Article

Bianchi Type V Scalar Field Cosmological Models in *f*(*R*,*T*) Theory of Gravity

T. Kanakavalli[1], G. Ananda Rao[2] & D. R. K. Reddy[3*]

[1]Department of Mathematics, GITAM University, Visakhapatnam, India
[2]Department of Mathematics, ANITS, Visakhapatnam, India
[3]Department of Applied Mathematics, Andhra University, Visakhapatnam -530 003, India

Abstract

This paper is devoted to investigation of anisotropic Bianchi type-V model in $f(R,T)$ theory of gravity proposed by Harko et al. (Physics review D 84, 024020, 2011) with scalar field (quintessence or phantom). Here R is Ricci scalar and T is the trace of energy momentum tensor. The field equations have been solved using the fact that scalar expansion is proportional to the shear scalar of the space-time. Some physical properties of our model are also obtained and discussed.

Keywords: Scalar field, cosmology, f (R, T) gravity, Bianchi Type III, modified gravity.

1. Introduction

In recent years the concepts of dark matter and dark energy have dominated the discussions in modern cosmology .This is because of the fact that the recent scenario of accelerated expansion of the universe (Perlmutter et al. [1]; Tegmark et al. [2]; Riess et al. [3]) is supposed to be driven by new energy with negative pressure known as dark energy. However, the nature and behavior of DE is still a mystery. Currently, there are two main approaches for the explanation of this accelerated expansion. One way is to introduce scalar field models like phantom (Caldwell [4]; Nojiri and Odintsov [5]), quintessence (Sahni and Starobinsky [6]; Padmanabhan [7]), anisotropic fluids (Akarsu and Kilinc [8]; Sharif and Zubair [9]) and etc. An alternative approach is to modify Einstein-Hilbert action to obtain alternate theories of gravity like $f(R)$ theory of gravity which provides a natural unification of early-time inflation and late-time acceleration (Capozziello and Francaviglia [10]; Nojiri and Odintsov [11]). Among the other modified theories, theory of scale-Gauss-Bonnet gravity, so called $f(G)$ gravity (Nojiri and Odintsov [12]) and $f(T)$ gravity (Linder [13]), where T is the torsion have been proposed to explain the accelerated expansion of universe.

[*]Correspondence: Dr. D. R. K. Reddy, Mathematics Department, MVGR college of Engg.Vizianagaram. India.
E-mail: reddy_einstein@yahoo.com

Recently, Harko et al. [14] proposed another modified theory known as $f(R,T)$ gravity, wherein the gravitational Lagrangian contains the Ricci scalar R and trace of the energy-momentum tensor T. It is well known that in the study of early stages of evolution of the universe anisotropic models play a vital role. Adhav [15], Reddy and Santhi Kumar [16], Reddy et al. [17], Houndjo [18] and Shri Ram & Chandel [19] are some of the authors who have reconstructed several cosmological models in f(R,T) gravity using different physical sources.

Scalar fields play a crucial role in particle physics and cosmology. Olive [20] has shown that, during inflation, the potential of a scalar field acts as a dynamical vacuum energy. This prominent role of scalar fields is also evident in models proposed to explain the late time accelerated expansion of the universe in vacuum energy and in evolving quintessence models ([21]-[23]). Further, it was recently proposed that a scalar field can also be the source of the anomalous acceleration [24]. Sharif and Zubair [25] have investigated the anisotropic universe models in $f(R,T)$ gravity in the presence of perfect fluid and scalar field. Singh and Singh [26] have obtained the Friedmann-Robertson-Walker (FRW) models with perfect fluid and scalar field in higher derivative theory. Sharif and Jawad [27] have studied reconstruction of scalar field dark energy models in Kaluza-Klein universe. Singh and Singh [28] have discussed the behavior of scalar field in modified $f(R,T)$ gravity within the framework of a flat FRW cosmological model. Later, Singh et al. [29] have investigated Bianchi type-I universe with scalar field and time varying cosmological constant in $f(R,T)$ gravity. Kanakavalli et al. [30] have studied a scalar field cosmological model in this modified theory of gravitation.

