**Article**

# Holographic Dark Energy Model with Generalized Chaplygin Gas in a Scalar-tensor Theory of Gravitation

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**Abstract**

The present work deals with a five-dimensional spherically symmetric space-time filled with two minimally interacting fluids: matter and holographic dark energy in Saez-Ballester (1) scalar tensor theory of gravitation. We have investigated geometric and kinematic properties of the model and the role of the holographic dark energy in the evolution of the spherically symmetric universe. We established a correspondence between the holographic dark energy model with the generalised Chaplygin gas dark energy model. We also reconstructed the potential and dynamics of the scalar field which describes the Chaplygin cosmology.

**Keywords:** Spherically symmetric metric, holographic dark energy, generalised Chaplygin gas.

## 1 Introduction

One of the outstanding developments in cosmology is the discovery of the accelerated expansion of the Universe which is believed to be driven by some exotic dark energy (Perlmutter et al. (2); Reiss et al. (3)). The nature and composition of dark energy is still an open problem. The thermo-dynamical studies of dark energy reveal that the constituents of dark energy may be massless particles (bosons or fermions) whose collective behavior resembles with a kind of radiation fluid having negative pressure. Also, it is commonly believed by the cosmological community that this unknown exotic physical entity known as dark energy is a kind of repulsive force which acts as antigravity responsible for gearing up the Universe. The Wilkinson Microwave Anisotropy Probe (WMAP) satellite experiment suggests that 73% content of the Universe is in the form of dark energy, 23% is in the form of non-baryonic dark matter and rest 4% is in the form of usual baryonic (normal) matter as well as radiation.

It has been conjectured that the simplest dark energy candidate is the cosmological constant, but it needs to be extremely fine tuned to satisfy the current value of the dark energy. Chaplygin gas as well as generalized Chaplygin gas has also been considered as possible candidates for dark energy due to negative pressure (Bertolami et al. (4); Bento et al. (5)). Some authors have also suggested that interacting & non-interacting two fluids scenario is possible dark energy

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candidates (Setare et al. (6); Pradhan et al. (7)) and some have considered modified gravitational action by adding a function f(R) (R being the Ricci scalar curvature) to Einstein-Hilbert Lagrangian where f(R) provides a gravitational alternative for dark energy causing late time acceleration of the Universe (Nojiri & Odinstov (8); Caroll et al. (9)). A review of modified gravity to explain dark energy is made available by Nojiri & Odintsov (10) and Copeland et al. (11). In spite of these attempts cosmic acceleration is, still, a challenge for modern cosmology.

Recently, a different idea has attracted considerable attention, proposing a model which has its roots on quantum gravity and is known as the holographic principle (t Hooft (13); Cohen et al. (14); Susskind (15)). The foundation of the holographic principle resides in the idea that the entropy of a given system does not depend upon the volume, but rather on the surface area surrounding it. In a cosmological context, the holographic principle establishes an upper limit for the entropy of the universe. Cohen et al. (14) conjecture that in a quantum field theory there exists a relationship between an ultraviolet (UV) cutoff and an infrared (IR) cutoff, L, which origins lies on the constraint imposed to a region of space preventing a black hole formation. The size of this region should not be larger than L, meaning that if there is an energy density $\rho_\Lambda$ (zero-point quantum energy density) in a region associated to the UV, then the total energy in a region of size L cannot overpass the mass of a black hole with the same size, so that

$$L^3\rho_\Lambda \leq LM_p^2. \tag{1.1}$$

In the cosmological context, the holographic energy density corresponds to the dark energy density (holographic dark energy) (Hsu (16); Li (17)). The maximum value L can take is obtained by considering the equality in equation (1.1), such that the holographic energy density is given by

$$\rho_\Lambda = 3c^2M_p^2L^{-2} \tag{1.2}$$

where $c$ is a numerical constant and $M_p^{-2} = 8\pi G$ is the reduced Planck mass.

