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Five-Dimensional Spherically Symmetrical Model in Lyra's Geometry

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Abstract

The Spherically Symmetric Model filled with perfect fluid in Lyra's geometry has been considered. We have derived two classes of exact solutions of the field equations in Lyra's geometry with a time- dependent displacement vector. The exact solutions to the corresponding field equations are investigated in quadrature form. The solutions to the Einstein field equations are obtained for power law and exponential form. The cosmological parameters are discussed in detail.

Keywords: Cosmological model, Lyra's geometry, cosmological parameters.

1. Introduction

After the discovery of Einstein general theory of relativity there has been numerous modification in it. In 1951 Lyra [1] presented modified Riemannian geometry in which the connection is metric preserving and length transfers are integrable as in Riemannian geometry. The Cosmological theories based on Lyra's geometry and Riemannian geometry lies in the fact that displacement field β has arises from geometry whereas cosmological term Λ is introduced in the usual treatment review paper [2,3] and references therein may be referred for further motivation about researcher based on Lyra's geometry.This geometry was utilized by Sen and Dunn [4], Halford [5] introduced a scalar tensor theory based on Lyra's geometry in which both the scalar and tensor field have intrinsic geometrical significance . Halford, Bhamra [6], Karade & Borikar [7], Reddy &Innaiah [8], probe into some of the aspects of the cosmological models in Lyra's geometry with a constant displacement field . However, this restriction of the displacement field to be constant is merely one of convenience and there is no a priori reason for it. Soleng[9] discussed that the cosmologies based on Lyra geometry with a constant gauge vector $\emptyset_i$ will either include a creation field and be equal to Hoyle's creation field cosmology [10-12] or contain a special vacuum field together with gauge vector term may be a considered as a cosmological term .

Beesham [13] have investigated FRW cosmological models in Lyra's geometry with a time –dependent displacement vector field. Singh and Singh [14, 15], Singh and Desikan [16] have

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observed anisotropic cosmological models in this geometry with a time - dependent displacement vector field and have made a comparative study of Robertson Walker models with constant deceleration parameter in Einstein's theory and in the cosmological theory based on Lyra's geometry . Khadekar and Nagpure [17], Sri Ram and Singh [18] have obtained exact solutions of the field equations in vacuum and in the presence e of stiff-matter for an anisotropic Bianchi type V cosmological models in the normal gauge with a time – dependent displacement vector field. Singh[19] considered flat FRW model in Lyra's geometry by using varying adiabatic equation of state and solved the field equations for the early phases of evolution of universe . Kumar and Singh [20] have presented Bianchi type –I models in Lyra's geometry. Samanta and Debata [21] have constructed five dimensional Bianchi type –I string cosmological models in Lyra manifold. Singh et al [22] have studied higher dimensional homogenous cosmological models with variable G and bulk viscosity in Lyra's geometry. Chaubey [23] have also obtained the exact solution for Kantowski – Sachs space – times in Lyra's geometry.

Moreover, in recent years there has been an immense interest in the study of higher dimensional space times because of underlined idea that the cosmos at its early stage of evolution might have had a higher dimensional era. The extra space reduced to volume with passage of time which is beyond the ability of experimental observations at the moment .There are several authors such as Khadekar et al [24], Samanta et al, Katore et al [25], Reddy et al [26], Tade and Sambhe [27] (and the references therein) have studied higher dimensional cosmological models in the framework of various theories of gravitation.

In this paper we derived two classes of exact solutions of the field equations in Lyra's geometry with perfect a time – dependent displacement vector in the normal gauge for the Spherically Symmetric Model with perfect fluid as matter content. The exact solutions to the Einstein field equations are obtained for power law and exponential form. The cosmological parameters have been discussed in detail.

We consider five dimensional metric of the form,

$$ds^2 = dt^2 - {a_1}^2 dr^2 - {a_2}^2\left[{d\theta_1}^2 + sin^2\theta_1 {d\theta_2}^2 + sin^2\theta_1 sin^2\theta_2 {d\theta_3}^2\right] \tag{1}$$

where the metric functions a_1 , a_2 are functions of cosmic time 't' only .

Defining the expression for the average scale factor and volume scale factor, we define the generalized Hubble's parameter H by analogy with a flat FRW model. We derive, by using the variation – law of Hubble's parameter, two different forms of average scale factor.

The average scale factor 'a' and spatial volume 'V' of the Spherically Symmetric model (Eq.1) are defined as

$$a = (a_1 {a_2}^3)^{\frac{1}{4}} \tag{2}$$

$$V = a^4 = a_1 {a_2}^3 \tag{3}$$

We define the generalized Hubble's parameter 'H' as

$$H = \frac{1}{4}(H_0 + 3H_1) \tag{4}$$

Where $H_0 = \frac{\dot{a_1}}{a_1}$ *and* $H_1 = \frac{\dot{a_2}}{a_2}$ are the directional Hubble's parameter. A overhead dot denotes the differentiation with respect to cosmic time t.

