Article

More about Induced Spinor Fields & TGD Counterpart for Higgs

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Abstract

The well-definedness of em charge has turned out to be very powerful guideline in the understanding of the details of fermionic dynamics. Although induced spinor fields have also a part assignable space-time interior, the spinor modes at string world sheets determine the fermionic dynamics in accordance with strong form of holography (SH). The twistor lift combined with strong form of holography (SH) and the condition that em charge is well-defined allows to formulate the action principle for induced spinor fields in more detail. Well-definedness of em charge implies that em charged particles can be assigned with string world sheets accompanying magnetic flux tubes assignable to homologically non-trivial geodesic sphere and neutrinos with those associated with homologically trivial geodesic sphere. This explains why neutrinos are so light and why dark energy density corresponds to neutrino mass scale, and provides also new insight about color confinement. The formalism works only for imbedding space dimension $D = 8$. This is due the fact that the number of vector components is the same as the number of spinor components of fixed chirality for $D = 8$ and corresponds directly to the octonionic triality. p-Adic thermodynamics predicts elementary particle masses in excellent accuracy without Higgs vacuum expectation: the problem is to understand fermionic Higgs couplings. The observation that CP_2 part of the modified gamma matrices gives rise to a term mixing M^4 chiralities contain derivative allows to understand the mass-proportionality of the Higgs-fermion couplings at QFT limit.

Keywords: Induced spinor field, TGD counterpart, Higgs particle, fermionic dynamics.

1 Introduction

The understanding of the modified Dirac equation and of the possible classical counterpart of Higgs field in TGD framework is not completely satisfactory. The emergence of twistor lift of Kähler action [4] [5] inspired a fresh approach to the problem and it turned out that a very nice understanding of the situation emerges.

More precise formulation of the Dirac equation for the induced spinor fields is the first challenge. The well-definedness of em charge has turned out to be very powerful guideline in the understanding of the details of fermionic dynamics. Although induced spinor fields have also a part assignable space-time interior, the spinor modes at string world sheets determine the fermionic dynamics in accordance with strong form of holography (SH).

The well-definedness of em charged is guaranteed if induced spinors are associated with 2-D string world sheets with vanishing classical W boson fields. It turned out that an alternative manner to satisfy the condition is to assume that induced spinors at the boundaries of string world sheets are neutrino-like and that these string world sheets carry only classical W fields. Dirac action contains 4-D interior term and 2-D term assignable to string world sheets. Strong form of holography (SH) allows to interpret 4-D spinor modes as continuations of those assignable to string world sheets so that spinors at 2-D string world sheets determine quantum dynamics.

Twistor lift combined with this picture allows to formulate the Dirac action in more detail. Well-definedness of em charge implies that charged particles are associated with string world sheets assignable to the magnetic flux tubes assignable to homologically non-trivial geodesic sphere and neutrinos with

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those associated with homologically trivial geodesic sphere. This explains why neutrinos are so light and why dark energy density corresponds to neutrino mass scale, and provides also a new insight about color confinement.

A further important result is that the formalism works only for imbedding space dimension $D = 8$. This is due the fact that the number of vector components is the same as the number of spinor components of fixed chirality for $D = 8$ and corresponds directly to the octonionic triality.

p-Adic thermodynamics predicts elementary particle masses in excellent accuracy without Higgs vacuum expectation: the problem is to understand fermionic Higgs couplings. The observation that CP_2 part of the modified gamma matrices gives rise to a term mixing M^4 chiralities contain derivative allows to understand the mass-proportionality of the Higgs-fermion couplings at QFT limit.

2 More precise view about modified Dirac equation

Consistency conditions demand that modified Dirac equation with modified gamma matrices Γ^α defined as contractions $\Gamma^\alpha = T^{\alpha k}\gamma_k$ of canonical momentum currents $T^{\alpha k}$ associated with the bosonic action with imbedding space gamma matrices γ_k [2, 3]. The Dirac operator is not hermitian in the sense that the conjugation for the Dirac equation for Ψ does not give Dirac equation for $\overline{\Psi}$ unless the modified gamma matrices have vanishing covariant divergence as vector at space-time surface. This says that classical field equations are satisfied. This consistency condition holds true also for spinor modes possibly localized at string world sheets to which one can perhaps assign area action plus topological action defined by Kähler magnetic flux. The interpretation is in terms of super-conformal invariance.

