

Article**Bianchi Type-V Two-Fluid Dark Energy Cosmological Models in Saez-Ballester Theory of Gravitation**T. Vinutha^{*}, V. U. M. Rao, M. Vijaya Santhi, Y. Aditya & Molla Mengesha Nigus

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Abstract

This paper deals with a spatially homogeneous and anisotropic Bianchi-V universe filled with barotropic fluid and dark energy in the frame work of scalar tensor theory of gravitation proposed by Saez and Ballester (1986). We consider both the cases when the dark energy is minimally coupled to barotropic fluid as well as direct interaction with it. The exact solutions of the corresponding field equations are obtained and also some important features of the models, thus obtained, have been discussed in detail.

Keywords: Bianchi-V metric, Saez-Ballester theory, two-fluids, dark energy, EoS Parameter.

1. Introduction

In the study of modern cosmology, we consider that the total energy density of the universe is dominated by the densities of two components: the dark matter and the dark energy. The recent observational data strongly motivate to study general properties of the cosmological models containing more than one fluid. These universes are modeled with perfect fluids and with mixtures of non interacting fluids under the assumption that there is no energy transfer among the components. But, such scenarios are not confirmed by observational data. This motivates us to study cosmological models containing fluids which interact with each other. In recent years there has been immense interest in cosmological models with dark energy in general relativity because of the fact that our observable universe is undergoing a phase of accelerated expansion, which has been confirmed by several cosmological observations such as type 1a supernova by several authors (Riess et al. 1998; Perlmutter et al. 1999). Caldwell (2002) and Huang (2006) have discussed cosmic microwave background (CMB) anisotropy and Daniel et al. (2008) have studied the large scale structure and strongly indicate that dark energy dominates the present universe, causing cosmic acceleration. Based on these observations, cosmologists have accepted the idea of dark energy, which is a fluid with negative pressure, making up around 70 % of the present universe energy content to be responsible for this acceleration due to repulsive gravitation.

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Cosmologists have proposed many candidates for dark energy to fit the current observations such as cosmological constant, Tachyon, quintessence, phantom and so on. Evolution of the equation of state (EoS) of dark energy $w_{de} = p_{de} / \rho_{de}$ transfers from $w_{de} > -1$ in the near past (quintessence region) to $w_{de} < -1$ at recent stage (phantom region). Akarsu and Kilinc (2010), Yadav (2011), Yadav and Yadav (2011), Pradhan et al. (2011a, b), Pradhan and Amirhashchi (2011) and Yadav et al. (2011) have investigated different aspects of dark energy models in general relativity with variable EoS parameter. The concept of dark energy was proposed for understanding this currently accelerating expansion of the universe, and then its existence was confirmed by several high precision observational experiments (Tegmark et al. 2004), especially the Wilkinson Microwave Anisotropy Probe (WMAP) satellite experiment. The WMAP shows that dark energy occupies about 73% of the energy of the universe, and dark matter about 23%. The usual baryon matter, which can be described by our known particle theory, occupies only about 4% of the total energy of the universe.

This motivates us to study cosmological models containing fluids which interact with each other. Cataldo et al. (2008) have considered the simplest non-trivial cosmological scenarios for an interacting mixture of two cosmic fluids described by power-law scale factors, i.e. the expansion as a power-law in time. Whereas, an interacting and non-interacting two-fluid scenario for dark energy in an FRW universe with constant deceleration parameter have been described by Pradhan et al. (2011). Adhav et al. (2011) have investigated interacting cosmic fluids in LRS Bianchi type-I cosmological model. Saha et al. (2012) revisited two-fluid scenario for dark energy models in an FRW universe investigated by Amirhashchi et al. (2011). Reddy and Santhi Kumar (2013) have discussed two fluid scenario for dark energy model in a scalar-tensor theory of gravitation. Amirhashchi et al. (2013) have studied interacting two-fluid viscous dark energy models in a non-flat universe. Recently Rao et al. (2016) have discussed two-fluid scenario for higher dimensional dark energy cosmological model in Saez-Ballester theory of gravitation

Saez and Ballester (1986) formulated a scalar-tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field an antigravity regime appears. This theory also suggests a possible way to solve missing matter problem in non flat FRW cosmologies.

