Article

Tsirelson Bound, Entanglement & Gravity

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Abstract

This paper illustrates an isomorphism between the Tsirelson bound of quantum mechanics and the spacetime metric. This illustrates how spacetime could be seen as quantum mechanics in disguise. The extension of this with gauge fields and gravitation illustrates further how spacetime can be seen as the result of quantum mechanics or quantum fields.

Keywords: Tsirelson bound, quantum mechanics, quantum field, spacetime metric.

1 Tsirelsion bound and flat spacetime

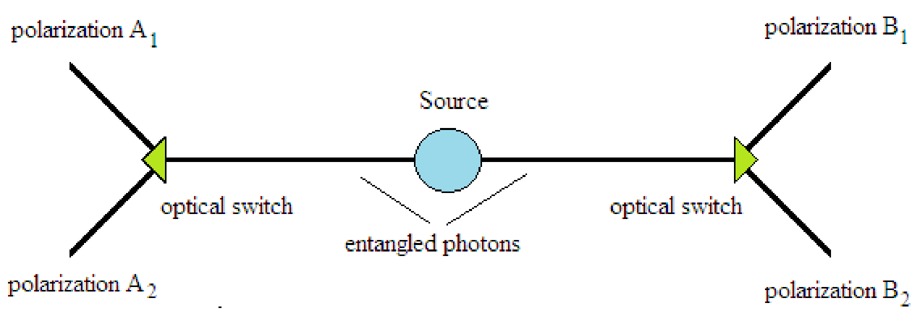
Suppose we have four operators A_1, A_2, B_1, B_2 such that:

$$A_i^2 \ = \ B_i^2 \ = \ 1$$

and

$$[A_i, B_i] = 0$$

These 4 operators correspond to the observables in Aspect's experiment [1]. A single source of photons emits pairs of photons to the left and right measuring apparatuses. At the measurement station a rapidly moving mirror pushes the photons to be measured for polarization either on direction A_1 or A_2 for the left detectors, or B_1 or B_2 for the right detector. The outcomes are $+1$ or -1 (thus $A^2 = B^2 \ = \ 1$). The A_i commute with B_i because they are spatially separated. This set up is diagrammatically the same as used in the Bell theorem as diagrammed below



Now define an operator $\hat{C}$ as follows:

$$\hat{C} \ = \ A_1 B_1 \ + \ A_2 B_1 \ + \ A_2 B_2 \ - \ A_1 B_2 \tag{1.1}$$

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and it is not hard to show that:

$$\hat{C}^2 \ = \ 4 \ + \ [A_1, A_2][B_1, B_2]$$

By using the triangle inequality it is not hard to see that

$$|\hat{C}^2| \ \leq \ 4 \ + \ 4, \ |C| \ \leq \ 2\sqrt{2}$$

This is a derivation of the Tsirelson bound [2].

The operator C appears very similar to the Lorentz metric,

$$x \cdot y \ = \ -x_0y_0 + x_1y_1 + x_2y_2 + x_3y_3,$$

which is the metric distance with Lorentz geometry or $SO(3,1)$. In the case of the Riemann sphere $\mathbb{C}P^1$ the set of conformal transformations are linear fractional transformations

$$z \ \rightarrow \ \frac{az+b}{cz+d}$$

where this transformation is isomorphic to $PSL(2,\mathbb{C})$. The heavenly sphere is then the case of the null metric distance, or equivalently the projective light cone. The product space V of $dim \ = \ n$ contains the Jordan algebra is the $v^2 \ = \ \langle v, v\rangle$ ($v \in V$, $\langle u, v\rangle$ is the $\mathbb{R}^n$ inner product) so that a spin factor $J(V) \ \sim \ V \oplus \mathbb{R}$ (space plus time) such that

$$(u, \alpha) \odot (v, \beta) \ = \ (\alpha v + \beta u, \langle u, v\rangle - \alpha\beta). \tag{1.2}$$

Then $J(V)$ is isomorphic to Minkowski spacetime [3]. This Clifford algebra defined on the right is the spacetime metric $\langle u, v\rangle \ - \ \alpha\beta$.

Now let spacetime V_c be represented with the basis elements

$$\begin{aligned} u_1 \ &= \ (A_1, 0, 0), u_2 \ = \ (0, B_1, 0), u_3 \ = \ (0, 0, A_2) \\ v_1 \ &= \ (B_1, 0, 0), v_2 \ = \ (0, A_2, 0), v_3 \ = \ (0, 0, B_2) \end{aligned} \tag{1.3}$$

and the real line R containing the two elements (A_1, B_2), it is then easy to see that the C operator can be expressed according to the Clifford algebra.