The challenge is to formulate this picture more precisely and here I have not achieved a satisfactory formulation. The question has been whether interior spinor field Ψ are present at all, whether only Ψ is present and somehow becomes singular at string world sheets, or whether both stringy spinors Ψ_s and interior spinors Ψ are present. Both Ψ and Ψ_s could be present and Ψ_s could serve as source for interior spinors with the same H-chirality.

The strong form of holography (SH) suggests that interior spinor modes Ψ_n are obtained as continuations of the stringy spinor modes $\Psi_{s,n}$ and one has $\Psi = \Psi_s$ at string world sheets. Dirac action would thus have a term localized at strong world sheets and bosonic action would contain similar term by the requirement of super-conformal symmetry. Can one realize this intuition?

1. Suppose that Dirac action has interior and stringy parts. For the twistor lift of TGD [5] the interior part with gamma matrices given by the modified gamma matrices associated with the sum of Kähler action and volume action proportional to cosmological constant Λ. The variation with respect to the interior spinor field Ψ gives modified Dirac equation in the interior with source term from the string world sheet. The H-chiralites of Ψ and Psi_s would be same. Quark like and leptonic H-chiralities have different couplings to Kähler gauge potential and mathematical consistency strongly encourages this.

   What is important is that the string world sheet part, which is bilinear in interior and string world sheet spinor fields Ψ and Ψ_s and otherwise has the same form as Dirac action. The natural assumption is that the stringy Dirac action corresponds to the modified gamma matrices assignable to area action.

2. String world sheet must be minimal surface: otherwise hermiticity is lost. This can be achieved either by adding to the Kähler action string world sheet area term. Whatever the correct option is, quantum criticality should determine the value of string tension. The first string model inspired guess is that the string tension is proportional to gravitational constant $1/G = 1/l_P^2$ defining the radius fo M^4 twistor sphere or to $1/R^2$, R CP_2 radius. This would however allow only strings not much longer than l_P or R. A more natural estimate is that string tension is proportional to the cosmological constant Λ and depends on p-adic length scale as $1/p$ so that the tension becomes

small in long length scales. Since Λ coupling contant type parameter, this estimate looks rather reasonable.

3. The variation of stringy Dirac action with action density

$$L = [\overline{\Psi}_s D_s^{\rightarrow}\Psi - \overline{\Psi}_s D_s^{\leftarrow}\Psi]\sqrt{g_2} + h.c. \tag{2.1}$$

with respect to stringy spinor field Ψ_s gives for Ψ Dirac equation $D_s\Psi = 0$ if there are no Lagrange multiplier terms (see below). The variation in interior gives $D\Psi = S = D_s\Psi_s$, where the source term S is located at string world sheets. Ψ satisfies at string world sheet the analog of 2-D massless Dirac equation associated with the induced metric. This is just what stringy picture suggests.

The stringy source term for D equals to $D_s\Psi_s$ localized at string world sheets: the construction of solutions would require the construction of propagator for D, and this does not look an attractive idea. For $D_s\Psi_s = 0$ the source term vanishes. Holomorphy for Ψ_s indeed implies $D_s\Psi = 0$.

4. $\Psi_s = \Psi$ would realize SH as a continuation of Ψ_s from string world sheet to Ψ in the interior. Could one introduce Lagrange multiplier term

$$L_1 = \overline{\Lambda}(\Psi - \Psi_s) + h.c.$$

to realize $\Psi_s = \Psi$? Lagrange multiplier spinor field Λ would serve a source in the Dirac equation for $\Psi = \Psi_s$ and Ψ should be constructed at string world sheet in terms of stringy fermionic propagator with Λ as source. The solution for Ψ_s would require the construction of 2-D stringy propagator for Ψ_s but in principle this is not a problem since the modes can be solved by holomorphy in hypercomplex stringy coordinate. The problem of this option is that the H-chiralities of Λ and Ψ would be opposite and the coupling of opposite H-chiralities is not in spirit with H-chirality conservation.

