. The conditions for S_{Im} relate the values of action in the Euclidian and Minkowskian regions. If α_K is real, one has $S_{Im}(M) \simeq A_0+n2\pi$ and $S_{Re}(E)$ small if negative and Euclidian and Minkowskian regions effectively decouple in the conditions. It seems that complex values of α_K are indeed needed.

4. $S_{Re}(E) = S_{Re}(M) + S_{Re}(E)$ receives a positive contribution from Euclidian regions. Minkowskian regions a contributions for complex value α_K. Both positive and negative contributions are present and the character of these contributions depends on sign of the imaginary part of α_K. Depending on the sign factor ± 1 of $Im(1/\alpha_K)$ Minkowskian regions give negative (positive) contribution from the space-time regions dominated by Kähler electric fields and positive (negative) contribution from the volume term and the regions dominated by Kähler magnetic field.

The option "+" for which Kähler magnetic action and volume term give positive contribution to $S_{Re}(M)$ looks physically attractive. "+" option would have no problems in ZEO since the contribution to S_{Re} would be automatically positive but bounded by the finite size of CD: this would give a deep reason for the notion of CD (also the realization of super-symplectic symmetries gives it). For "-" option Minkowskian regions containing Kähler electric fields would be essential in order to obtain $S_{Re} > 0$: Kähler magnetic fields would not be favored and the unavoidable volume term would give wrong sign contribution to $S_{Re} > 0$.

3.4 The condition $S_{Im} \leq \pi/2$ is not realistic

One can look what the mere volume term contributes to S_{Im} assuming $S_{Im} \leq \pi/2$. Volume term dominates for near to vacuum extremals with a small Kähler action: in particular, for string like objects $X^2 \times S^2$, S^2 a homologically trivial geodesic sphere with vanishing induced Kähler form. It turns out that these conditions are not physically plausible and that $S_{Im} \simeq A_0 + n2\pi$ is the only realistic option.

1. Cosmological constant (parametrizable using the scale L) together with the finite size of CD gives a very stringent upper bound for the volume term of the action: $A = vol(X^4)/L^4$. The rough estimate is that for the largest CDs involved the volume action is not much larger than $L^4\pi/2$ in the recent cosmology. In the recent cosmology L would be only about 40 μm so that the bound is extremely strong! and suggests that $S_{Im} < \pi/2$ is not a realistic condition.

2. $L(CD) = L$ is certainly excluded. Can one have $L(CD) = L_\Lambda$? How can one achieve space-time volume not much larger that L^4 for space-time surfaces with duration $L(CD)$? Could magnetic flux tubes help! For the simplest string like objects $X^2 \times Y^2$, where $X^2 \subset M^4$ is minimal surface and Y^2 a 2-D surface (complex sub-manifold of CP_2) the volume action is essentially

$$Action = \frac{V}{l_P^2 L_\Lambda^2} = \frac{Area(X^2)}{L_\Lambda^2} \times \frac{Area(Y^2)}{l_P^2} \quad . \tag{3.2}$$

   The conservative condition for the absence of destructive interference is roughly $Action < \pi/2$.

3. To get a more concrete idea about the situation one can use the parametrization

$$Area(string) = L(CD) \times L(string) \quad , \quad Area(Y^2) = x \times 4\pi R^2 \quad . \tag{3.3}$$

   x is a numerical parameter, which can be quite large for deformations of cosmic strings with thick transversal M^4 projection. The condition for the absence of destructive interference is roughly

$$\frac{L(CD)\times L(string))}{L_\Lambda^2} \times x \times \frac{4\pi R^2}{l_P^2} < \frac{\pi}{2} \quad . \tag{3.4}$$

   For $L(string) \ll L(CD)$ one can have space-time surfaces of temporal duration $L(CD) = L_\Lambda$. For these the condition reduces to

$$\begin{aligned} & y \times x < \pi \frac{l_P^2}{4\pi R^2} = 2^{-13}\pi \quad , \\ & y \equiv \frac{L(string)}{L_\Lambda} \quad . \end{aligned} \tag{3.5}$$

For deformations the transversal area of string like object can be also chosen to be considerably larger than the area of geodesic sphere. For flux tubes of length of order 1 AU the one have $y \sim 10^{-16}$. This would require $x \leq 10^{13}$. This would correspond to a radius $L(Y^2)$ about $10^6 R$ much smaller than required.

For $L(string) \sim L$ this would give $y \sim 10^{-31}$ giving $x \leq 10^{28}$ $L(Y^2) \leq 10^{14} R$, which corresponds to elementary particle scale. Still this fails to fit with intuitive expectations, which are of course inspired by the standard positive energy ontology.

4. One could try to invent mechanisms making volume term small. The required reduction would be enormous. This does look sensible. One can have vacuum extremals of Kähler action for which CP_2 projection is a geodesic line: $\Phi = \omega t$. The time component $g_{tt} = 1 - R^2\omega^2$ of the flat metric can be arbitrarily small so that the volume proportional to $\sqrt{gtt}$ can be arbitrarily small. One expects that this happens in early cosmology but as a general mechanism this is not plausible. Also very rapidly rotating string like objects with small area of string world sheet are in principle possible but do not represent a realistic option.

The cautious conclusion is that Bohr quantization $S_{Im} \simeq A_0 + n2\pi$ is the only sensible option. The hypothesis that the coupling constant evolution for $1/\alpha_K$ is given in terms of zeros of Riemann zeta seems to be consistent with this picture and correlates the values of actions in Minkowskian and Euclidian regions.

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