The field equations given by Saez- Ballester (1986) for the combined scalar and tensor fields (using geometrized units with $c = 1, 8\pi G = 1$) are

$$G_{ij} - \omega \phi^r \left(\phi_{,i} \phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k} \phi^{,k} \right) = -T_{ij} \quad (1.1)$$

and the scalar field ϕ satisfies the equation

$$2\phi^r \phi^i_{,i} + r\phi^{r-1} \phi_{,k} \phi^k = 0, \tag{1.2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}R g_{ij}$ is an Einstein tensor, R is the scalar curvature, ω and r are constants, T_{ij} is the stress energy tensor of the matter.

The energy conservation equation is

$$T^{ij}_{,j} = 0. \tag{1.3}$$

The study of cosmological models in the framework of scalar-tensor theories has been the active area of research for the last few decades. In particular, Rao et al. (2007), Rao et al. (2008), Katore et al. (2010), Rao et al. (2011), Pradhan et al. (2013), Reddy et al. (2014) and Rao & Jayasudha (2015) are some of the authors who have investigated several aspects of the cosmological models in Saez-Ballester (1986) scalar-tensor theory.

Bianchi type cosmological models are important in the sense that these are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view anisotropic universe has a greater generality than isotropic models. The simplicity of the field equations and relative ease of solutions made Bianchi space times useful in constructing models of spatially homogeneous and anisotropic cosmologies. Bianchi types V and VII_h represent the generalized open FRW models.

Motivated by above investigations, in this paper we will discuss Bianchi type-V space-time filled with two fluids (barotropic and dark energy) in a scalar-tensor theory of gravitation proposed by Saez-Ballester (1986) theory of gravitation. The paper is organized as follows, in section 2 we discuss metric and field equations. In section 3 and 4, we obtained solutions for interacting and non-interacting cosmological models respectively. In section 5, we discuss some other important features of the obtained models. The last section contains some discussions and conclusions.

2. Metric and field equations

We consider spatially homogeneous Bianchi type-V metric in the form

$$ds^2 = dt^2 - a_1^2 dx^2 - e^{-2\alpha x} (a_2^2 dy^2 + a_3^2 dz^2) \tag{2.1}$$

where a_1 , a_2 and a_3 are functions of time ‘ t ’ only and α is a constant.

The total energy momentum tensor for two fluids is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{2.2}$$

where $\rho = \rho_m + \rho_{de}$ and $p = p_m + p_{de}$. Here ρ_m and p_m are energy density and pressure of barotropic fluid and ρ_{de} and p_{de} are energy density and pressure of dark fluid respectively, u^i is the four-velocity of the fluid satisfying the following condition,

$$g_{ij} u^i u^j = 1 \tag{2.3}$$

In a commoving coordinate system, we get

$$T_1^1 = T_2^2 = T_3^3 = -p, T_4^4 = \rho \text{ and } T_j^i = 0 \text{ for } i \neq j \tag{2.4}$$

where the quantities ρ and p are functions of ‘ t ’ only.

The field equations (1.1) for the metric (2.1) with the help of (2.2) to (2.4), can be written as

$$\frac{\alpha^2}{a_1^2} - \frac{\ddot{a}_2}{a_2} - \frac{\ddot{a}_3}{a_3} - \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{\omega}{2} \phi^r \dot{\phi}^2 = w_m \rho_m + w_{de} \rho_{de} \tag{2.5}$$

$$\frac{\alpha^2}{a_1^2} - \frac{\ddot{a}_1}{a_1} - \frac{\ddot{a}_3}{a_3} - \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} + \frac{\omega}{2} \phi^r \dot{\phi}^2 = w_m \rho_m + w_{de} \rho_{de} \tag{2.6}$$

$$\frac{\alpha^2}{a_1^2} - \frac{\ddot{a}_1}{a_1} - \frac{\ddot{a}_2}{a_2} - \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\omega}{2} \phi^r \dot{\phi}^2 = w_m \rho_m + w_{de} \rho_{de} \tag{2.7}$$