The connection between the null condition and the Tsirelson bound might be made by defining the elements of the real line $\mathbb{R}$ as $(iA_1 + \sqrt{2\sqrt{2}}, iB_2 \ + \ \sqrt{2\sqrt{2}})$ with the product is the real valued so this is $-B_2A_1 \ + \ 2\sqrt{2}$. In that way the modified $|C|^2$ would be zero if it is at the Tsirelson bound, and similar to a choice of metric signature is negative if outside the Tsirelson bound.

2 Gauge fields and gravitation

In this section a gauge covariant formulation is considered. This starts by looking at transformations of the operator $\hat{C}$ due to a unitary group generated by Lie algebraic elements. This will result in an result that is not physically correct, which will require a covariant of this theory.

Consider transformations on the four operators A_i, B_i such that $A'_i \ = \ gA_i$ and $B'_i \ = \ g'B_i$. This is a group theoretic form of the textbook gauge transformation $\vec{A}' \ = \ \vec{A} \ + \nabla\chi$[4]. The group elements are $g \ = \ exp(i\epsilon_\mu E^\alpha U^\mu_\alpha)$ and $g' \ = \ exp(i\epsilon_\mu E^\alpha V^\mu_\alpha)$, where E_α is a root vector in a Lie algebra and U^μ_α a vierbein. For small parameter ϵ the group elements are $g \ = \ 1 \ + \ i\epsilon_\mu E^\alpha U^\mu_\alpha$ and $g' \ = \ 1 \ + \ i\epsilon_\mu E^\alpha V^\mu_\alpha$. The commutator of the transformed operators is

$$[A'_i, \ B'_j] \ = \ [gA_i, \ g'B_j] \ = \ gg'[A_i, \ B_j] \ + \ [g, \ g']B_jA_i \ = \ gg'[A_i, \ B_j] \ + \ \epsilon_\mu\epsilon_\nu[E^\alpha, \ E^\beta]B_jA_iU^\mu_\alpha V^\nu_\beta.$$

The commutators are taken between observables at different locations, for those at the same location will be rotated by the same transformation with no relative difference resulting between them. The commutator in the ϵ small limit is

$$[g,\ g']B_jA_i\ =\ \epsilon_\mu\epsilon_\nu[E^\alpha,\ E^\beta]U^\mu_\alpha V^\nu_\beta B_jA_i.$$

Physically this transformation is a local gauge transformation which occurs between the $A_1,\ B_1$ side of the experiment and $A_2,\ B_2$ side.

An alternative transformation on the operators is with $g\ =\ (\epsilon\cdot\nabla_u)$ and $g'\ =\ (\epsilon\cdot\nabla_v)$. This transformation is a parallel translation rule for the state according to a geodesic flow. In this case we have that the commutator above is

$$[A'_i,\ B'_j]\ =\ gg'[A_i,\ B_j]\ +\ \epsilon\wedge\epsilon\cdot[\nabla_u,\ \nabla_v]B_jA_i\ =\ \mathcal{R}(U,\ V)B_jA_i,$$

where $\mathcal{R}(U,\ V)$ is a curvature operator

The entire operator C is then

$$C^2\ =\ 4+[A'_1,A'_2][B'_1,B'_2]+\epsilon_\mu\epsilon_\nu[E^\alpha,\ E^\beta]U^\mu_\alpha V^\nu_\beta(B'_1A'_1A'_2B'_2+B'_2A'_2A'_1B'_1+B'_1A'_1A'_2B'_2+B'_2A'_2A'_1B'_1).$$

The commutators are then $[E^\alpha,\ E^\beta]\ =\ N^{\alpha\beta}E_{\alpha+\beta}$. For the geodesic flow version this is

$$C^2\ =\ 4\ +\ [A'_1,A'_2][B'_1,B'_2]$$

$$+\ \epsilon^2\Big(B'_1\langle A'_1\mathcal{R}(U,\ V)A'_2\rangle B'_2+A'_2\langle B'_2\mathcal{R}(U,\ V)A'_1\rangle B'_1+B'_1\langle A'_1\mathcal{R}(U,\ V)B'_2\rangle A'_2+A'_2\rangle B'_2\mathcal{R}(U,\ V)B'_1\rangle A'_1\Big).$$

The Riemann curvature is defined by $\langle A'_1,R(U,\ V)\ A'_2\rangle\ =R_{\alpha\mu\beta\nu}{A'_1}^\alpha U^\mu V^\nu {A'_2}^\beta$. These two cases correspond to first an internal gauge transformation and in the second a spacetime transformation. The apparatus in this case is held together by materials that provide strength against the curvature.