Several approaches for the IR cutoff $L$ have been proposed in the literature (Li (17); Gao et al. (18); Granda and Oliveros (19)). For instance, in Nojiri and Odintsov (20), a generalized holographic dark energy model was proposed, identifying the IR cutoff with combination of FRW parameters. In this work, we consider the one given by Granda and Oliveros (19), where the authors took the holographic dark energy density to depend on the usual quadratic term in the Hubble parameter (H) as well as on the time derivative of H, i.e., $\rho_\Lambda = M_p^2(\beta_1 H^2 + \beta_2\dot{H})$. This model has been widely studied by the scientific community in various contexts and it has been known as the new holographic dark energy model (NHDE).

There has been a number of works done concerning the reconstruction of the holographic scalar field models of dark energy. For instance, Setare have studied the correspondence between the holographic dark energy and each one of Chaplygin gas (Setare (21)) and generalized Chaplygin gas (Setare (22)) in FRW universe. On the other hand, Setare (23) has shown a correspondence between the interacting generalized Chaplygin gas and phantom dark energy model in non-flat FRW universe. Sarkar (25) has discussed holographic dark energy model with linearly varying deceleration parameter and generalized Chaplygin gas dark energy model in Bianchi type-I universe. Jalil (26) has investigated interacting holographic generalized cosmic Chaplygin gas model. Kiran et al. (27), Reddy et al. (28), (29) have studied Bianchi type minimally interacting holographic dark energy models in Brans-Dicke and Saez-Ballester theories of gravitation. Santhi et al. (30) have investigated LRS Bianchi type-$V$ universe with variable modified Chaplygin gas in Brans-Dicke scalar-tensor theory of gravitation. Rao and Sireesha (31) have studied axially symmetric holographic dark energy model with generalized Chaplygin gas in Brans-Dicke Theory of gravitation. Santhi et al. (32) have discussed spherically symmetric universe with holographic dark energy and generalized Chaplygin gas in Brans-Dicke theory of gravitation.

One of the interesting concept is the theory of extra dimensions due to Kaluza (33) and Klein (34). They used an extra dimensions in addition to usual four spacetime dimensions to unify gravity and electromagnetism. Kaluza-Klein model has been used in many literature for studying the models of cosmology as well as particle physics (35), (36). It may be pointed out here that one does not need to insert matter by hand because the matter is induced in 4D by 5D vaccum theory (37)-(39) in a five-dimensional noncompact Kaluza-Klein theory. It is known from Campbells theorem (40) also that the curvature of five-dimensional spacetime induces effective properties of matter in four dimensions. Faraz (41) and Darabi (42) further studied Kaluza-Klein cosmology by including matter as well. But the old Kaluza-Klein-approach does not work well. However, the higher dimensional theories revived once again in recent times due to the advent of string theory which led a paradigm shift in higher-dimensional cosmology. Samanta and Debata (44) have discussed two-fluid cosmological models in Kaluza-Klein space time. Recently, Ghose et al. (43) have studied holographic dark energy with generalized Chaplygin gas in higher dimensions. Rao and Jayasudha (45) have investigated five dimensional spherically symmetric cosmological models in different scalar-tensor theories of gravitation.

Inspiring by above investigations, we consider higher dimensional spherically symmetric holographic dark energy model correspondence with generalized Chaplygin gas in Saez-Ballester (1) scalar-tensor theory of gravitation. In section 2, we present Saez-Ballester field equations for spherically symmetric metric. In section 3, we obtain the solution of the equations and properties of the obtained model. The correspondence between the holographic dark energy with generalized Chaplygin gas is shown in section 4. Conclusions of the obtained model are presented in section 5.

## 2 Metric and the field equations

We consider the five dimensional spherically symmetric space time in the form

$$ds^2 = dt^2 - e^{\lambda}(dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2) - e^{\mu} d\psi^2 \tag{2.1}$$

where $\lambda$ and $\mu$ are cosmic scale factors and functions of time only.
The energy momentum tensor for matter and the holographic dark energy are defined as

$$\begin{aligned} T_{ij} &= \rho_m u_i u_j \\ \overline{T}_{ij} &= (p_\Lambda + \rho_\Lambda) u_i u_j - p_\Lambda g_{ij} \end{aligned} \tag{2.2}$$

where $p_\Lambda$, $\rho_\Lambda$ are the pressure and energy density of holographic dark energy and $\rho_m$ is the energy density of the matter.