Motivated by the above discussions, we study in this paper Bianchi type-V space time in presence of scalar field within the framework of $f(R,T)$ theory of gravitation proposed by Harko et al. [14]. The plan of the paper as follows: Sect. 2 describes $f(R,T)$ gravity formalism in the presence of scaler field. Sect. 3 is devoted to the derivation of field equations and solutions of field equations leading to scalar field model. Sect. 4 contains a detailed physical discussion of the model. Summery and conclusions are presented in the last section.

2. $f(R,T)$ gravity formalism with scalar field

The field equations of $f(R,T)$ gravity are derived from the Hilbert-Einstein type variation principle. The action for the $f(R,T)$ gravity with scalar field is [28]

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\,d^4x + \int L_\phi \sqrt{-g}\,d^4x, \tag{1}$$

where $f(R,T)$ is an arbitrary function of Ricci scalar R, T is the trace of stress-energy tensor (T_{ij}) of the matter and L_ϕ is the matter Lagrangian of scalr field.

The energy momentum tensor T_{ij} is defined as

$$T_{ij} = -(\frac{2}{\sqrt{-g}})\frac{\delta(\sqrt{-g})L_\phi}{\delta g^{ij}}. \tag{2}$$

Here we consider that the dependence of matter Lagrangian is merely on the metric tensor g_{ij} rather than on its derivatives and we obtain

$$T_{ij} = g_{ij}L_\phi - \frac{\partial L_\phi}{\partial g^{ij}}. \tag{3}$$

Now varying the action S with respect to metric tensor g_{ij}, $f(R,T)$ gravity field equations are obtained as

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\Omega - \nabla_i\nabla_j)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij}, \tag{4}$$

where

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_\phi - 2g^{\alpha\beta}\frac{\partial^2 L_\phi}{\partial g^{ij}\partial g^{\alpha\beta}}. \tag{5}$$

Here $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$, $f_T(R,T) = \frac{\partial f(R,T)}{\partial T}$ and $\Omega = \nabla^\mu\nabla_\mu$, where ∇_μ denotes the covariant derivative.

Here we assume that the universe is filled with scalar field minimally coupled to gravity. Therefore, the energy-momentum tensor of a scalar field ϕ with self-interacting scalar field potential $\psi(\phi)$ has the form

$$T_{ij} = \varepsilon\phi_{,i}\phi_{,j} - g_{ij}\left(\frac{\varepsilon}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - \psi(\phi)\right) \tag{6}$$

where $\varepsilon = \pm 1$ correspond to quintessence and phantom scalar fields respectively. The trace of the energy-momentum tensor $T = g^{ij}T_{ij}$ is given by

$$T = -\varepsilon\dot{\phi}^2 + 4\psi(\phi) \tag{7}$$

hereafter dot denotes differentiation with respect to time t. The matter Lagrangian of the scalar field is given by

$$L_\phi = -\frac{1}{2}\varepsilon\dot{\phi}^2 + \psi(\phi). \tag{8}$$

Now from equations (5) and (8), we have

$$\Theta_{ij} = -2T_{ij} - g_{ij}\left(\frac{1}{2}\varepsilon\dot{\phi}^2 - \psi(\phi)\right). \tag{9}$$

Generally, the field equations also depend, through the tensor Θ_{ij}, on the physical nature of the matter field. Hence in the case of $f(R,T)$ gravity depending on the nature of the matter source, we obtain several theoretical models corresponding to different matter contributions for $f(R,T)$ gravity. However, Harko et al. [14] gave three classes of these models:

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T). \end{cases}$$

Here we consider the first case, i.e $f(R,T) = R + f(T)$, where $f(T)$ is an arbitrary function of the trace of stress-energy tensor T_{ij}. Using this relation $f(R,T)$ gravity field equations (4) reduced to

$$R_{ij} - \frac{1}{2} R g_{ij} = T_{ij} - 2(T_{ij} + \Theta_{ij}) f'(T) + f(T) g_{ij}, \tag{10}$$

where a prime denotes differentiation with respect to the argument.

3. Metric and Field equations

We consider the Bianchi type-V space-time in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{2x} dy^2 - C^2 e^{2x} dz^2 \tag{11}$$

where A, B and C are functions of cosmic time t only.