This redefines the Tsirelson bound. The norm with $|gA_i|\ \le\ |g||A_i|$ and with $|g|\ =\ 1$ we have the norm

$$|C^2|\ \le\ 8\ +\ 4\epsilon^2(F,\ R),$$

where $(F,\ R)$ is the choice of a field strength from the commutator of Lie algebraic roots, or from the curvature of spacetime.

This equation is motivating, but it is also wrong. The correspondence between the null metric and the Tsirelson bound at equality means the two must hold for arbitrary metric. A metric $g(X,\ Y)$ that deviated from the Minkowski case may be expanded in variations Y, $\delta Y\ =\ \nabla_xY\delta x$

$$g(X,\ Y')\ =\ g(X,\ Y)\ +\ g(X,\ \delta Y)\ +\ \frac{1}{2}g(X,\ \delta^2Y)$$

$$=\ g(X,\ Y)\ +\ g\left(X,\ \nabla_xY\right)\delta x\ +\ \frac{1}{2}g\left(X,\ R(V,\ Y)V\right)\delta x^2$$

The first order term vanishes and the second order term is the geodesic deviation equation. If the geodesic is null $ds\ =\ 0$, which will still correspond to $|C|\ =\ 2\sqrt{2}$

To correct for this we let $A_i\ \rightarrow\ A^\omega\ = A_i\ +\ i\alpha$ and $B_i\ \rightarrow\ B^\omega_i\ = B_i\ +\ i\beta$. The parameters α and β obey $\alpha\beta\ =\ \omega$ and $\alpha B\ +\ \beta B\ =\ 0$. In addition we have $\omega\ <<\ A_i,\ B_i$. The triangle inequalities in the Tsirelson bound then have

$$|AB|\ \rightarrow\ |\ |A^\omega B^\omega|\ =\ (A\ +\ i\alpha)(B\ +\ i\beta)|\ \le\ |AB|\ -\ |\omega|$$

so that The Tsirelson bound is then

$$|C^2| \;\leq\; 8 \;-\; 8|\omega| \;+\; 4\epsilon^2(F,\; R) \tag{2.1}$$

The Tsirelson bound is preserved for $|\omega| \;+\; \frac{1}{2}\epsilon^2(F,\; R) \;=\; 0$. This is a form of gauge invariance. In the case of the gauge field the term ω is a form of the Aharanov-Bohm effect. The instrument operators $A_i,\; B_i$ are gauged in a manner similar to $\mathbf{P} \;=\; \mathbf{p} \;+\; \frac{ie}{\hbar}\mathbf{A}$

3 Weyl Curvature and Symmetries

The two considerations, one a gauge transformation defined by the root vectors of a Lie algebra and the other gravitation, are considered with respect to each other. The Riemann curvature with vanishing Ricci curvature is the Weyl tensor. For sourceless region the curvature is purely vacuum and given by the Weyl tensor[5]. Given null vectors U^α the null vector $U^{\alpha\beta} \;=\; [U^\alpha,\; U^\beta]$ is defined. These are eigen-bivectors of the Weyl tensor [7]

$$\frac{1}{2}C^{\mu\nu}{}_{\alpha\beta}U^{\alpha\beta} \;=\; \lambda U^{\mu\nu}.$$

Consider the metric composed of the null vectors $x_\alpha,\; y_\alpha,\; z_\alpha$, wuch that $z^\alpha y_\alpha \;= x^\alpha y_\alpha \;=\; -1$ and $\bar{y}^\alpha y_\alpha \;=\; 1$,

$$g_{\alpha\beta} \;=\; x_\alpha z_\beta \;+\; z_\alpha x_\beta \;+\; y_\alpha\bar{y}_\beta \;+\; \bar{y}_\alpha y_\beta.$$