Saez-Ballester (1) field equations for combined scalar and tensor fields are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} - w\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,l}\phi^{,l}\right) = -(T_{ij} + \overline{T}_{ij}) \tag{2.3}$$

and the scalar field $\phi$ satisfies the equation

$$2\phi^n\phi^i_{;j} + n\phi^{n-1}\phi_{,l}\phi^{,l} = 0 \tag{2.4}$$

where $w$ is a dimensionless constant.
Also, the energy conservation equation is given by

$$T^{ij}_{;j} + \overline{T}^{ij}_{;j} = 0. \tag{2.5}$$

In a commoving co-ordinate system, Saez-Ballester field equations (2.3) and (2.4) for the metric (2.1) with the help of (2.2) can be written as

$$\ddot{\lambda} + \frac{3}{4}\dot{\lambda}^2 + \frac{1}{2}\ddot{\mu} + \frac{1}{4}\dot{\mu}^2 + \frac{1}{2}\dot{\mu}\dot{\lambda} - \frac{w}{2}\phi^n\dot{\phi}^2 = -p_\Lambda \tag{2.6}$$

$$\frac{3}{2}\left(\ddot{\lambda} + \dot{\lambda}^2\right) - \frac{w}{2}\phi^n\dot{\phi}^2 = -p_\Lambda \tag{2.7}$$

$$\frac{3}{4}\left(\dot{\lambda}^2 + \dot{\lambda}\dot{\mu}\right) + \frac{w}{2}\phi^n\dot{\phi}^2 = (\rho_\Lambda + \rho_m) \tag{2.8}$$

$$\ddot{\phi} + \dot{\phi}\left(3\frac{\dot{\lambda}}{2} + \frac{\dot{\mu}}{2}\right) + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0 \tag{2.9}$$

The energy conservation equation (2.5) is given by

$$\dot{\rho_m} + \dot{\rho_\Lambda} + \left(3\frac{\dot{\lambda}}{2} + \frac{\dot{\mu}}{2}\right)(\rho_m + \rho_\Lambda + p_\Lambda) = 0 \tag{2.10}$$

For minimally interaction between the matter and dark energy, we can write the continuity equation of matter and dark energy as

$$\dot{\rho_m} + \left(3\frac{\dot{\lambda}}{2} + \frac{\dot{\mu}}{2}\right)\rho_m = 0 \tag{2.11}$$

$$\dot{\rho_\Lambda} + \left(3\frac{\dot{\lambda}}{2} + \frac{\dot{\mu}}{2}\right)(1 + \omega_\Lambda)\rho_\Lambda = 0 \tag{2.12}$$

where $p_\Lambda = \omega_\Lambda\rho_\Lambda$ is equation of state (EoS) parameter and an overhead dot denotes differentiation with respect to time.

# 3 Solution of the field equations

In order to solve the system of four independent field equations (2.6)-(2.9), we use the relation between the metric potential given by

$$\lambda = k\mu \tag{3.1}$$

where $k \neq 0$ is a constant.
Now using above condition in equations (2.6) and (2.7), we get

$$\mu = ln\left[\frac{3k+1}{2}(k_1 t + k_2)\right]^{\frac{2}{3k+1}} \tag{3.2}$$

Consequently from equation (3.1), we get

$$\lambda = ln\left[\frac{3k+1}{2}(k_1 t + k_2)\right]^{\frac{2k}{3k+1}}. \tag{3.3}$$

Now the metric (2.1) can be rewritten as

$$ds^2 = dt^2 - \left[\frac{3k+1}{2}(k_1 t + k_2)\right]^{\frac{2k}{3k+1}}(dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2) - \left[\frac{3k+1}{2}(k_1 t + k_2)\right]^{\frac{2}{3k+1}} d\psi^2. \tag{3.4}$$

