

Article

New Study of Cosmological Model with Cosmological Constant

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Abstract

Einstein's field equations with variable cosmological constant is considered in the presence of a perfect fluid for a Bianchi type-I universe by assuming that the cosmological term is proportional to the square of the Hubble parameter. The variation law for vacuum density was recently proposed by many researchers on the basis of quantum field estimation in a curved expanding background. The cosmological term tends asymptotically to a genuine cosmological constant and the model tends to a de-sitter universe. Here we wish to desire some results by using a slightly different method from that of fore researchers and obtain that the present universe is accelerating with a large fraction of cosmological density in the form of a cosmological term.

Keywords: Bianchi Type-I, spacetime, bulk viscosity, shear scalar, expansion scalar.

Introduction

In recent most remarkable observational discoveries, the cosmological constant problem is very interesting to researchers. The cosmological constant Λ was originally given by Einstein by his field equations. In an evolving universe, it appears natural to look at this constant as a function of time. The Λ term arises naturally in general relativistic quantum field theory where it is interpreted as the energy density of the vacuum [1-4]. Some authors [5-9] argued for the dependence Λ . Cosmological models with variable G and Λ have been studied by a number of researchers [10-17] for a homogeneous and isotropic FRW line element. Also Bianchi type-I models are studied by using variable G and Λ [18-24]. Schutzhold [25, 26] recently proposed a vacuum energy density proportional to the Hubble parameter, this leads to vacuum energy density decaying as $\Lambda \approx m^3 H$, where $m \approx 150 MeV$ is the energy scale of the chiral phase transmission of QCD.

Also, Borges and Carnerio [27] have considered an isotropic and homogeneous flat space filled with matter and a cosmological term proportional to H obeying the equation of state of the vacuum. Recently Tiwari and Divya Singh [28] investigated the anisotropic Bianchi type-I model with a varying Λ term. Tiwari and Sonia [29] investigated the non-existence of shear in Bianchi type-III string cosmological models with bulk viscosity and time-dependent Λ . Also

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Tiwari and Sonia [30] also investigated the Bianchi type-I string cosmological model with bulk viscosity and time-dependent Λ term. For studying the possible effects of anisotropy in the early universe on present day observations many researchers [31-35] have investigated Bianchi type-I models from different point of view.

In the present paper, we investigate the homogeneous anisotropic Bianchi type-I space time with variable Λ containing matter in the form of a perfect fluid. We obtain the solution of the Einstein field equations assuming that cosmological term is proportional to Hubble parameter for stiff matter.

Metric & Field Equations

The line element for spatially homogeneous and anisotropic Bianchi type-I space time is described by

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2 \quad (1)$$

where A, B, C are functions of t only.

We assume that cosmic matter is represented by the energy momentum tensor of a perfect fluid

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij} \quad (2)$$

where ρ and p are energy density and thermodynamic pressure, and v_i is the four velocity vector of the fluid satisfying the relation $v_i v^i = -1$. We assume that the matter content obeys an equation of state,

$$p = \omega \rho, 0 \leq \omega \leq 1 \quad (3)$$

The Einstein's equations with varying Λ in suitable units are

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} + \Lambda g_{ij} \quad (4)$$

Spatial volume V as an average scale factor of the model (1) may be defined as

$$V = R^3 = ABC \quad (5)$$

Hubble parameter H in anisotropic models may be defined as

$$H = \frac{\dot{R}}{R} = \frac{1}{3} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \quad (6)$$

where a dot stands for ordinary time derivative of the concerned quantity

$$H = \frac{1}{3}(H_1 + H_2 + H_3)$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble factors in the x, y, and z directions, respectively.

For the metric (1) and energy moment tensor (2) in the commoving system of coordinates, the field equation (4) yields

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -p + \Lambda \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -p + \Lambda \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -p + \Lambda \tag{10}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = \rho + \Lambda \tag{11}$$

In view of the vanishing divergence of the Einstein tensor, we have

$$\left[\dot{\rho} + (\rho + p) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \right] + \dot{\Lambda} = 0 \tag{12}$$

The non-vanishing component of shear tensor σ_{ij} defined by $\sigma_{ij} = u_{i;j} + u_{j;i} - \frac{2}{3} g_{ij} u^k_k$ are obtained as

$$\sigma_1^1 = \frac{4}{3} \frac{\dot{A}}{A} - \frac{2}{3} \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \tag{13}$$

$$\sigma_2^2 = \frac{4}{3} \frac{\dot{B}}{B} - \frac{2}{3} \left(\frac{\dot{C}}{C} + \frac{\dot{A}}{A} \right) \tag{14}$$

$$\sigma_3^3 = \frac{4}{3} \frac{\dot{C}}{C} - \frac{2}{3} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) \tag{15}$$