There are then three possible null bivectors

$$U_{\alpha\beta} \;=\; -X_{[\alpha}y_{\beta]},\; V_{\alpha\beta} \;=\; z_{[\alpha}y_{\beta]},\; W_{\alpha\beta} \;=\; -y_{[\alpha}\bar{y}_{\beta]} \;-X_{[\alpha}y_{\beta]},$$

so the Weyl tensor is composed as

$$\begin{aligned}C_{\alpha\beta\mu\nu} \;=\; &\Psi_0 U_{\alpha\beta}U_{\mu\nu}\\ &+\;\Psi_1(U_{\alpha\beta}W_{\mu\nu} \;+\; W_{\alpha\beta}U_{\mu\nu})\\ &+\;\Psi_2(V_{\alpha\beta}U_{\mu\nu} \;+\; U_{\alpha\beta}V_{\mu\nu} \;+\; W_{\alpha\beta}W_{\mu\nu}),\\ &+\;\Psi_3(V_{\alpha\beta}W_{\mu\nu} \;+\; W_{\alpha\beta}V_{\mu\nu})\\ &+\;\Psi_4 V_{\alpha\beta}V_{\mu\nu}\end{aligned}$$

for Ψ_i Weyl scalars. These define different physics; Ψ_2 gives the vacuum around a central source, such as a black hole, Ψ_4 are transverse modes and $\Psi_1,\; \Psi_3$ are in and out directed longitudinal modes.

Each of the bivectors may be expressed according to a vierbein $U^a \;=\; E^\alpha u^a_\alpha$, where now the Latin indices refer to spacetime and Greek indices correspond to an internal space given by the root vectors of a Lie algebra. We can then see that $U_{ab} \;= 2[E^\alpha,\; E^\beta]U^{[b}_\beta U^{a]}_\alpha$. The Weyl tensor is then

$$\begin{aligned}C^{abcd} \;=\; &2\Psi_0[E^\alpha,\; E^\beta][E^{\alpha'},\; E^{\beta'}]U^{ab}_{\alpha\beta}U^{cd}_{\alpha'\beta'}\\ &+\;2\Psi_1[E^\alpha,\; E^\beta][E^{\alpha'},\; E^{\beta'}](U^{ab}_{\alpha\beta}W^{cd}_{\alpha'\beta'} \;+\; W^{ab}_{\alpha\beta}U^{cd}_{\alpha'\beta'})\\ &+\;2\Psi_2[E^\alpha,\; E^\beta][E^{\alpha'},\; E^{\beta'}](V^{ab}_{\alpha\beta}U^{cd}_{\alpha'\beta'} \;+\; U^{ab}_{\alpha\beta}V^{cd}_{\alpha'\beta'} \;+\; W^{ab}_{\alpha\beta}W^{cd}_{\alpha'\beta'}),\\ &+\;2\Psi_3[E^\alpha,\; E^\beta][E^{\alpha'},\; E^{\beta'}](V^{ab}_{\alpha\beta}W^{cd}_{\alpha'\beta'} \;+\; W^{ab}_{\alpha\beta}V^{cd}_{\alpha'\beta'})\\ &+\;2\Psi_4[E^\alpha,\; E^\beta][E^{\alpha'},\; E^{\beta'}]V^{ab}_{\alpha\beta}V^{cd}_{\alpha'\beta'}\end{aligned} \tag{3.1}$$

where the bi-vierbeins $U^{ab}_{\alpha\beta}$ are evidently defined. The nature of the gauge field or gauge-like field associated with these Lie algebraic roots is discussed in the last section.

The root vectors E^α obey the commutators

$$[E^\alpha,\ E^\beta]\ =\ N^{\alpha\beta}H^{\alpha+\beta},$$

where $H^{\alpha+\beta}$ are the weights. With any Lie algebra there are elements that are analogous to $a\ a^\dagger$ for the harmonic oscillator, which are the standard roots E^α and $a^\dagger a$ that correspond to the weights H^α. For the gauge theoretic description of the $\hat{C}$ operators the result is linear in the weight. For gravitation the operator is quadratic in the weights. The Weyl tensor for type D and II solutions are eigenvaled with $C_{abcd}U^bU^c\ =\ \lambda U^aU^d$. It is possible to see the Weyl tensor for these eigenvalued Petrov types obeys

$$C_{abcd}U^bU^c\ =\ \lambda E_\alpha E_\delta U^\alpha_a U^\delta_d.$$

Consequently the Weyl tensor for these eigenvalued Petrov types obeys

$$C_{abcd}U^bU^c\ =\ N_{\alpha\beta}N_{\gamma+\delta}H_{\alpha+\beta}H_{\gamma+\delta}U^\alpha_a U^\beta_b U^\gamma_c U^\delta_d U^b U^c$$
$$=\ N_{\alpha\beta}N_{\gamma+\delta}H_{\alpha+\beta}H_{\gamma+\delta}E^\beta E^\gamma U^\alpha_a U^\delta_d\ =\ \lambda E_\alpha E_\delta U^\alpha_a U^\delta_d.$$