From equations (2.9), (2.11), (3.2) and (3.3), we get scalar field ($\phi$) and energy density of matter ($\rho_m$) as

$$\phi = \left[\frac{n+2}{(3k+1)(k_1 t + k_2)^2}\right]^{\frac{2}{n+2}} \tag{3.5}$$

$$\rho_m = \frac{2\rho_0}{(3k+1)(k_1 t + k_2)} \tag{3.6}$$

Using (3.2), (3.3) and (3.6) in the field equations (2.6)-(2.8), we get energy density ($\rho_\Lambda$) and pressure ($p_\Lambda$) of holographic dark energy as

$$\rho_\Lambda = \frac{k_1^2(k^2 + k + 2w)}{(3k+1)^2(k_1 t + k_2)^2} - \frac{2\rho_0}{(3k+1)(k_1 t + k_2)} \tag{3.7}$$

$$p_\Lambda = \frac{2(5k+1)(3k+1) + k_1^2(4w - (9k^2 + 2k + 1))}{8(3k+1)^2(k_1 t + k_2)^2} \tag{3.8}$$

and the EoS parameter ($\omega_\Lambda$) of holographic dark energy is given by

$$\omega_\Lambda = \frac{2(5k+1)(3k+1) + k_1^2(4w - (9k^2 + 2k + 1))}{8k_1^2(k^2 + k + 2w) - 16\rho_0(3k+1)(k_1 t + k_2)} \tag{3.9}$$

Thus the metric (3.4) together with (3.5)-(3.9) constitutes five dimensional spherical symmetric holographic cosmological model in Saez-Ballester scalar tensor theory of gravitation.

The coincidence parameter $(r)$ is given by

$$r = \frac{\rho_\Lambda}{\rho_m} = \frac{k_1^2(k^2 + k + 2w) - 2\rho_0(3k+1)(k_1 t + k_2)}{2\rho_0(3k+1)(k_1 t + k_2)} \tag{3.10}$$

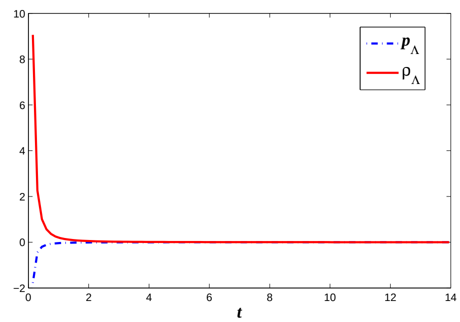


Figure 1: Plot of energy density and pressure of holographic dark energy versus time.

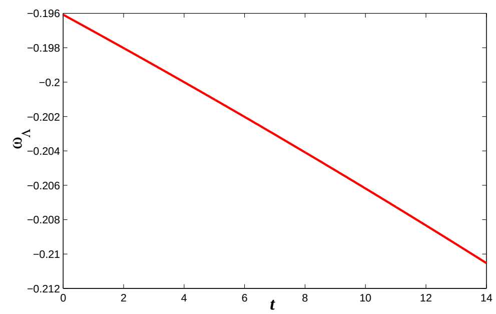


Figure 2: Plot of holographic dark energy EoS parameter versus time.

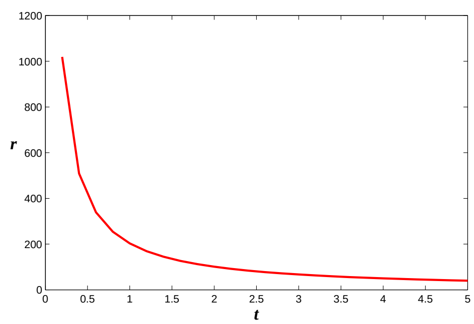


Figure 3: Plot of coincidence parameter $(r)$ versus time.

Figure 1 depicts holographic pressure and energy density in terms of time. It is observed that energy density $(\rho_\Lambda)$ is decreasing positive function of $t$, whereas pressure $(p_\Lambda)$ is increasing negative function of time and both are vanish for sufficiently large values of time. From figure 2 we observed that the EoS parameter $(\omega_\Lambda)$ of holographic dark energy is always varying in quintessence region $(\omega_\Lambda > -1)$.

Figure 3 explains the variation of coincidence parameter in terms of cosmic time $t$. We observed that coincidence parameter $r$ is varies in very early stage of evolution, but after some finite time $t$ it converges to a constant value and remains constant throughout the evolution of the universe. To avoid the coincidence problem recent observations predicted that the coincidence parameter $r$ remains constant in spite of their different rates of time evolution.