$$\frac{3\alpha^2}{a_1^2} - \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} - \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} - \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1} - \frac{\omega}{2} \phi^r \dot{\phi}^2 = -\rho_m - \rho_{de} \tag{2.8}$$

$$2\alpha \frac{\dot{a}_1}{a_1} - \alpha \frac{\dot{a}_2}{a_2} - \alpha \frac{\dot{a}_3}{a_3} = 0 \tag{2.9}$$

$$\ddot{\phi} + \dot{\phi} \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \right) + \frac{r}{2\phi} \dot{\phi}^2 = 0 \tag{2.10}$$

The conservation equation yields

$$\dot{\rho}_m + 3(1 + w_m)\rho_m \frac{\dot{a}_1}{a_1} + \dot{\rho}_{de} + 3(1 + w_{de})\rho_{de} \frac{\dot{a}_1}{a_1} = 0 \tag{2.11}$$

3. Non-interacting two -fluid model

Here we consider that two fluids do not interact with each other. Hence the general form of conservation equation (2.11) leads us to write the conservation equations for the dark energy and barotropic fluid separately as

$$\dot{\rho}_m + 3\frac{\dot{a}_1}{a_1}(\rho_m + p_m) = 0 \tag{3.1}$$

and
$$\dot{\rho}_{de} + 3\frac{\dot{a}_1}{a_1}(\rho_{de} + p_{de}) = 0 \tag{3.2}$$

The EoS parameter of barotropic fluid w_m is constant (Akarsu and Kilinc 2010),

$$\text{i.e., } w_m = \frac{p_m}{\rho_m} = \text{constant.} \tag{3.3}$$

So, equation (3.1) is integrable, while w_{de} is a function of time.

In order to get a deterministic solution we take the following plausible physical condition, the shear scalar σ is proportional to scalar expansion θ , which leads to the linear relationship between the metric potentials a_2 and a_3 , i.e.,

$$a_2^n = a_3 \tag{3.4}$$

where a_2 and a_3 are metric potentials and n is positive constant.

From equation (2.9), we get $a_1^2 = c_1 a_2 a_3$,

without loss of generality, by taking $c_1 = 1$, we get

$$a_1^2 = a_2 a_3 \tag{3.5}$$

From equations (2.5), (2.6), (2.7) and (3.5), we get

$$\frac{\ddot{a}_3}{\dot{a}_3} + \left(\frac{3n+1}{2}\right)\frac{\dot{a}_3}{a_3} = 0. \tag{3.6}$$

From equation (3.6), we get

$$a_3 = (c_2t + c_3)^{\frac{2}{3(n+1)}}, \tag{3.7}$$

where $c_2 = \frac{3(n+1)}{2}k_1$ and $c_3 = \frac{3(n+1)}{2}k_2$.

k_1, k_2 are constants of integration.

From equations (3.5) and (3.7), we get

$$a_2 = (c_2t + c_3)^{\frac{2n}{3(n+1)}}. \tag{3.8}$$

From equations (3.4), (3.7) and (3.8) we get

$$a_1 = (c_2t + c_3)^{\frac{1}{3}}. \tag{3.9}$$

From (2.10) we obtain the scalar field as

$$\phi^{\frac{r+2}{2}} = \frac{(r+2)c_4}{2c_2} \log(c_2t + c_3) \tag{3.10}$$

where c_4 is an integrating constant.

Now, the metric (2.1), in this case, can be written as

$$ds^2 = dt^2 - (c_2t + c_3)^{\frac{2}{3}} dx^2 - e^{-2\alpha x} [(c_2t + c_3)^{\frac{4n}{3(n+1)}} dy^2 + (c_2t + c_3)^{\frac{4}{3(n+1)}} dz^2] \tag{3.11}$$

From equation (3.3), (3.1) can be written as

$$\rho_m = \rho_0 a_1^{-3(1+w_m)} \tag{3.12}$$

where ρ_0 is an integrating constant.