The commutators of the Lie algebraic roots concern the observables A_i and B_i on either side of the apparatus. In the case of a Lie algebra a gauge transformation, or the introduction of a force, transforms these operators relative to each other. This results then in a modification of the Tsirelson bound. Similarly, for gravitation the parallel translation of these operators on either side of the apparatus does the same. This has lead to a modification of the assocated line element expressed according to the heavenly sphere.

4 Holography, gauge-gravity correspondence and information

The Aspect apparatus is a guiding diagram for the following. We may think of the A and B states as set by the optical switch where Alice and Bob measure an entangled pair. The general situation for an entanglement is of course that the density matrix $\rho(a,a',b,b')\ =\ \psi(a,b)\psi^*(a'b')$ and what Alice or Bob observe is given by the trace over the b or a states respectively. The state $\psi(a,b)\ =\ \phi(a)\chi(b)$ as a product means that the trace over Bob's states

$$Tr_B\rho(a,a',b,b')\ =\ \sum_{b,b'}\psi(a,b)\psi^*(a'b')\ =\ \phi(a)\phi^*(a')\sum_{bb'}\chi(b)\chi^*(b)$$
$$=\ \phi(a)\phi^*(a')\sum_{bb'}\delta_{bb'}\ =\ \rho(a,a').$$

The composite or entangled system is one where neither the A or B states are known; the maximal knowledge of the system is the entangled state as a whole [8]. This is knowledge is determined from the preparation of a state as the source. If the separation between Alice and Bob is spacelike neither can know directly anything about what the other observes. We may compare this case to the accelerated frame. If Alice and Bob are on accelerated frames so that Bob is in region I and Alice is in region II. Then the motion of the two observers with

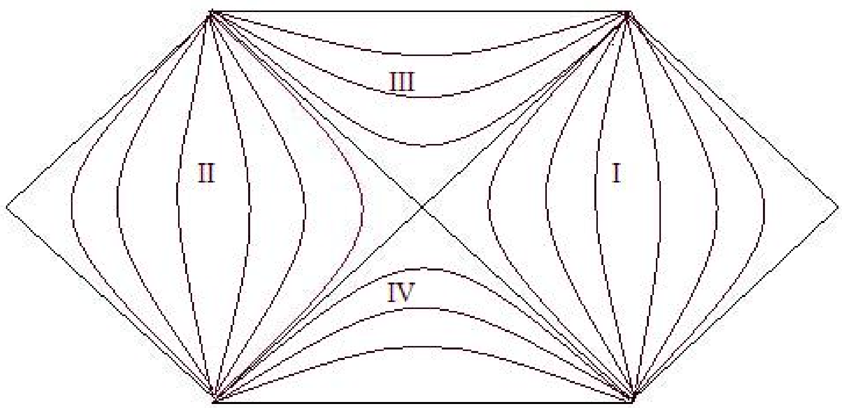
$$t_{ab}\ =\ \rho\ sinh(\omega),\ x_b\ =\ \rho\ cosh(\omega), x_a\ =\ -\rho\ cosh(\omega)$$

is such that on this frame Alice and Bob will never observe each other entangled pair[8].

The metric for the accelerated frame

$$ds^2\ =\ -\rho^2 d\omega^2\ +\ d\omega^2\ +\ dy^2\ +\ dz^2$$

describes a path that is ρ distance form the Rindler wedge horizon. This distance is related to the temperature according to $\rho\ =\ 1/2\pi T$. The observer in either region I or II observes close to the horizon virtual particle loops that cross the Rindler horizon into all regions. The two observers however observe a virtual particle leaving the horizon from the asymptotic past infinity, or $\omega = \frac{3\pi}{4}$ and then approach the horizon as $t \to \infty$ or $\omega \to \frac{\pi}{4}$ in region I and similarly leave from $\omega = \frac{5\pi}{4}$ and approach $\omega \to \frac{7\pi}{4}$ in region II. This metric is an approximation to the near horizon condition for the Schwarzschild metric. The near horizon time-angle ω and time in the asymptotic region is $\omega = t/4GM$ and there is a Weyl curvature for the vacuum spacetime. The commutators $[E^\alpha,\ E^\beta] = N^{\alpha\beta}E^{\alpha+\beta}$ correspond to an internal gauge-like field that parametrizes the relative parallel translations of the field vectors of an entangled pair in regions I and II.