# 4 Some important properties of the model

The spatial volume and average scale factor are given by

$$V = \left(\frac{3k+1}{2}\right)(k_1 t + k_2) \quad and \quad a = \left[\left(\frac{3k+1}{2}\right)(k_1 t + k_2)\right]^{1/4} \tag{4.1}$$

The directional Hubble's parameters and average Hubble's parameter are given by

$$H_1 = H_2 = H_3 = \frac{kk_1}{(3k+1)(k_1 t + k_2)}, \;\; H_4 = \frac{k_1}{(3k+1)(k_1 t + k_2)} \;\; and \;\; H = \frac{k_1}{k_1 t + k_2} \tag{4.2}$$

The expansion scalar ($\theta$) is given by

$$\theta = \frac{4k_1}{k_1 t + k_2} \tag{4.3}$$

The shear scalar is

$$\sigma^2 = \frac{k_1^2(5 - 18k - 3k^2)}{16(3k+1)^2(k_1 t + k_2)^2} \tag{4.4}$$

Average anisotropic parameter

$$A_h = 4\left(\frac{k-1}{3k+1}\right)^2 \tag{4.5}$$

The deceleration parameter $q = 3(> 0)$. This shows that universe in five dimensions decelerates in the standard way. Viswakarma (46) has shown that decelerating models are also consistent with recent cosmic background observations made by WAMP as well as with the high-redshift supernova Ia data including SN 1997 ff at Z = 1.775.

# 5 Correspondence between the holographic and generalized Chaplygin gas model of dark energy

Here we establish the correspondence between the holographic dark energy with generalized Chaplygin gas dark energy model, for that we compare the EoS and the dark energy density for the corresponding models of dark energy.
The pressure and the energy density of the generalized Chaplygin gas are given by

$$p_{GCG} = -\frac{A_1}{\rho_{GCG}^{\alpha}} \tag{5.1}$$

$$\rho_{GCG} = \left[A_1 + \frac{A_2}{a^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} \tag{5.2}$$

where $a$ is the average scale factor of the universe and $A_1$, $A_2$ and $\alpha$ are positive constants with $0 < \alpha \leq 1$.

Now following Setare (47) we assume that the origin of the dark energy is a scalar field $\Phi$, so

$$\rho_\Phi = \frac{1}{2}\dot{\Phi}^2 + V(\Phi) = \left[A_1 + \frac{A_2}{a^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} \tag{5.3}$$

$$p_\Phi = \frac{1}{2}\dot{\Phi}^2 - V(\Phi) = \frac{-A_1}{\left[A_1 + \frac{A_2}{a^{3(1+\alpha)}}\right]^{\frac{\alpha}{1+\alpha}}} \tag{5.4}$$

$$\omega_{GCG} = \frac{p_{GCG}}{\rho_{GCG}} = \frac{-A_1}{\rho_{GCG}^{\alpha+1}} = \frac{-A_1}{A_1 + \frac{A_2}{a^{3(1+\alpha)}}} \tag{5.5}$$

Now from equations (5.3) and (5.4), we get

$$\dot{\Phi}^2 = \left[A_1 + \frac{A_2}{a^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} - \frac{A_1}{\left[A_1 + \frac{A_2}{a^{3(1+\alpha)}}\right]^{\frac{\alpha}{1+\alpha}}} \tag{5.6}$$

$$V(\Phi) = \frac{1}{2}\left[A_1 + \frac{A_2}{a^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} + \frac{A_1}{2\left[A_1 + \frac{A_2}{a^{3(1+\alpha)}}\right]^{\frac{\alpha}{1+\alpha}}} \tag{5.7}$$

Now we assume that the holographic dark energy density is equivalent to the generalized Chaplygin gas energy density. Therefore using equations (3.7) and (5.3), we get

$$A_2 = a^{3(1+k)}\left[\left(\frac{k_1^2(k^2+k+2w)}{(3k+1)^2(k_1t+k_2)^2} - \frac{2\rho_0}{(3k+1)(k_1t+k_2)}\right)^{1+\alpha} - A_1\right] \tag{5.8}$$

Again from equations (3.9) and (5.5), we get

$$\begin{aligned}\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} &= \frac{2(5k+1)(3k+1) + k_1^2(4w - (9k^2+2k+1))}{8k_1^2(k^2+k+2w) - 16\rho_0(3k+1)(k_1t+k_2)} \\ &= \frac{-A_1}{A_1 + \frac{A_2}{a^{3(1+\alpha)}}}\end{aligned} \tag{5.9}$$