Using (3.9) in (3.12), we get

$$\rho_m = \rho_0 (c_2 t + c_3)^{-(1+w_m)} \tag{3.13}$$

From (3.3) and (3.13), we get

$$p_m = \rho_0 w_m (c_2 t + c_3)^{-(1+w_m)} \tag{3.14}$$

From equations (3.10), (2.5) - (2.8) and (3.14), we get pressure and energy density for dark energy as

$$p_{de} = \frac{\omega}{2} c_4^2 (c_2 t + c_3)^{-2} + \frac{2c_2^2 (n^2 + 4n + 1)(c_2 t + c_3)^{-2}}{9(n+1)^2} + \alpha^2 (c_2 t + c_3)^{-2/3} - \rho_0 w_m (c_2 t + c_3)^{-(1+w_m)} \tag{3.15}$$

$$\rho_{de} = \frac{\omega}{2} c_4^2 (c_2 t + c_3)^{-2} + \frac{2c_2^2 (n^2 + 4n + 1)(c_2 t + c_3)^{-2}}{9(n+1)^2} - 3\alpha^2 (c_2 t + c_3)^{-2/3} - \rho_0 (c_2 t + c_3)^{-(1+w_m)} \tag{3.16}$$

By using equations (3.15) and (3.16), we find the EoS parameter $\left(w_{de} = \frac{P_{de}}{\rho_{de}} \right)$ of dark energy as

$$w_{de} = \left(\frac{\frac{\omega}{2} c_4^2 (c_2 t + c_3)^{-2} + \frac{2c_2^2 (n^2 + 4n + 1)(c_2 t + c_3)^{-2}}{9(n+1)^2} + \alpha^2 (c_2 t + c_3)^{-2/3} - \rho_0 w_m (c_2 t + c_3)^{-(1+w_m)}}{\frac{\omega}{2} c_4^2 (c_2 t + c_3)^{-2} + \frac{2c_2^2 (n^2 + 4n + 1)(c_2 t + c_3)^{-2}}{9(n+1)^2} - 3\alpha^2 (c_2 t + c_3)^{-2/3} - \rho_0 (c_2 t + c_3)^{-(1+w_m)}} \right) \tag{3.17}$$

Thus the metric (3.11) together with (3.10) and (3.12)-(3.17) constitutes Bianchi type-V non-interacting two fluid cosmological model in Saez - Ballester theory of gravitation.

Fig.1: Plot of density of ordinary matter and dark energy versus time in non-interacting model. Here $n = 2$, $k_1 = c_4 = \rho_0 = 1$, $k_2 = 0$, $\alpha = 0.1$, $w_m = \omega = 2$.

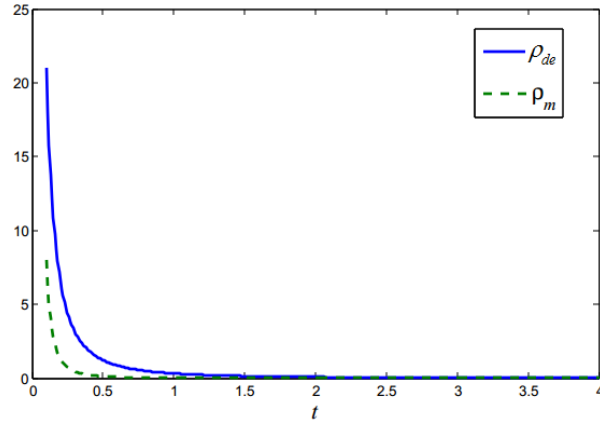
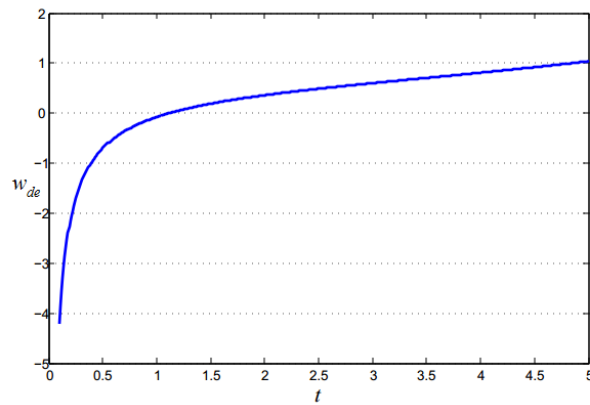


Fig.2: Plot of EoS parameter of dark energy versus time for non-interacting model. Here $n = 2$, $k_1 = c_4 = \rho_0 = 1$, $k_2 = 0$, $\alpha = 0.1$, $w_m = \omega = 2$.