It is evident the spacetime metric is a form of the Tsirelson bound. The Schwarzschild metric is then an entanglement structure between quantum states in the two spacelike regions in the conformal diagram. The Rindler wedge with the correspondence to the near horizon condition of a black hole leads to a correspondence with the Penrose diagram for the Schwarzschild black hole. This connects then with the emergence of spacetime from quantum entanglements[9]. The spacetime metric as a form of the Tsirelson bound defines spacetime according to entanglements across event horizons.

The elements $U^\mu = E_\alpha U^\mu_\alpha$ define the Weyl curvature and the commutators for the entanglement of states divided by the event horizon. The root vectors correspond to gauge-like action that parametrized spacetime. The nature of this gauge field can be considered by looking at the group $U(2,2) \sim O(4,2)$, which is the isometry group for the AdS_5 spacetime. Consider the irreducible representation

$$U(2,\ 2) \to (\mathbf{2},\ \mathbf{2}) \otimes (\mathbf{2},\ \mathbf{2}) = (\mathbf{3},\ \mathbf{3}) \oplus (\mathbf{3},\ \mathbf{1}) \oplus (\mathbf{1},\ \mathbf{3}) \oplus (\mathbf{1},\ \mathbf{1})$$

The $(\mathbf{3},\ \mathbf{3})$ is spin 2 for gravitation. The $(\mathbf{3},\ \mathbf{1}) \oplus (\mathbf{1},\ \mathbf{3})$ is a spin 1 gauge field, and the last $(\mathbf{1},\ \mathbf{1})$ are a scalar fields, such as the dilaton and axion.

This representation has the following correspondences

$$(\mathbf{3},\mathbf{3}) \to \frac{U(2,2)}{U(2)\times U(2)},\ (\mathbf{1},\mathbf{3}) \to \frac{U(2,2)}{U(1)\times U(3)},\ \mathbf{1} \to \frac{U(2,2)}{U(2,2)}.$$

The first two in complexified coordinates define the spaces $G_2(V^{2,2})$ and $\mathbb{C}\mathbf{P}^4$ with complex dimensions 6 and 4 respectively. With $\mathbb{C}\mathbf{P}^3 \subset \mathbb{C}\mathbf{P}^4$ or $C\mathbf{P}^4 = \mathbb{C}\mathbf{P}^3 \cup \mathbb{C}^3$ the split $(\mathbf{3},\mathbf{1})$ corresponds to $\mathbb{C}\mathbf{P}^{3,1}$ as the projective space defined by the Hopf fibration of the boundary. The relevant dynamics is then the action of $\mathbb{C}\mathbf{P}^3$ on $\mathbb{C}^3$. These two spaces are mapped from the flag manifold $\mathbb{F}_{12}$ as

$$\mathbb{C}\mathbf{P}^3 ~=~ \mathbb{F}_1 ~\leftarrow~ \mathbb{F}_{12} ~\rightarrow~ \mathbb{F}_2 ~=~ G_2(V^{2,2})$$

This defines twistor space[10]. The $(\mathbf{3},~\mathbf{1}) \oplus (\mathbf{1},~\mathbf{3})$ defines $\mathbb{C}\mathbf{P}^3$ and $\mathbb{C}\mathbf{P}^{*3}$, for the second defined for the double fibration on the dual flag manifold $\mathbb{F}^*_{12}$, which defines ambitwistor space. The remaining scalar fields $(\mathbf{1}, \mathbf{1})$ define a dual set of $(E,~\phi)$ and $(E^*,~\phi^*)$, for E and E^* holomorphic vector bundles. This is then a form of Higgs bundles of Hitchens[11].