From equations (5.8) and (5.9), we get

$$\begin{aligned}A_1 &= \left[\frac{2(5k+1)(3k+1) + k_1^2(4w - (9k^2+2k+1))}{8k_1^2(k^2+k+2w) - 16\rho_0(3k+1)(k_1t+k_2)}\right] \\ &\qquad \left(\frac{k_1^2(k^2+k+2w) - 2\rho_0(3k+1)(k_1t+k_2)}{(3k+1)^2(k_1t+k_2)^2}\right)^{1+\alpha}\end{aligned} \tag{5.10}$$

Using these values of $A_1$ and $A_2$ in equations (5.6) and (5.7) we get the potential and dynamics of the scalar field as

$$\Phi = \int \left\{\frac{k_1^2(17k^2+10k+12w+1) - 2(3k+1)[8\rho_0(k_1t+k_2) + 5k+1]}{8(3k+1)^2(k_1t+k_2)^2}\right\}^{\frac{1}{2}} dt \tag{5.11}$$

$$V(\Phi) = \frac{k_1^2(17k^2 + 10k + 12w + 1) - 2(3k+1)[8\rho_0(k_1 t + k_2) + 5k + 1]}{16(3k+1)^2(k_1 t + k_2)^2} \tag{5.12}$$

# 6 Conclusions

In this paper, we have studied the behavior of five dimensional spherically symmetric cosmological model filled with minimally interacting matter and holographic dark energy in Saez-Ballester scalar-tensor theory of gravitation. It should be pointed out that evidence was recently provided by Abell-Cluster A586 in support of interaction between dark energy and dark matter (Bertolami et al. (48); Le Delliou et al. (49)).

- We observed that the scale factors and the spatial volume of the model are zero at $t = t^* = \frac{-k_2}{k_1}$ and all the remaining parameters are diverge. Therefore, the model has point-type singularity at $t = t^*$. which shows that the Universe starts evolving with zero volume at $t = t_*$ with an infinite rate of expansion.

- As $t \to \infty$, spatial volume becomes infinite and expansion scalar $\theta$ and Hubble's parameter $H$ are vanish. Thus the rate of expansion slows down with increase in time. The energy density and pressure have infinite values at $t = t^*$ but as at $t \to \infty$, energy density of matter as well as holographic dark energy vanishes whereas holographic pressure becomes negative. The EoS parameter of holographic dark energy in this is varying in quintessence region only.

- The deceleration parameter is positive and hence the model obtained is decelerating initially and will accelerate in finite time due to cosmic re-collapse (Nojiri and Odintsov (8)). So the obtained higher dimensional holographic cosmological model is useful to explain evolution of the universe at early stages.

- Out of the different alternate candidate of the dark energy, the generalized Chaplygin gas model is an interesting candidate for the unification of the dark matter and dark energy. An attractive feature of these models is that at early times, the energy density behaves as a matter and as a cosmological constant at a later stage. Using these results, we have established a correspondence between the holographic dark energy models with a scalar field which described the generalised Chaplygin gas cosmology in the higher dimensional spherically symmetric universe. We have also reconstructed the potentials and the dynamics of the scalar field for this accelerating universe.

Received October 26, 2016; Accepted November 6, 2016

# References

[1] Saez, D., Ballester, V.: J. Phys. Lett. A **113**, 467 (1986).

[2] Perlmutter, S., et al.: Astrophys. J. **517**, 5 (1999).

[3] Reiss, A., et al.: Astron. J. **116**, 1009 (1998).

[4] Bertolami, O., et al.: Mon. Not. Roy. Astron. Soc. **353**, 329 (2004).

[5] Bento, M.C., et al.: Phys. Rev. D **66**, 043507 (2002).

[6] Setare, M. R., et al.: Phys. Lett. B **673**, 241 (2009).

[7] Pradhan, A., et al.: Astrophys. Space Sci. **334**, 249 (2011).

[8] Nojiri, S., Odintsove, S.D.: Phys. Rev. D **68**, 123512 (2003).

[9] Caroll, S.M., et al.: Phys. Rev. D **70**, 043528 (2004).

[10] Nojiri, S., Odintsov, S.D.: Int. J. Geom. Meth. Mod. Phys. **4**, 115 (2007).

