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Kantowski-Sachs Models with Domain Walls in $f(R,T)$ Theory of Gravity

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Abstract

Kantowski-Sachs space time is considered in $f(R,T)$ theory of gravity proposed by Harko et al. (2011) in the presence of domain walls. Here, we have considered the functional $f(R,T)$ to be in the form $f(R,T) = f_1(R) + f_2(T)$ with $f_1(R) = \lambda_1 R$, $f_2(T) = \lambda_2 T$ and $f_1(R) = \lambda_1 R$, $f_2(T) = \lambda_2 T^2$. The exact solutions to the corresponding field equations are obtained from the relationship between the shear scalar and expansion scalar of the model. This leads to the condition $A = B^k$, where A & B are metric potentials and k is an arbitrary constant. The physical and geometrical properties of the corresponding cosmological models also discussed.

Keywords: Kantowski-Sachs metric, $f(R,T)$ gravity, domain walls.

1 Introduction:

In modern cosmology, the most attractive topic is the accelerated expansion of the Universe which is based on the recent astronomical data (Riess et al. 1998; Perlmutter et al. 1999). The root cause for this phenomenon is still under extensive investigation. However, it is said an invincible cosmic fluid called dark energy with a huge negative pressure is responsible for this expansion. There are two major approaches to tackle this problem of cosmic acceleration either by introducing a dark energy component in the Universe and study its dynamics or by interpreting it as a failure of general relativity and consider modifying Einsteins theory of gravitation termed as modified gravity approach. Noteworthy amongst them are $f(R)$ modified theory of gravity formulated by Nojiri and Odintsov (2003). Harko et al. (2011) developed $f(R,T)$ theory of gravity, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and the trace T of the stress-energy tensor.

In $f(R,T)$ gravity, the field equations are obtained from the Hilbert-Einstein type variational principle. Using gravitational units (by taking $G\&c$ as unity) the corresponding field equations of $f(R,T)$ gravity are obtained by varying the action principle with respect to g_{ij} as (Harko et al. 2011)

$$\frac{\partial f(R,T)}{\partial R}R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\nabla^{\mu}\nabla_{\mu} - \nabla_i\nabla_j)\frac{\partial f(R,T)}{\partial R} = 8\pi T_{ij} - \frac{\partial f(R,T)}{\partial R}(T_{ij} + \Theta_{ij}), \quad (1.1)$$

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where

$$\Theta_{ij} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}}. \tag{1.2}$$

Here ∇_i is the covariant derivative and T_{ij} is usual matter energy-momentum tensor derived from the Lagrangian L_m. Rao and Sireesha (2013), Rao and Neelima (2013a), Sahoo et al. (2014), Reddy et al. (2014a), Mishra & Sahoo (2014) and Rao et al. (2015) have discussed some cosmological models in $f(R,T)$ theory of gravity. Aditya et al. (2016) have discussed Bianchi type-II, $VIII$ and IX cosmological models in a modified theory of gravity with variable Λ.

The topological defects namely domain walls, monopoles, cosmic strings and textures (Kibble 1976; Vilenkin and Shellard 1994) are formed when the Universe endure a series of phase transitions. In particular, the appearance of domain wall is associated with the breaking of a discrete symmetry i.e. the vacuum manifold M consists of several disconnected components. So, the homotopy group $\phi_0(M)$ is non-trivial ($\phi_0(M) \neq 1$) (Vilenkin and Shellard 1994). Formations of galaxies are due to domain walls produced during a phase transition after the time of recombination of matter and radiation. Reddy et al. (2006) have studied on plane symmetric domain walls and cosmic strings in Bimetric theory. Very recently, Biswal et al. (2015) have discussed Kaluza-Klein cosmological model in $f(R,T)$ gravity with domain walls. Katore et al. (2015) have investigated domain walls in $f(R,T)$ theory of gravitation.