A possible cure is to replace the Lagrange multiplier term with

$$L_1 = \overline{\Lambda}^k\gamma_k(\Psi - \Psi_s) + h.c. \ . \tag{2.2}$$

The variation with respect to the spin 3/2 field Λ^k would give 8 conditions - just the number of spinor components for given H-chirality - forcing $\Psi = \Psi_s$! $D = 8$ would be in crucial role! In other imbedding space dimensions the number of conditions would be too high or too low.

One would however obtain

$$D_s\Psi = D_s\Psi_s = \Lambda^k\gamma_k \ . \tag{2.3}$$

One could of course solve Ψ at string world sheet from $\Lambda^k\gamma_k$ by constructing the 2-D propagator associated with D_s. Conformal symmetry for the modes however implies $D_s\Psi = 0$ so that one has actually $\Lambda^k = 0$ and Λ^k remains mere formal tool to realize the constraint $\Psi = \Psi_s$ in mathematically rigorous manner for imbedding space dimension $D = 8$. This is a new very powerful argument in favor of TGD.

5. At the string world sheets Ψ would be annihilated both by D and D_s. The simplest possibility is that the actions of D and D_s are proportional to each other at string world sheets. This poses conditions on string world sheets, which might force the CP_2 projection of string world sheet to

belong to a geodesic sphere or circle of CP_2. The idea that string world sheets and also 3-D surfaces with special role in TGD could correspond to singular manifolds at which trigonometric functions representing CP_2 coordinates tend to go outside their allowed value range supports this picture. This will be discussed below.

(a) For the geodesic sphere of type II induced Kähler form vanishes so that the action of 4-D Dirac massless operator would be determined by the volume term (cosmological constant). Could the action of D reduce to that of D_s at string world sheets? Does this require a reduction of the metric to an orthogonal direct sum from string world sheet tangent space and normal space and that also normal part of D annihilates the spinors at the string world sheet? The modes of Ψ at string world sheets are locally constant with respect to normal coordinates.

(b) For the geodesic sphere of type I induced Kähler form is non-vanishing and brings an additional term to D coming from CP_2 degrees of freedom. This might lead to trouble since the gamma matrix structures of D and D_s would be different. One could however add to string world sheet bosonic action a topological term as Kähler magnetic flux. Although its contribution to the field equations is trivial, the contribution to the modified gamma matrices is non-vanishing and equal to the contraction $J^{\alpha k}\gamma_k$ of half projection of the Kähler form with CP_2 gamma matrices. The presence of this term could allow the reduction of $D\Psi_s = 0$ and $D_s\Psi_s = 0$ to each other also in this case.

3 A more detailed view about string world sheets

In TGD framework gauge fields are induced and what typically occurs for the space-time surfaces is that they tend to go out from CP_2. Could various lower-D surfaces of space-time surface correspond to sub-manifolds of space-time surface?

1. To get a concrete idea about the situation it is best to look what happens in the case of sphere $S^2 = CP_1$. In the case of sphere S^2 the Kähler form vanishes at South and North poles. Here the dimension is reduced by 2 since all values of ϕ correspond to the same point. $sin(\Theta)$ equals to 1 at equator - geodesic circle - and here Kähler form is non-vanishing. Here dimension is reduced by 1 unit. This picture conforms with the expectations in the case of CP_2 These two situations correspond to 1-D and 2-D geodesic sub-manifolds.

2. CP_2 coordinates can be represented as cosines or sines of angles and the modules of cosine or sine tends to become larger than 1 (see `http://tinyurl.com/z3coqau`). In Eguchi-Hanson coordinates (r, Θ, Φ, Ψ) the coordinates r and Θ give rise to this kind of trigonometric coordinates. For the two cyclic angle coordinates (Φ, Ψ) one does not encounter this problem.

3. In the case of CP_2 only geodesic sub-manifolds with dimensions $D = 0, 1, 2$ are possible. 1-D geodesic submanifolds carry vanishing induce spinor curvature. The impossibility of 3-D geodesic sub-manifolds would suggest that 3-D surfaces are not important. CP_2 has two geodesic spheres: S^2_I is homologically non-trivial and S^2_{II} homologically trivial (see `http://tinyurl.com/z3coqau`).