This decomposition can be seen according to the Clifford algebra $CL(4)$ with dimension $2^4 ~=~ 16$ that by binomial distribution is $16 ~=~ 1~4~6~4~1$, this assignments of dimensions above. For a more general unified field, such as expected of $E_8 \times E_8$ consider the Clifford $CL(8)$ with the decomposition

$$2^8 ~=~ 16^2 ~=~ 1~8~28~56~70~56~28~8~1.$$

The $16 ~=~ 8 ~+~ 8$ is the $U(2,2) ~\sim~ O(4,2)$. By extension $Cl(16) ~=~ Cl(8) ~\times~ Cl(8)$ contains E_8 and has graded structure. $CL(16)$ is

$$2^{16} ~=~ 2^8 \times 2^8 ~=~ 1~16~120~560~\dots~560~120~16~1.$$

With $\mathfrak{e}_8 ~+~ so(16) \oplus (\mathbf{64} \oplus \mathbf{64})$ the exceptional $\mathfrak{e}_8$ is constructed from $so(16)$ and two spinors $\mathbf{64}$. The spinor structure of $CL(16)$ has size $(64 ~+~ 64) \times (128 ~+~ 128) =~ 32768$ which is half of the Clifford algebra. This permits the embedding of two exceptional E_8 or $E_8 \times E_8$ in heterotic string theory.

The Tsirelson bound and the metric is generically a form of the Fisher distance defined by the multiplication of different elements[12]. In this manner the information content of qubits and their relationship with spacetime may best be formulated according to Fisher information. Gauge fields and gravity then transform the metric in such a way that information is conserved. Fisher information and its definition according to the sort of product that defines the $\hat{C}$ operator is then conserved.

The equivalency or morphism between the Tsirelson bound and the metric is a map between magmas. Magmas are universal algebras with a binary operation μ on a set S, $\mu : S \times S ~\rightarrow~ S$, is a class of algebras of groups, quasigroups (division algebra) and semi-groups and groupoids. These define a class call magmas. The product operations in equations 1 and 2 define magmas, and the representation of the spacetime with basis elements in equation 3 illustrate a mapping from one magma to another as a morphism. A monoid is a category with one element, and the category of groups may similarly be defined in monoids. This is then a demonstration on how spacetime and quantum mechanics, when looked according to group properties, are categorically the same. This opens the door for the foundations of physics to be a formulated as a categorical cohomology, such as Grothendiek theory.

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References

[1] A. Aspect, P. Grangier, G. Roger, "Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A New Violation of Bell's Inequalities," Phys. Rev. Lett. **49** 2, pp 914 (1982)

[2] B. S. Tsirel'son, "Quantum Generalizations of Bell's Inequality," Lett. Math. Phys. **4**, 93 (1980)

[3] R. Penrose, W. Rindler, "Spinors and Space-Time," Cambridge U. Press, Cambridge, 1985-86

[4] L. D. Faddeev, A. A. Slanov, Gauge Fields: An Introduction to Quantum Theory, Addison Wesley Publishing, Redwood City, CA USA, (1991)

[5] S. W. Hawking, G. F. R. Ellis, Large Scale Structure of Space-time, Cambridge p41 (1973)

[6] H. Stephani, D. Kramer, M. MacCallum, C. Hoenselaers, E. Herlt, "Exact Solutions of Einstein's Field Equations (2nd edn.)." Cambridge: Cambridge University Press. (2003).

[7] C. A. Mantica, L. G. Molinari. "Weyl compatible tensors," Int. J. Geom. Methods Mod. Phys. **11**, 1450070 (2014) http://arxiv.org/pdf/1212.1273

[8] J. Lindesay, L. Susskind, An Introduction to Black Holes, Information and the String Revolution, World Scientific, Singapore, (2005)

[9] M. Van Raamsdonk, "Building up spacetime with quantum entanglement," Gen.Rel.Grav.**42** 2323-2329 (2010), Int.J.Mod.Phys.D19 2429-2435 (2010) http://arxiv.org/abs/1005.3035v1

[10] R. S. Ward, R. O. Wells, Twistor Geometry and Field Theory, p19-45, Cambridge University Press (1990)

[11] N. Hitchin, "Stable bundles and integrable systems", Duke Mathematical Journal **54** (1): 91114 (1987)

[12] B. R. Frieden, R. A. Gatenby, "Principle of maximum Fisher information from Hardy's axioms applied to statistical systems". Physical Review E. **88** (2013) https://arxiv.org/abs/1405.0007

[13] T. Kepka, P. Nmec, "Simple balanced groupoids," Acta Universitatis Palackianae Olomucensis. Facultas Rerum Naturalium. Mathematica, **35**, 1 5360 (1996) http://dml.cz/bitstream/handle/10338.dmlcz/120353/ActaOlom$_3 5 - 1996 - 1_7.pdf$