The behavior of ρ_m and ρ_{de} in terms of cosmic time t are shown in Fig. 1. Both are positive decreasing function of time and converge to zero for sufficiently large times.

Figure 2 depicts the behavior of the EoS parameter of dark energy w_{de} versus time. It is observed that w_{de} begins in the phantom region, increases and becomes -1 around $t = 0.5$ then passes into the quintessence region and tends to a constant which is in the quintessence region.

The expressions for the matter-density and dark-energy density are given by

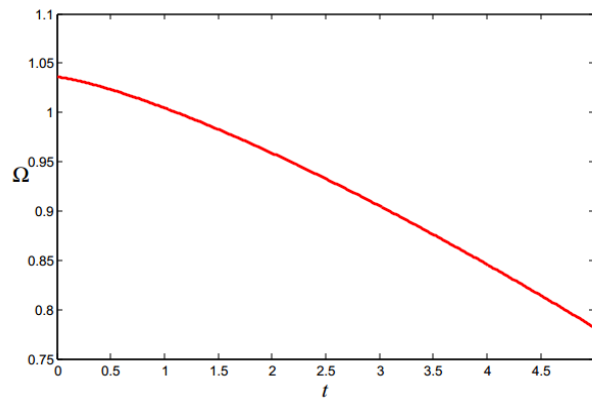
$$\Omega_m = \frac{3\rho_0(c_2t + c_3)^{1-w_m}}{c_2^2} \tag{3.18}$$

$$\Omega_{de} = \frac{3\omega c_4^2}{2c_2^2} + \frac{2(n^2 + 4n + 1)}{3(n + 1)^2} - \frac{9\alpha^2}{c_2^2}(c_2t + c_3)^{4/3} - \frac{3\rho_0(c_2t + c_3)^{(1-w_m)}}{c_2^2} \tag{3.19}$$

And the total density parameter Ω is given by

$$\Omega = \frac{3\omega c_4^2}{2c_2^2} + \frac{2(n^2 + 4n + 1)}{3(n + 1)^2} - \frac{9\alpha^2}{c_2^2}(c_2t + c_3)^{4/3} \tag{3.20}$$

Fig. 3: Plot of overall density parameter versus time in non-interacting model. Here $n = 2$, $k_1 = c_4 = \rho_0 = 1$, $k_2 = 0$, $\alpha = 0.1$, $w_m = \omega = 2$.



The variation of total energy density parameter with cosmic time is shown in Figure 3. It is observed that density parameter (Ω) is decreasing function of time and varying in positive region throughout the evolution of the universe. This result is compatible with the observational results.

4. Interacting two-fluid model

Now, here we consider the interaction between dark energy and barotropic fluids. For this purpose we can write the continuity equations for dark fluid and barotropic fluids as

$$\dot{\rho}_m + 3\frac{\dot{a}_1}{a_1}(\rho_m + p_m) = Q \tag{4.1}$$

and
$$\dot{\rho}_{de} + 3\frac{\dot{a}_1}{a_1}(\rho_{de} + p_{de}) = -Q \tag{4.2}$$

The quantity Q expresses the interaction between the dark energy components. Since we are interested in an energy transfer from the dark energy to dark matter, we consider $Q > 0$ which ensures that the second law of thermodynamics is fulfilled (Pavon and Wang 2009). Following Amendola et al. (2007) and Guo et al. (2007), we consider

$$Q = 3H\sigma\rho_m \tag{4.3}$$

where σ is a coupling constant.