[11] Copeland, E.J., et al.: Int. J. Mod. Phys. D **15**, 1753 (2006)

[12] Bekenstein, J.D.: Phys. Rev. D **7**, 2333 (1973).

[13] 't Hooft, G.: gr-qc/9310026 (1993).

[14] Cohen, A.G., Kaplan, D.B., Nelson, A.E.: Phys. Rev. Lett. **82**, 4971 (1999).

[15] Susskind, L.: J. Math. Phys. **36**, 6377 (1995).

[16] Hsu, S.D.H.: Phys. Lett. B **594**, 13 (2004).

[17] Li, M.: Phys. Lett. B **603**, 1 (2004).

[18] Gao, C., Chen, X., Shen, Y.G.: Phys. Rev. D **79**, 043511 (2009).

[19] Granda, L.N., Oliveros, A.: Phys. Lett. B **669**, 275 (2008).

[20] Nojiri, S., Odintsov, S.D.: Gen. Relativ. Gravit. **38**, 1285 (2006).

[21] Setare, M.R.: Phys. Lett. B **648**, 329 (2007).

[22] Setare, M.R.: Phys. Lett. B **654**, 1 (2007).

[23] Setare, M.R.: Eur. Phys. J. C **52**, 689 (2007).

[24] Setare, M.R., Vanegas, E.C.: Int. J. Mod. Phys. D **18**, 147 (2009).

[25] Sarkar, S.: Astrophys. Space Sci. **349**, 985 (2014).

[26] Jalil, N.: Astrophys. Space Sci. **350**, 385 (2014).

[27] Kiran, M., Reddy, D.R.K., Rao, V.U.M.: Astrophys. Space Sci. **354**, 577 (2014).

[28] Reddy, D.R.K., et al.: Amaravathi Univ. Res. J. Spl. Issue, 1 (2015)

[29] Reddy, D.R.K., et al.: Prespace time J. **7**, 100 (2016)

[30] Santhi, M.V., et al.: Can. J. Phys. (2016) doi.org/10.1139/cjp-2016-0099.

[31] Rao, V.U.M., Sireesha, K.V.S.: Can. J. Phys. (2016) doi: 10.1139/cjp-2016-0339

[32] Santhi, M.V., et al.: Prespacetime J. 7, 537 (2016)

[33] Kaluza, T.: Sitz. Preuss. Acad. Wiss. F1 966, (1921).

[34] Klein, O.: Z. Phys. **37**, 895 (1926).

[35] Chien Lee, H.: An Introduction to Kaluza Klein Theories (World Scientific, Singapore, 1984).

[36] Appelquist, T., Chodos, A., Freund, P.G.O.: Modern Kaluza Klein Theories (Addison-Wesley, 1984).

[37] Wesson, P.S.: Gen. Relativ. Gravit. **16**, 193 (1984).

[38] Bellini, M.: Nucl. Phys. B **660**, 389 (2003).

[39] Wesson, P.S.: Space-Time Matter: Modern Higher-Dimensional Cosmology (World Scientific, 1999).

[40] Liu, H.U., Wesson, P.S.: Astrophys. J. **562**, 1 (2001).

[41] Farajollah, H., Amir, H.: Int. J. Mod. Phys. D **19**, 1823 (2010).

[42] Darabi, F: Mod. Phys. Lett. A **25**, 1635 (2010).

[43] Ghose, S., Saha, A., Paul, B.C.: Int. J. Mod. Phys. D **23**, 1450015 (2014).

[44] Samanta, G.C., Debata, S.: Int. J. Theor. Phys. **52**, 3999 (2013).

[45] Rao, V.U.M., Jayasudha, L.: Astrophys. Space Sci. **358**, 29 (2015).

[46] Viswakarma, R. G.: Mon. Not. R. Astron. Soc. **345**, 545 (2003).

[47] Setare, M.R.: Phys. Lett. B, **648**, 329 (2007).

[48] Bertolami, O., Gil Pedro, F., Le Delliou, M.: Phys. Lett. B **654**, 165 (2007).

[49] Le Delliou, M., Bertolami, O., Gil Pedro, F.: AIP Conf. Proc. **957**, 421 (2007).