The Kantowski-Sachs cosmologies have two symmetry properties, the spherical symmetry and the invariance under spatial translations. Kantowski-Sachs space time is the simplest model with anisotropic background. Wang (2005) investigated string cosmological models with bulk viscosity in Kantowski-Sachs space time. Recently, Rao and Neelima (2013b), Reddy et al. (2014b), Rao and Suryanarayana (2015), Rao and Prasanthi (2015a, 2015b, 2016) have investigated different aspects of Kantowski-Sachs cosmological models. Motivated by above investigations in this paper, we have studied the Kantowski-Sachs metric with domain walls in $f(R,T)$ theory of gravity proposed by Harko et al. (2011).

2 Field equations and the models:

We consider the spatially homogeneous and anisotropic Kantowski-Sachs space-time in the form

$$ds^2 = dt^2 - A^2dr^2 - B^2(d\psi^2 + sin^2\psi d\varphi^2), \tag{2.1}$$

where A and B are functions of cosmic time t only.
The energy momentum tensor of the domain walls is given by

$$T_{ij} = \rho\,(g_{ij} + w_iw_j) + pg_{ij} \tag{2.2}$$

where ρ and p are energy density and pressure of the domain walls respectively which includes normal matter ρ_m and p_m as well as tension η and they are related by $p = p_m - \eta$ and $\rho = \rho_m - \eta$ where $p_m = (\gamma - 1)\rho_m$, $1 \leq \gamma \leq 2$.
Using comoving coordinate system $w^i = (0, 0, 0, 1)$ satisfying $w_iw^i = -1$, from equation (2.2), the non-vanishing components of T_{ij} can be written as

$$T^1_1 = T^2_2 = T^3_3 = \rho \quad and \quad T^4_4 = -p \tag{2.3}$$

where p and ρ are functions of time 't' only.

Here we consider a special choice of function $f(R,T) = f_1(R)+f_2(T)$, then the field equations (1.1) can be written as

$$\left(R_{ij} - \frac{1}{2}Rg_{ij}\right)\frac{\partial f_1(R)}{\partial R} = T_{ij}\left(8\pi + \frac{\partial f_2(T)}{\partial T}\right) + \left(p + \frac{T}{2}\right)\frac{\partial f_2(T)}{\partial T}g_{ij}, \tag{2.4}$$

where R_{ij} is Ricci tensor, T_{ij} is energy-momentum tensor and T is trace of energy-momentum tensor.

2.1 If $f_1(R) = \lambda_1 R$ and $f_2(T) = \lambda_2 T$:

In this case we consider the linear form of the functions $f_1(R) = \lambda_1 R$ and $f_2(T) = \lambda_2 T$ where λ_1 and λ_2 are arbitrary parameters, so that $f(R,T) = \lambda_1 R + \lambda_2 T$.
Using the above condition in equation (2.3), we have

$$R_{ij} - \frac{1}{2}Rg_{ij} = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right)T_{ij} + \frac{\lambda_2}{\lambda_1}\left(p + \frac{T}{2}\right)g_{ij}, \tag{2.5}$$

Now with the help of (2.3), the field equations (2.5) for the metric (2.1) can be written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right)\rho + \frac{\lambda_2}{2\lambda_1}(p + 3\rho) \tag{2.6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right)\rho + \frac{\lambda_2}{2\lambda_1}(p + 3\rho) \tag{2.7}$$

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{1}{B^2} = -\left(\frac{8\pi + \lambda_2}{\lambda_1}\right)p + \frac{\lambda_2}{2\lambda_1}(p + 3\rho) \tag{2.8}$$

where an overhead dot indicates ordinary differentiation with respect to time t.

By taking the transformation $dt = AB^2 d\tau$, the above field equations (2.6)-(2.8) can be written as

$$\frac{1}{A^2B^4}\left[2\frac{B''}{B} - \frac{A'B'}{AB} - \frac{3B'^2}{B^2}\right] + \frac{1}{B^2} = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right)\rho + \frac{\lambda_2}{2\lambda_1}(p + 3\rho) \tag{2.9}$$

$$\frac{1}{A^2B^4}\left[\frac{A''}{A} + \frac{B''}{B} - \frac{2A'B'}{AB} - \frac{A'^2}{A^2} - \frac{2B'^2}{B^2}\right] = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right)\rho + \frac{\lambda_2}{2\lambda_1}(p + 3\rho) \tag{2.10}$$

$$\frac{1}{A^2B^4}\left[\frac{2A'B'}{AB} + \frac{2B'^2}{B^2}\right] + \frac{1}{B^2} = -\left(\frac{8\pi + \lambda_2}{\lambda_1}\right)p + \frac{\lambda_2}{2\lambda_1}(p + 3\rho) \tag{2.11}$$

where prime denotes ordinary differentiation with respect to τ.