   (a) Let us consider S^2_I first. CP_2 has 3 poles, which obviously relates to SU(3), and in Eguchi Hanson coordinates (r, θ, Φ, Ψ) the surface $r = \infty$ is one of them and corresponds - not to a 3-sphere - but homologically non-trivial geodesic 2- sphere, which is complex sub-manifold and orbits of $SU(2) \times U(1)$ subgroup. Various values of the coordinate Ψ correspond to same point as those of Φ at the poles of S^2. The Kähler form J and classical Z^0 and γ fields are non-vanishing whereas W gauge fields vanish leaving only induced γ and Z^0 field as one learns by studying the detailed expressions for the curvature of spinor curvature and vierbein of CP_2.

String world sheet could have thus projection to S^2_I but both γ and Z^0 would be vanishing except perhaps at the boundaries of string world sheet, where Z^0 would naturally vanish in the picture provided by standard model. One can criticize the presence of Z^0 field since it would give a parity breaking term to the modified Dirac operator. SH would suggest that the reduction to electromagnetism at string boundaries might make sense as counterpart for standard model picture. Note that the original vision was that besides induced Kähler form and em field also Z^0 field could vanish at string world sheets.

(b) The homologically trivial geodesic sphere S^2_{II} is the orbit of $SO(3)$ subgroup and not a complex manifold. By looking the standard example about S^2_I, one finds that the both J, Z_0, and γ vanish and only the W components of spinor connection are non-vanishing. In this case the notion of em charge would not be well-defined for S^2_{II} without additional conditions. Partonic 2-surfaces, their light-like orbits, and boundaries of string world sheets could do so since string world sheets have 1-D intersection with with the orbits. This picture would make sense for the minimal surfaces replacing vacuum extremals in the case of twistor lift of TGD.

Since em fields are not present, the presence of classical W fields need not cause problems. The absence of classical em fields however suggests that the modes of induced spinor fields at boundaries of string worlds sheets must be em neutral and represent therefore neutrinos. The safest but probably too strong option would be right-handed neutrino having no coupling spinor connection but coupling to the CP_2 gamma matrices transforming it to left handed neutrino. Recall that ν_R represents a candidate for super-symmetry.

Neither charged leptons nor quarks would be allowed at string boundaries and classical W gauge potentials should vanish at the boundaries if also left-handed neutrinos are allowed: this can be achieved in suitable gauge. Quarks and charged leptons could reside only at string world sheets assignable to monopole flux tubes. This could relate to color confinement and also to the widely different mass scales of neutrinos and other fermions as will be found.

To sum up, the new result is that the distinction between neutrinos and other fermions could be understood in terms of the condition that em charge is well-defined. What looked originally a problem of TGD turns out to be a powerful predictive tool.

4 Classical Higgs field again

A motivation for returning back to Higgs field comes from the twistor lift of Kähler action.

1. The twistor lift of TGD [4] [5] brings in cosmological constant as the coefficient of volume term resulting in dimensional reduction of 6-D Kähler action for twistor space of space-time surface realized as surface in the product of twistor space of M^4 and CP_2. The radius of the sphere of M^4 twistor bundle corresponds to Planck length. Volume term is extremely small but removes the huge vacuum degeneracy of Kähler action. Vacuum extremals are replaced by 4-D minimal surfaces and modified Dirac equation is just the analog of massless Dirac equation in complete analogy with string models.

2. The well-definedness and conservation of fermionic em charges and SH demand the localization of fermions to string world sheets. The earlier picture assumed only em fields at string world sheets. More precise picture allows also W fields.

3. The first guess is that string world sheets are minimal surfaces and this is supported by the previous considerations demanding also string area term and Kähler magnetic flux tube. Here gravitational constant assignable to M^4 twistor space would be the first guess for the string tension.

What one can say about the possible existence of classical Higgs field?

1. TGD predicts both Higgs type particles and gauge bosons as bound states of fermions and antifermions and they differ only in that their polarization are in M^4 resp. CP_2 tangent space. p-adic thermodynamics [1] gives excellent predictions for elementary particle masses in TGD framework. Higgs vacuum expectation is not needed to predict fermion or boson masses. Standard model gives only a parametrization of these masses by assuming that Higgs couplings to fermions are proportional to their masses, it does not predict them.