Thus the shear scalar σ is given by

$$\sigma^2 = \frac{1}{3} \left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC} \right) \quad (16)$$

$$\therefore \frac{\dot{\sigma}}{\sigma} = - \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = -3H \quad (17)$$

The Einstein's field equations (8)-(11) in terms of Hubble parameter H , shear scalar σ and deceleration parameter q can be written as

$$H^2(2q-1) - \sigma^2 = p - \Lambda \quad (18)$$

$$3H^2 - \sigma^2 = \rho + \Lambda \quad (19)$$

where

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{R\ddot{R}}{\dot{R}^2} \quad (20)$$

From equation (8), (9) and (10) and integrating, we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{ABC} = \frac{k_1}{R^3} \quad (21)$$

$$\frac{\dot{A}}{A} - \frac{\dot{C}}{C} = \frac{k_2}{ABC} = \frac{k_2}{R^3} \quad (22)$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_3}{ABC} = \frac{k_3}{R^3} \quad (23)$$

where k_1, k_2, k_3 are constants of integration. We now assume energy conservation equation $T_{ij}=0$ yields

$$\dot{\rho} + \rho(1+\omega) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0 \quad (24)$$

Using eq. (5) & (24), we get

$$\rho = \frac{k_4}{R^{3(1+\omega)}} \quad (25)$$

where k_4 is the constant of integration.

Again integrating eq. (21-23), we get

$$\frac{A}{B} = m_1 \exp\left(k_1 \int \frac{1}{R^3} dt\right) \quad (26)$$

$$\frac{A}{C} = m_2 \exp\left(k_2 \int \frac{1}{R^3} dt\right) \quad (27)$$

$$\frac{B}{C} = m_3 \exp\left(k_3 \int \frac{1}{R^3} dt\right) \quad (28)$$

where m_1, m_2, m_3 are integration constants.

From eq. (20), we obtain

$$\frac{3\sigma^2}{\theta^2} = 1 - \frac{3\rho}{\theta^2} - \frac{3\Lambda}{\theta^2} \quad (29)$$

$$\text{implying } \Lambda \geq 0, 0 < \frac{\sigma^2}{\theta^2} \leq \frac{1}{3}, 0 < \frac{\rho}{\theta^2} < \frac{1}{3}$$

Thus the presence of positive Λ lowers the upper limit of anisotropy whereas a negative value of Λ gives more room for anisotropy.

Eq. (29) can be written as

$$\frac{3\sigma^2}{3H^2} = 1 - \frac{\rho}{3H^2} - \frac{\Lambda}{3H^2} = 1 - \frac{\rho}{\rho_c} - \frac{\rho_v}{\rho_c} \quad (30)$$

where $\rho_c = 3H^2$ is the critical density and $\rho_v = \Lambda$ is the vacuum density.

$$\frac{d\theta}{dt} = -\frac{3}{2}(\rho + p) - 3\sigma^2 \quad (31)$$

Showing that the rate of volume expansion decreases during time evolution and the presence of positive Λ slows down the rate of decrease whereas a negative Λ would promote it. From eq. (19) and (20),

$$\Lambda = (2 - q)H^2 - \frac{(1 - \omega)\rho}{2} \quad (32)$$

Implying $\Lambda \leq 0$ for $q \geq 2$.

The system of equations (3), (8-11) are five independent equations in six unknowns A, B, C, ρ , p and Λ . Therefore, one extra condition is needed to solve the system completely. For this we take the cosmological term which is proportional to the Hubble parameter, since many authors considered as Λ decay. Schutzhold [26] consider variation law for vacuum density, Borges and Carnerio [27], R. K. Tiwari and Divya Singh [28], Tiwari and Sonia [29, 30] have considered a cosmological term proportional to H. Thus we take the decaying vacuum energy density

$$\Lambda = \beta H^2 \tag{33}$$

where β is the positive constant.

Let $\Omega = \Lambda/\rho$ be the ratio between the vacuum and matter densities. From eq. (19) and (33), we get

$$\beta = \frac{3\Omega}{1+\Omega} \left(1 - \frac{\sigma^2}{27\theta^2} \right) \tag{34}$$

Thus the value of β in an anisotropic background is smaller in comparison to its value in isotropic.