For the particular choice of the function $f(T) = \lambda T$ (Harko et al. [14]), where λ is a constant, the field equations (10) for the metric (11) using (6) and (9) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} = \left(\frac{1+2\lambda}{2}\right)\varepsilon\dot{\phi}^2 - (4\lambda+1)\psi(\phi) \tag{12}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = \left(\frac{1+2\lambda}{2}\right)\varepsilon\dot{\phi}^2 - (4\lambda+1)\psi(\phi) \tag{13}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \left(\frac{1+2\lambda}{2}\right)\varepsilon\dot{\phi}^2 - (4\lambda+1)\psi(\phi) \tag{14}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3}{A^2} = -\left(\frac{1+2\lambda}{2}\right)\varepsilon\dot{\phi}^2 - (4\lambda+1)\psi(\phi) \tag{15}$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - \frac{2\dot{A}}{A} = 0. \tag{16}$$

Integration of Eq. (16) yields

$$A^2 = k\,B\,C$$

where *k* is a constant of integration which can be chosen as unity without loss of generality, so that we have

$$A^2 = BC \tag{17}$$

Equations (12)-(15) are a set of three independent equations with four unknowns (A, B, ϕ and $\psi(\phi)$). Therefore, we need an additional condition to solve the above system. Here we use the physical condition that the expansion scalar θ is proportional to the shear scalar σ , which leads to (Collins [31])

$$B = C^m \tag{18}$$

where $m \neq 1$ is a positive constant.

Now from equations (12), (13), (17) and (18), we get the metric coefficients as

$$A = \left[\frac{3(m+1)}{2}(k_1 t + k_2)\right]^{\frac{2}{3}}$$

$$B = \left[\frac{3(m+1)}{2}(k_1 t + k_2)\right]^{\frac{2m}{3(m+1)}} \tag{19}$$

$$C = \left[\frac{3(m+1)}{2}(k_1 t + k_2)\right]^{\frac{2}{3(m+1)}}$$

where k_1 and k_2 are integrating constants. Now using the proper choice of integrating constants ($k_1 = 1$ and $k_2 = 0$) the metric (11) can be written as

$$ds^2 = dt^2 - \left[\frac{3(m+1)}{2}\ t\right]^{\frac{4}{3}} dx^2 - \left[\frac{3(m+1)}{2}\ t\right]^{\frac{4m}{3(m+1)}} e^{2x} dy^2 - \left[\frac{3(m+1)}{2}\ t\right]^{\frac{4}{3(m+1)}} e^{2x} dz^2 \tag{20}$$

4. Physical discussion

Equation (20) represents Bianchi type-*V* scalar field cosmological model in *f(R,T)* theory of gravity with the following physical and geometrical parameters which are significant in the discussion of cosmology.

The spatial volume is

$$V^3 = \frac{3}{2}(m+1)t \tag{23}$$

The scalar expansion is

$$\theta = \frac{1}{t} \tag{24}$$

The Hubble parameter is

$$H = \frac{1}{3t} \tag{25}$$

The average anisotropy parameter is

$$A_h = \frac{14m^2+20m+14}{27(m+1)^2} \tag{26}$$

The shear scalar is

$$\sigma^2 = \frac{7m^2+10m+7}{9(m+1)^2t^2} \tag{27}$$

The deceleration parameter

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = 2 \tag{28}$$

From equations (12)-(15) and (19), we get scalar field potential as

$$\psi(\phi) = \frac{2}{(4\lambda+1)}\left[\frac{3(m+1)}{2}\ t\right]^{\frac{-2}{3}} \tag{29}$$

and the scalar field ϕ as

$$\phi = \int\left\{\frac{1}{3\varepsilon(2\lambda+1)}\left[6\left(\frac{3(m+1)}{2}\ t\right)^{-2/3} - \frac{4}{3t^2}\left(\frac{m^2+4m+1}{(m+1)^2}\right)\right]\right\}^{\frac{1}{2}} dt \tag{30}$$

It may be observed that the spatial volume increases with time and vanishes at t=0. This shows that the universe evolves from zero volume. The parameters $H, \theta, \sigma^2, \psi$ will vanish as t approaches infinity while they alldiverge at t=0.The anisotropy parameter is a constant and the deceleration parameter is positive This shows that the universe initially decelerates.However, it will accelerate in finite time due to ‘cosmic re-collase’[32].