The field equations (2.9) to (2.11) are three independent equations with four unknowns A, B, ρ and p. In order to get a deterministic solution we take the following plausible physical conditions, the shear scalar σ is proportional to scalar expansion θ, which leads to the linear relationship between the metric potentials as

$$A = B^k \tag{2.12}$$

where $k \neq 0$ is a constant.

Using equations (2.12) in (2.9) and (2.10), we have

$$\begin{aligned} A &= \left[\frac{c_4}{c_2} sec(c_3\tau + c_4)\right]^{\frac{k}{k+1}} \\ B &= \left[\frac{c_4}{c_2} sec(c_3\tau + c_4)\right]^{\frac{1}{k+1}} \end{aligned} \tag{2.13}$$

where $c_3 = \frac{c_1}{k-1}$, $c_4 = \frac{c_1 c_2}{\sqrt{k^2-1}}$, c_1 and $c_2 \neq 0$ are integrating constants.
Thus, the metric (2.1) can be written as

$$\begin{aligned} ds^2 = \left[\frac{c_4}{c_2} sec(c_3\tau + c_4)\right]^{k+2} d\tau^2 - \left[\frac{c_4}{c_2} sec(c_3\tau + c_4)\right]^{\frac{2k}{k+1}} dr^2 \\ - \left[\frac{c_4}{c_2} sec(c_3\tau + c_4)\right]^{\frac{2}{k+1}} (d\psi^2 + sin^2\psi d\varphi^2) \end{aligned} \tag{2.14}$$

Using equation (2.13) in the equations (2.9) to (2.11), we obtain the energy density ρ_m and the pressure p_m of the normal matter as

$$\rho_m = \frac{c_3^2 \lambda_1 \left\{ \left[k^2 + 4k - 3\right] sec(c_3\tau + c_4) - [8k + 6] \right\} - \left[\frac{c_4}{c_2} sec(c_3\tau + c_4)\right]^2}{2\gamma(8\pi + \lambda_2)(k+1)^2 \left[\frac{c_4}{c_2} sec(c_3\tau + c_4)\right]^{\frac{2k+4}{k+1}}} \tag{2.15}$$

$$p_m = \frac{c_3^2 (\gamma - 1)\lambda_1 \left\{ \left[k^2 + 4k - 3\right] sec(c_3\tau + c_4) - [8k + 6] \right\} - \left[\frac{c_4}{c_2} sec(c_3\tau + c_4)\right]^2}{2\gamma(8\pi + \lambda_2)(k+1)^2 \left[\frac{c_4}{c_2} sec(c_3\tau + c_4)\right]^{\frac{2k+4}{k+1}}} \tag{2.16}$$

and the tension η of the domain walls as

$$\eta = \frac{c_3^2 \lambda_1 \left[d_1 sec(c_3\tau + c_4) + d_2\right] - \left[\frac{c_4}{c_2} sec(c_3\tau + c_4)\right]^2}{2\gamma(8\pi + \lambda_2)(8\pi + 2\lambda_2)(k+1) \left[\frac{c_4}{c_2} sec(c_3\tau + c_4)\right]^{\frac{2k+4}{k+1}}} \tag{2.17}$$

where $d_1 = 8\pi \left[2\gamma(k^3 + 3) - (k^2 + 4k - 3)\right] - 2\lambda_2 \left[\gamma(k^2 + 2k - 10) - (k^2 + 4k - 3)\right]$ and $d_2 = 64\pi(1 - \gamma) + 48\pi + 2\lambda_2 \left[\gamma(27 - 3k) + k + 6\right]$.

