

Does GRT Really Allow Gravitational Radiation?

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Abstract

The general view in General Relativity is that pure gravitational radiation corresponds to a situation in which curvature tensor equals to Weyl tensor. The problem is that gravitational radiation would correspond to vanishing energy momentum tensor and vanishing energy and momentum. This is additional argument against GRT space-time besides the loss of standard conservation laws assignable to Poincare symmetries, which led to TGD. In TGD framework the so called "massless extremals" (MEs) are extremals of practically any general coordinate invariant action such that curvature tensor reduces to Weyl tensor and interpretation as gravitational radiation is possible. Energy momentum tensor is however non-vanishing and light-like as expected and assigns well-defined four-momentum to gravitational radiation which is also classically quantized to massless quanta.

Keywords: General relativity, gravitational radiation, Weyl tensor, spacetime, TGD framework.

In Facebook discussion Niklas Grebäck mentioned Weyl tensor and I learned something that I should have noticed long time ago. Wikipedia article (see https://en.wikipedia.org/wiki/Weyl_tensor) lists the basic properties of Weyl tensor as the traceless part of curvature tensor, call it R . Weyl tensor C is vanishing for conformally flat space-times. In dimensions $D=2,3$ Weyl tensor vanishes identically so that they are always conformally flat: this obviously makes the dimension $D = 3$ for space very special. Interestingly, one can have non-flat space-times with nonvanishing Weyl tensor but the vanishing Schouten/Ricci/Einstein tensor and thus also with vanishing energy momentum tensor.

The rest of curvature tensor R can be expressed in terms of so called Kulkarni-Nomizu product $P \cdot g$ of Schouten tensor P and metric tensor g : $R = C + P \cdot g$, which can be also transformed to a definition of Weyl tensor using the definition of curvature tensor in terms of Christoffel symbols as the fundamental definition. Kulkarni-Nomizu product \cdot is defined as tensor product of two 2-tensors with symmetrization with respect to first and second index pairs plus antisymmetrization with respect to second and fourth indices.

Schouten tensor P is expressible as a combination of Ricci tensor Ric defined by the trace of R with respect to the first two indices and metric tensor g multiplied by curvature scalar s (rather than R in order to use index free notation without confusion with the curvature tensor). The expression reads as

$$P = \frac{1}{D-2} \left[Ric - \frac{s}{2(D-1)} g \right] .$$

Note that the coefficients of Ric and g differ from those for Einstein tensor. Ricci tensor and Einstein tensor are proportional to energy momentum tensor by Einstein equations relate to the part.

Weyl tensor is assigned with gravitational radiation in GRT. What I see as a serious interpretational problem is that by Einstein's equations gravitational radiation would carry no energy and momentum in absence of matter. One could argue that there are no free gravitons in GRT if this interpretation is adopted! This could be seen as a further argument against GRT besides the problems with the notions of energy and momentum: I had not realized this earlier.

Interestingly, in TGD framework so called massless extremals (MEs) [1, 3, 2] are four-surfaces, which are extremals of Kähler action, have Weyl tensor equal to curvature tensor and therefore would have interpretation in terms of gravitons. Now these extremals are however non-vacuum extremals.

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1. Massless extremals correspond to graphs of possibly multi-valued maps from M^4 to CP_2 . CP_2 coordinates are arbitrary functions of variables $u = k \cot m$ and $w = \epsilon \cdot m$. k is light-like wave vector and ϵ space-like polarization vector orthogonal to k so that the interpretation in terms of massless particle with polarization is possible. ME describes in the most general case a wave packet preserving its shape and propagating with maximal signal velocity along a kind of tube analogous to wave guide so that they are ideal for precisely targeted communications and central in TGD inspired quantum biology. MEs do not have Maxwellian counterparts. For instance, MEs can carry light-like gauge currents parallel to them: this is not possible in Maxwell's theory.
2. I have discussed a generalization of this solution ansatz so that the directions defined by light-like vector k and polarization vector ϵ orthogonal to it are not constant anymore but define a slicing of M^4 by orthogonal curved surfaces (analogs of string world sheets and space-like surfaces orthogonal to them). MEs in their simplest form at least are minimal surfaces and actually extremals of practically any general coordinate invariance action principle. For instance, this is the case if the volume term suggested by the twistorial lift of Kähler action [4] and identifiable in terms of cosmological constant is added to Kähler action.
3. MEs carry non-trivial induced gauge fields and gravitational fields identified in terms of the induced metric. I have identified them as correlates for particles, which correspond to pairs of wormhole contacts between two space-times such that at least one of them is ME. MEs would accompany to both gravitational radiation and other forms or radiation classically and serve as their correlates. For massless extremals the metric tensor is of form

$$g = m + a\epsilon \otimes \epsilon + bk \otimes k + c(\epsilon \otimes kv + k \otimes \epsilon) ,$$

where m is the metric of empty Minkowski space. The curvature tensor is necessarily quadrilinear in polarization vector ϵ and light-like wave vector k (light-like ifor both M^4 and ME metric) and from the general expression of Weyl tensor C in terms of R and g it is equal to curvature tensor: $C = R$.

Hence the interpretation as graviton solution conforms with the GRT interpretation. Now however the energy momentum tensor for the induced Kähler form is non-vanishing and bilinear in velocity vector k and the interpretational problem is avoided.

What is interesting that also at GRT limit cosmological constant saves gravitons from reducing to vacuum solutions. The deviation of the energy density given by cosmological term from that for Minkowski metric is identifiable as gravitonic energy density. The mysterious cosmological constant would be necessary for making gravitons non-vacuum solutions. The value of graviton amplitude would be determined by the continuity conditions for Einstein's equations with cosmological term. The p-adic evolution of cosmological term predicted by TGD is however difficult to understand in GRT framework.

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