Using equation (4.3) in equation (4.1) and integrating, we obtain

$$\rho_m = \rho_0 a_1^{-3(1+w_m-\sigma)} \tag{4.4}$$

where ρ_0 is an integrating constant.

Using (3.9) in (4.4), we get

$$\rho_m = \rho_0(c_2t + c_3)^{-(1+w_m-\sigma)} \tag{4.5}$$

Now, the barotropic pressure is given by

$$p_m = \rho_0 w_m (c_2t + c_3)^{-(1+w_m-\sigma)} \tag{4.6}$$

From equations (3.10), (2.5) - (2.8) and (4.5), we get pressure and energy density for dark energy as

$$p_{de} = \frac{\omega}{2} c_4^2 (c_2t + c_3)^{-2} + \frac{2c_2^2(n^2 + 4n + 1)(c_2t + c_3)^{-2}}{9(n+1)^2} + \alpha^2 (c_2t + c_3)^{-2/3} - \rho_0 w_m (c_2t + c_3)^{-(1+w_m-\sigma)} \tag{4.7}$$

$$\rho_{de} = \frac{\omega}{2} c_4^2 (c_2t + c_3)^{-2} + \frac{2c_2^2(n^2 + 4n + 1)(c_2t + c_3)^{-2}}{9(n+1)^2} - 3\alpha^2 (c_2t + c_3)^{-2/3} - \rho_0 (c_2t + c_3)^{-(1+w_m-\sigma)} \tag{4.8}$$

Using (4.7) and (4.8) we get the EoS parameter $\left(w_{de} = \frac{P_{de}}{\rho_{de}} \right)$ of dark energy as

$$w_{de} = \left(\frac{\frac{\omega}{2} c_4^2 (c_2t + c_3)^{-2} + \frac{2c_2^2(n^2 + 4n + 1)(c_2t + c_3)^{-2}}{9(n+1)^2} + \alpha^2 (c_2t + c_3)^{-2/3} - \rho_0 w_m (c_2t + c_3)^{-(1+w_m-\sigma)}}{\frac{\omega}{2} c_4^2 (c_2t + c_3)^{-2} + \frac{2c_2^2(n^2 + 4n + 1)(c_2t + c_3)^{-2}}{9(n+1)^2} - 3\alpha^2 (c_2t + c_3)^{-2/3} - \rho_0 (c_2t + c_3)^{-(1+w_m-\sigma)}} \right) \tag{4.9}$$

Thus the metric (3.11) together with (3.10) and (4.5) - (4.9) constitutes Bianchi type-V interacting two fluid cosmological model in Saez - Ballester theory of gravitation.

Fig. 4: Plot of energy density of dark energy and barotropic fluid in interacting model. Here $n = 2, k_1 = c_4 = \rho_0 = 1, k_2 = 0, \alpha = 0.1, w_m = \omega = 2, \sigma = 0.4$.

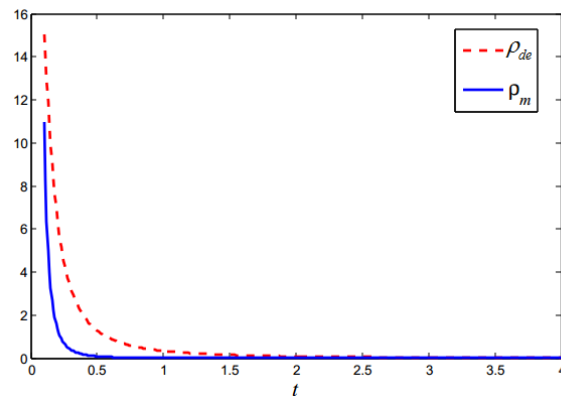
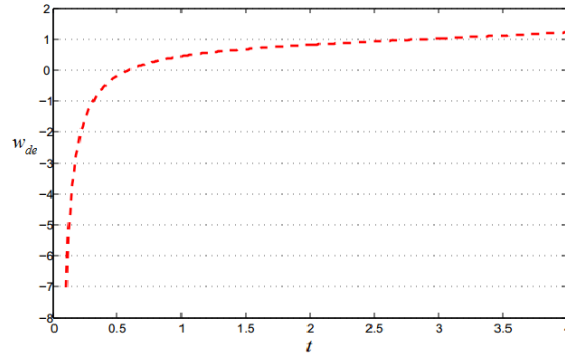


Fig.5: Plot of EoS parameter of dark energy versus time for interacting model. Here $n = 2$, $k_1 = c_4 = \rho_0 = 1$, $k_2 = 0$, $\alpha = 0.1$, $w_m = \omega = 2$, $\sigma = 0.4$.