Thus, the metric (2.14) together with (2.15)-(2.17) constitutes a Kantowski-Sachs cosmological model with domain walls in $f(R,T)$ theory of gravity proposed by Harko et al. (2011) for a particular choice of function $f(R,T) = \lambda_1 R + \lambda_2 T$.

2.2 If $f_1(R) = \lambda_1 R$ and $f_2(T) = \lambda_2 T^2$:

The field equations (2.4) of $f(R,T)$ gravity with special choice of the functions $f_1(R) = \lambda_1 R$ and $f_2(T) = \lambda_2 T^2$ can be written as

$$R_{ij} - \frac{1}{2} R g_{ij} = \left(\frac{8\pi + 2\lambda_2 T}{\lambda_1}\right) T_{ij} + \frac{\lambda_2}{\lambda_1} \left(2pT + \frac{T^2}{2}\right) g_{ij}, \tag{2.18}$$

where λ_1 and λ_2 are arbitrary parameters.

The field equations (2.18) for the metric (2.1) with the help of (2.3), by taking the transformation $dt = AB^2 d\tau$, can be written as

$$\frac{1}{A^2B^4}\left[2\frac{B''}{B} - \frac{\dot{A'}B'}{AB} - \frac{3B'^2}{B^2}\right] + \frac{1}{B^2} = \frac{8\pi}{\lambda_1}\rho + \frac{\lambda_2}{2\lambda_1}(3\rho - p)(7\rho - 3p) \quad (2.19)$$

$$\frac{1}{A^2B^4}\left[\frac{A''}{A} + \frac{B''}{B} - \frac{2\dot{A'}B'}{AB} - \frac{A'^2}{A^2} - \frac{2B'^2}{B^2}\right] = \frac{8\pi}{\lambda_1}\rho + \frac{\lambda_2}{2\lambda_1}(3\rho - p)(7\rho - 3p) \quad (2.20)$$

$$\frac{1}{A^2B^4}\left[\frac{2\dot{A'}B'}{AB} + \frac{2B'^2}{B^2}\right] + \frac{1}{B^2} = -\frac{8\pi}{\lambda_1}p + \frac{\lambda_2}{2\lambda_1}(p - 3\rho)^2 \quad (2.21)$$

where prime denotes ordinary differentiation with respect to τ.

The above set of field equations give the values of metric potential A and B same as that of equations (2.13). We assume a barotropic equation of state described through the relationship $p = \omega\rho$. Here ω is the equation of state (EoS) parameter and for the sake of simplicity, it is assumed to be constant quantity. Using metric potentials given by equation (2.13) in the equations (2.19) to (2.21), we obtain the energy density ρ_m and the pressure p_m of the normal matter as

$$\rho_m = \frac{2c_3^2\lambda_1(\omega - 1)\left\{[14\omega + 5k\omega - 13k - 10]\, tan^2(c_3\tau + c_4) + sec^2(c_3\tau + c_4)\right\}}{24\pi(k^2 - 1)^2(1 + 2\omega - \omega^2)\left[\frac{c_4}{c_2}sec(c_3\tau + c_4)\right]^{\frac{2k+4}{k+1}}} \quad (2.22)$$

$$p_m = \frac{2c_3^2\lambda_1(\omega - 1)\left\{[14\omega + 5k\omega - 13k - 10]\, tan^2(c_3\tau + c_4) + sec^2(c_3\tau + c_4)\right\}}{24\gamma\pi(k^2 - 1)^2(1 + 2\omega - \omega^2)\left[\frac{c_4}{c_2}sec(c_3\tau + c_4)\right]^{\frac{2k+4}{k+1}}} \quad (2.23)$$

and the tension η of the domain walls as

$$\eta = \frac{4c_3^2\lambda_1(\gamma - \omega - 1)\left\{[14\omega + 5k\omega - 13k - 10]\, tan^2(c_3\tau + c_4) + sec^2(c_3\tau + c_4)\right\}}{24\pi\gamma(k^2 - 1)^2(1 + 2\omega - \omega^2)\left[\frac{c_4}{c_2}sec(c_3\tau + c_4)\right]^{\frac{2k+4}{k+1}}} \quad (2.24)$$