   The experimental fact is however that the couplings of Higgs are proportional to fermion masses and TGD should be able to predict this and there is a general argument for the proportonality, which however should be deduced from basic TGD. Can one achieve this?

2. Can one imagine any candidate for the classical Higgs field? There is no covariantly constant vector field in CP_2, whose space-time projection could define a candidate for classical Higgs field. This led years ago before the model for how bosons emerge from fermions to the wrong conclusion that TGD does not predict Higgs.

   The first guess for the possibly existing classical counterpart of Higgs field would be as CP_2 part for the divergence of the space-time vector defined modified gamma matrices expressible in terms of canonical momentum currents having natural interpretation as a generalization of force for point like objects to that for extended objects. Higgs field in this sense would however vanish by above consistency conditions and would not couple to spinors at all.

   Classical Higgs field should have only CP_2 part being CP_2 vector. What would be also troublesome that this proposale for classical Higgs field would involve second derivatives of imbedding space coordinates. Hence it seems that there is no hope about geometrization of classical Higgs fields.

3. The contribution of the induced Kähler form gives to the modified gamma matrices a term expressible solely in terms of CP_2 gamma matrices. This term appears in modified Dirac equation and mixes M^4 chiralities - a signal for the massivation. This term is analogous to Higgs term expect that it contains covariant derivative.

   The question that I have not posed hitherto is whether this term could at QFT limit of TGD give rise to vacuum expectation of Higgs. The crucial observation is that the presence of derivative, which in quantum theory corresponds roughly to mass proportionality of chirality mixing coupling at QFT limit. This could explain why the coupling of Higgs field to fermions is proportional to the mass of the fermion at QFT limit!

4. For S^2_{II} type string world sheets assignable to neutrinos the contribution to the chirality mixing coupling should be of order of neutrino mass. The coefficient $1/L^4$ of the volume term defining cosmological constant [5] separates out as over all factor in massless Dirac equation and the parameter characterizing the mass scale causing the mixing is of order $m = \omega_1\omega_2 R$. Here ω_1 characterizes the scale of gradient for CP_2 coordinates. The simplest minimal surface is that for which CP_2 projection is geodesic line with $\Phi = \omega_1 t$. ω_2 characterizes the scale of the gradient of spinor mode.

   Assuming $\omega_1 = \omega_2 \equiv \omega$ the scale m is of order neutrino mass $m_\nu \simeq .1$ eV from the condition $m \sim \omega^2 R \sim m_\nu$. This gives the estimate $\omega \sim \sqrt{m_{CP_2} m_\nu} \sim 10^2 m_p$ from $m_{CP_2} \sim 10^{-4} m_P$, which is weak mass scale and therefore perfectly sensible. The reduction $\Delta c/c$ of the light velocity from maximal signal velocity due the replacement $g_{tt} = 1 - R^2\omega^2$ is $\Delta c/c \sim 10^{-34}$ and thus completely negligible. This estimate does not make sense for charged fermions, which correspond to S^2_I type string world sheets.

   A possible problem is that if the value of the cosmological constant Λ evolves as $1/p$ as function of the length mass scale the mass scale of neutrinos should increase in short scales. This looks strange unless the mass scale remains below the cosmic temperature so that neutrinos would be always effectively massless.

5. For S^2_I type string world sheets assignable to charged fermions Kähler action dominates and the mass scales are expected to be higher than for neutrinos. For S^2_I type strings the modified gamma matrices contain also Kähler term and a rough estimate is that the ratio of two contributions is the ratio of the energy density of Kähler action to vacuum energy density. As Kähler energy density exceeds the value corresponding to vacuum energy density $1/L^4$, $L \sim 40\ \mu$m, Kähler action density begins to dominate over dark energy density.

To sum up, this picture suggest that the large difference between the mass scales of neutrinos and em charged fermions is due to the fact that neutrinos are associated with string world sheet of type II and em charged fermions with string world sheets of type I. Both strings world sheets would be accompanied by flux tubes but for charged particles the flux tubes would carry Kähler magnetic flux. Cosmological constant forced by twistor lift would make neutrinos massive and allow to understand neutrino mass scale.

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