For stiff fluid ($\omega=1$), eq.'s (18), (19) and (33) leads to a differential equation

$$\dot{H} + (3 - \beta)H^2 = 0 \tag{35}$$

Integrating, we get

$$R = [(3 - \beta)(c_1t + c_2)]^{\frac{1}{3-\beta}} \tag{36}$$

where c_1 and c_2 are the constant of integration.

$$H = \frac{\dot{R}}{R} = c_1 [(3 - \beta)(c_1t + c_2)]^{-1} \tag{37}$$

$$A = [(3 - \beta)(c_1t + c_2)]^{\frac{1}{3-\beta}} \exp \left[\frac{2k_1 + k_2}{6 \{(3 - \beta)(c_1t + c_2)\}^{\frac{3}{3-\beta}}} \right]$$

$$B = [(3 - \beta)(c_1t + c_2)]^{\frac{1}{3-\beta}} \exp \left[\frac{k_2 - k_1}{3 \{(3 - \beta)(c_1t + c_2)\}^{\frac{3}{3-\beta}}} \right]$$

$$C = [(3-\beta)(c_1t + c_2)]^{\frac{1}{3-\beta}} \exp \left[\frac{2k_2 - k_1}{2\{(3-\beta)(c_1t + c_2)\}^{\frac{3}{3-\beta}}} \right]$$

Therefore, the metric (1) reduces to

$$ds^2 = -dt^2 + [(3-\beta)(c_1t + c_2)]^{\frac{2}{3-\beta}} \left\{ \begin{aligned} & \exp \left[\frac{2k_1 + k_2}{3\{(3-\beta)(c_1t + c_2)\}^{\frac{3}{3-\beta}}} \right] dx^2 \\ & + \exp \left[\frac{2(k_2 - k_1)}{3\{(3-\beta)(c_1t + c_2)\}^{\frac{3}{3-\beta}}} \right] dy^2 + \exp \left[\frac{2k_2 - k_1}{\{(3-\beta)(c_1t + c_2)\}^{\frac{3}{3-\beta}}} \right] dz^2 \end{aligned} \right\} F$$

or this model, the matter density ρ , pressure p , cosmological term Λ , shear scalar σ and expansion scalar θ are given by

$$\rho = p = k_4 \{(3-\beta)(c_1t + c_2)\}^{-\frac{6}{3-\beta}}$$

$$\Lambda = \beta c_1^2 \{(3-\beta)(c_1t + c_2)\}^{-2}$$

$$\theta = \frac{H}{3} = \frac{c_1}{3} \{(3-\beta)(c_1t + c_2)\}^{-1}$$

$$\sigma = (c_1t + c_2)^{\frac{-3c_1^2}{3-\beta}} c_3$$

The ratio between the vacuum and matter densities is given by

$$\Omega = \frac{\Lambda}{\rho} = \frac{\beta c_1^2}{k_4} \{(3-\beta)(c_1t + c_2)\}^{\frac{2\beta}{3-\beta}}$$

The deceleration parameter q for this model is

$$q = 2 - \beta$$

The vacuum energy density ρ_v and the critical density ρ_c are given by

$$\rho_v = \beta c_1^2 \{(3-\beta)(c_1t + c_2)\}^{-1}$$

$$\rho_c = 3c_1^2 \{(3-\beta)(c_1t + c_2)\}^{-2}$$

Special volume

$$V = R^3 = \left\{ (3 - \beta)(c_1 t + c_2) \right\}^{\frac{3}{3-\beta}}$$

The special volume V is zero at $t = -c_2/c_1$ and expansion scalar is infinite at $t = -c_2/c_1$. It shows that the universe starts evolving with zero volume with an infinite rate of expansion. The scale factor R is also zero at $t = -c_2/c_1$, it means that during initial age the space time exhibits a point type singularity. At $t = -c_2/c_1$, $\rho \rightarrow \infty$, $\sigma \rightarrow \infty$. As time increases, the scale factor R and special volume V increases but the expansion scalar decreases i.e. rate of expansion slows down. When time $t \rightarrow \infty$, then $R \rightarrow \infty$, $V \rightarrow \infty$, $\Lambda \rightarrow \infty$ and ρ , ρ_c , ρ_v all tends to zero. Therefore, model gives an empty universe for $t \rightarrow \infty$. This result is in agreement with observations obtained by many astronomers [12, 28, 30, 36]

In summary, we have investigated the Bianchi type-I cosmological model containing a stiff fluid with cosmological term $\Lambda = \beta H^2$. It is found that the deceleration parameter q for the model is 2 at $\beta = 0$, it is zero at $\beta = 2$ and decreases as β increases. The cosmological term Λ , being very large at the initial stage, later relaxes to a genuine cosmological constant which is in accordance with the recent observations [34-40]. The model asymptotically tends to the de-sitter universe.

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