Thus, the metric (2.14) together with (2.22)-(2.24) constitutes a Kantowski-Sachs cosmological model with domain walls in $f(R,T)$ theory of gravity proposed by Harko et al. (2011) for a particular choice of function $f(R,T) = \lambda_1 R + \lambda_2 T^2$. ,

3 Some other properties of the models:

Spatial volume and average scale factor of the models are given by

$$V = \sqrt{-g} = \left[\frac{c_4}{c_2}sec(c_3\tau + c_4)\right]^{\frac{k+2}{k+1}} sin\ \psi \quad (3.1)$$

$$a(t) = V^{\frac{1}{3}} = \left\{\left[\frac{c_4}{c_2}sec(c_3\tau + c_4)\right]^{\frac{k+2}{k+1}} sin\ \psi\right\}^{1/3} \quad (3.2)$$

The expressions for Hubbles parameter (H), expansion scalar (θ) and shear scalar (σ) are given by

$$H = \frac{\dot{a}}{a} = \frac{c_3(k+2)tan(c_3\tau + c_4)}{3(k+1)[sec(c_3\tau + c_4)]^{\frac{k+2}{k+1}}} \tag{3.3}$$

$$\theta = 3H = \frac{c_3(k+2)tan(c_3\tau + c_4)}{(k+1)[sec(c_3\tau + c_4)]^{\frac{k+2}{k+1}}} \tag{3.4}$$

$$\sigma^2 = \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 = \frac{c_3^2(k-1)^2tan^2(c_3\tau + c_4)}{9(k+1)^2[sec(c_3\tau + c_4)]^{\frac{2(k+2)}{k+1}}} \tag{3.5}$$

Anisotropic parameter is

$$A_h = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2(k-1)^2}{(k+2)^2} \tag{3.6}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = H_3 = \frac{\dot{B}}{B}$ are the directional Hubble parameters.
The deceleration parameter is

$$\begin{aligned} q &= \frac{-a\ddot{a}}{\dot{a}^2} \\ &= -\left[\frac{3(k+1)}{k+2}cosec^2(c_3\tau + c_4)\right] + 2 \end{aligned} \tag{3.7}$$

The tensor of rotation $u_{ij} = w_{i,j} - w_{j,i}$ is identically zero and hence this universe is non-rotational.

4 Summary and conclusions:

In this paper, we have studied a spatially homogeneous and anisotropic Kantowski-Sachs cosmological models with domain walls in $f(R,T)$ theory of gravity proposed by Harko et al. (2011) with an appropriate choice of a function $f(R,T) = \lambda_1 R + \lambda_1 T$ and $f(R,T) = \lambda_1 R + \lambda_1 T^2$.

- The models presented here are free from singularities and the volume of the models is constant at $\tau = \tau^*$ where $\tau^* = -\frac{c_4}{c_3}$. We observed that the expansion scalar, shear scalar and Hubble's parameter vanish at initial epoch, i.e., $\tau = \tau^*$ whereas they increase as τ increases, which shows that the Universe starts evolving with constant volume at $\tau = \tau^*$ with increasing rate of expansion. The pressure, energy density and tension of the domain walls are attain a constant value at initial epoch i.e., $\tau = \tau^*$ and vanishes for large values of τ in both the models.

- Since, the deceleration parameter q has negative values the obtained models represent accelerating expansion of the Universe, which is consistent with recent observations from the recent supernovae Ia (Riess et al. 1998; Perlmutter et al. 1999). As $A_h \neq 0$ (for $m \neq 1$) the models are anisotropy throughout the evolution of the Universe.

The models represent expanding, shearing, accelerating and non-rotating Universe.The solutions presented in this paper are new and different from other authors solutions. Our solutions may be useful for better understanding of the evolution of the Universe in Kantowski-Sachs Universe within the frame work of $f(R,T)$ modified theory of gravitation.

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