5. Conclusions

In this paper, we have considered Bianchi type-V space-time in the presence of a scalar field with self interacting scalar fied potential in the frame work of f(R,T) gravity[14]which is proposed to explain early deceleration and late time accelration of the universe. The gravitational field equations of the theory are solved using a relation between the metric potentials. It is observed that the model presented represents scalar field(quintessence or Phantom) model in this theory. It is found that all the dynamical parameters approach infinity at the initial epoch and vanish at infinitely large time. It can be seen that the scalar field, in the model, increases whereas the scalar potential decreases with time and tends to zero as $t \to \infty$. Since at the early stages of evolution of the universe scalar fields and anisotropic models play a vital role, the scalar field model obtained here will be useful for a better understanding of scalar field cosmology in this modified theory of gravitation.

Received October 8, 2016; Accepted October 22, 2016

References

[1] Perlmutter, S., et al.: Astrophys. J. 517, 565 (1999)
[2] Tegmark, M., et al.: Phys. Rev. D 69, 103501 (2004)
[3] Riess, A.G., et al.: Astrophys. J. 659, 98 (2007)
[4] Caldwell, R.R.: Phys. Lett. B 23, 545 (2002)
[5] Nojiri, S., Odintsov, S.D.: Phys. Lett. B 147, 562 (2003)
[6] Sahni, V., Starobinsky, A.A.: Int. J. Mod. Phys. D 9, 373 (2000)
[7] Padmanabhan, T.: Gen. Relativ. Gravit. 40, 529 (2008)
[8] Akarsu, O., Kilinc, B.C.: Gen. Relativ. Gravit. 42, 119 (2010)
[9] Sharif, M., Zubair, M.: Astrophys. Space Sci. 330, 399 (2010)
[10] Capozziello, S., Francaviglia, M.: Gen. Relativ. Grav. 40, 357 (2008).
[11] Nojiri, S., Odintsov, S. D.: Phys. Rep. 505, 59 (2011).
[12] Nojiri, S., Odintsov, S. D., Sami, M.: Physical Rev. D 74, 046004 (2006).
[13] Linder, E.V.: Phys. Rev. D 81, 127301 (2010).
[14] Harko, T., et al.: Physics review D 84, 024020 (2011).
[15] Adhav, K.S.: Astrophys. Space Sci. 339, 365 (2012).
[16] Reddy, D.R.K., Santi Kumar, R.: Astrophys. Space Sci. 344, 253 (2013).
[17] Reddy, D.R.K., et al.: Eur. Phys. J. Plus 129, 96 (2014).
[18] Houndjo, M.J.S.: Int. J. Mod. Phys. D 21, 1250003.
[19] Shri Ram, Chandel, S.: Astrophys Space Sci. 355, 195 (2015).
[20] Olive, K.A.: Phys. Rev. 190, 307 (1990).
[21] Bertolami, O.: Nuovo Cimento Soc. Ital. Fis. 93B, 36 (1986)
[22] Ratra, B., Peebles, P. J. E.: Phys. Rev. D 37, 3406 (1988)
[23] Caldwell, R.R., Dave, R., Steinhardt, P.J.: Phys. Rev. Lett. 80, 1582 (1998)
[24] Bertolami,O., Pa´ramos, J.: Classical Quantum Gravity 21, 3309 (2004).
[25] Sharif, M., Zubair, M.: J. Phys. Soc. Jpn. 81, 114005 (2012) .
[26] Singh, C.P., Singh, V.: Mod. Phy. Lett. A 26, 1495 (2011).
[27] Sharif, M., Abdul Jawad: Commun. Theor. Phys. 60, 183 (2013).
[28] Singh, V., Singh, C.P.: Astrophys. Space Sci. 355, 2183 (2014).
[29] Singh, G.P., et al.: Chinese Journal of Physics 54, 244 (2016).
[30] Kanakavalli, T., et al.: Prespacetime J., 7, 1623 (2016)
[31] Collins, C.B., et al.: Gen. Relativ. Gravit. 12, 805 (1980)
[32] Nojiri, S. Odintsov,S.D.: Phys. Rev. D **68**, 123512 (2003)