Figures 3 and 4 describe the behavior of density of dark energy & barotropic fluid and EoS parameter of dark energy in terms of time ‘t’. We observed that the behavior of ρ_{de} , ρ_m and w_{de} in this case is almost similar to non-interacting case.

5. Some other important properties of the models

The rate of expansion in the direction of x, y and z are given by

$$\begin{aligned}
 H_x &= \frac{\dot{a}_1}{a_1} = \frac{c_2}{3(c_2t + c_3)} , \\
 H_y &= \frac{\dot{a}_2}{a_2} = \frac{2nc_2}{3(n+1)(c_2t + c_3)} , \\
 H_z &= \frac{\dot{a}_3}{a_3} = \frac{2c_2}{3(n+1)(c_2t + c_3)} .
 \end{aligned}
 \tag{5.1}$$

The mean Hubble’s parameter (H), expansion scalar (θ) and shear scalar (σ^2) are given by

$$H = \frac{c_2}{3(c_2t + c_3)} , \tag{5.2}$$

$$\theta = \frac{c_2}{(c_2t + c_3)} , \tag{5.3}$$

$$\sigma^2 = \frac{(n-1)^2 c_2^2}{9(n+1)^2 (c_2t + c_3)^2} . \tag{5.4}$$

The spatial volume (V), mean anisotropy parameter (A_m) and deceleration parameter (q) are found to be

$$V = (c_2 t + c_3) e^{-2mx} , \tag{5.5}$$

$$A_m = \frac{2(n-1)^2}{3(n+1)^2} , \tag{5.6}$$

$$q = \frac{d}{dt} \left(\frac{1}{H} \right) - 1 = 2 . \tag{5.7}$$

From equations (5.3) and (5.4), we get

$$\frac{\sigma}{\theta} = \frac{(n-1)}{3(n+1)} . \tag{5.8}$$

6. Discussion and Conclusions

In this paper, we have obtained and presented spatially homogeneous Bianchi type-V cosmological model filled with barotropic fluid and dark energy in a scalar tensor theory of gravitation proposed by Saez-Ballester (1986). The following are the observations and conclusions:

- From (3.11) it can be observed that the models have no initial singularity i.e. at $t = 0$. The volume of the models vanishes at $t = -c_3/c_2$ and expansion scalar tends to infinity, which shows that the Universe starts evolving with zero volume at $t = -c_3/c_2$ with an infinite rate of expansion.
- The spatial volume increases and becomes infinitely large as $t \rightarrow \infty$. We observe that $H, \sigma^2, \theta, \rho_{de}$ and ρ_m diverge at $t = 0$ and they all vanish as $t \rightarrow \infty$. The scalar field (ϕ) is diverge as $t \rightarrow \infty$ and vanishes for small values of time i.e., at $t = 0$. From (5.6), one can observe that average anisotropy $A_m \neq 0$ (for $n \neq 1$) throughout the history of the Universe, this indicates that these models are anisotropic.
- It is observed that in both interacting and non-interacting cases the behavior of EoS parameter of dark energy is almost same, i.e., increasing function of cosmic time which explains the late time acceleration of the universe.

- From (5.7) we observed that the deceleration parameter is positive and hence the model obtained is decelerating initially and will accelerate in finite time due to cosmic re-collapse (Nojiri and Odintsove 2003).

The two models presented here are anisotropic, non-rotating, shearing and also accelerating in a standard way. Hence they represent not only the early stage of evolution but also the present universe.

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