By using Eqs. (2), (3) and (4), we obtain

$$H = \frac{1}{4}\frac{\dot{V}}{V} = \frac{\dot{a}}{a} = \frac{1}{4}\left(\frac{\dot{a_1}}{a_1} + 3\frac{\dot{a_2}}{a_2}\right) \tag{5}$$

The field equations in normal gauge for Lyra's geometry, as obtained by Sen [28] are

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\left(\emptyset_i\emptyset_j - \frac{1}{2}g_{ij}\emptyset_l\emptyset^l\right) = -8\pi G T_{ij} \tag{6}$$

where $\emptyset_i$ is the displacement vector defined as $\emptyset_i = \left(0,0,0,0,\beta(t)\right)$ and T_{ij} is the energy momentum tensor of the matter.

For a perfect fluid the energy momentum Tensor is given by,

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{7}$$

where ρ is the energy density of the fluid, p is the pressure , $u^i$is the velocity vector satisfying $u_i u^i = 1$

From Eq. (1) & (7) , the field Eq. (6) yield following Equations

$$3\frac{\ddot{a_2}}{a_2} + 3\left(\frac{\dot{a_2}}{a_2}\right)^2 + \frac{3}{{a_2}^2} + \frac{3}{4}\beta^2 = -p \tag{8}$$

$$\frac{\ddot{a_1}}{a_1} + 2\frac{\ddot{a_2}}{a_2} + \frac{2\dot{a_1}\dot{a_2}}{a_1 a_2} + \left(\frac{\dot{a_2}}{a_2}\right)^2 + \frac{1}{{a_2}^2} + \frac{3}{4}\beta^2 = -p \tag{9}$$

$$\frac{3\dot{a_1}\dot{a_2}}{a_1 a_2} + 3\left(\frac{\dot{a_2}}{a_2}\right)^2 + \frac{3}{{a_2}^2} - \frac{3}{4}\beta^2 = \rho \tag{10}$$

Assumed $8\pi G = 1$ in proper unit.

The Energy Conservation Equation $T^i_j; i = 0$ takes the form

$$\dot{\rho} + \frac{3}{2}\beta\dot{\beta} + \left[(\rho + p) + \frac{3}{2}\beta^2\right]\left(\frac{\dot{a_1}}{a_1} + 3\frac{\dot{a_2}}{a_2}\right) = 0 \tag{11}$$

Let us assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho\,, 0 \le \gamma \le 1 \tag{12}$$

where ρ is energy density of the fluid and p is pressure .

In order to solve the field equation, let us consider that the Hubble parameter H is related to the average scale factor 'a' by the relation

$$H = l a^{-n} \tag{13}$$

where $l > 0\ and\ n(\ge 0)$ are constants. Such type of relation has already been considered by Berman [29], Berman and Gomide [30] for solving FRW model and Ram et al .[31] for solving Bianchi type-V perfect fluid models in Lyra's geometry.

The physical quantities of observational interest in cosmology are the expansion parameter (θ), Shear Scalar (σ^2) and the mean anisotropy parameter (A). They are defined as [32, 33]

$$\theta = u^i; i = 4H \tag{14}$$

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} \tag{15}$$

$$A = \frac{1}{4}\sum_{i=0}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{16}$$

where $u^i = (0,0,0,0,1)$ is the matter four velocity vector , $\Delta H_i = H_i - H$ $(i = 0,1,2,3)$ and

$$\sigma_{ij} = \frac{1}{2}\left(u_i\, ;\alpha\, P_j^\alpha + u_j\, ;\alpha\, P_i^\alpha\right) - \frac{1}{3}\theta p_{ij} \tag{17}$$

Here the projection tensor P_{ij} has the form

$$P_{ij} = g_{ij} - u_i u_j \tag{18}$$

With the use of Eq. (14) & (15) , We can express the physical quantities for the Spherically Symmetric metric (Eqs.(1)) as ,

$$\theta = \left(\frac{\dot{a_1}}{a_1} + 3\frac{\dot{a_2}}{a_2}\right) = 4H \tag{19}$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{a_1}}{a_1}\right)^2 + 3\left(\frac{\dot{a_2}}{a_2}\right)^2\right] - \frac{\theta^2}{9} \tag{20}$$

and the deceleration parameter q which is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \tag{21}$$

By using Eqs.(5) and (13) , we get

$$\dot{a} = la^{-n+1} \tag{22}$$

$$\ddot{a} = -l^2(n-1)a^{-2n+1} \tag{23}$$

From Eqs.(21) , (22) and (23) , we get

$$q = n - 1 \tag{24}$$

We see that, under the law of variation of H in Eq. (13), the deceleration parameter q is constant. The sign of q indicates whether the model inflates or not. The positive sign of q corresponds to standard decelerating model whereas the negative sign indicates inflation. For n > 1, q > 0 , therefore the model represent a decelerating model whereas for $n \le 1$,We get $-1 \le q < 0$ which describes an accelerating model of universe.

By using Eq.(22) , we obtain the law of variation for average scale factor a as

$$a = (nlt + c_1)^{\frac{1}{n}}, n \neq 0 \tag{25}$$

$$a = c_2 e^{lt} \quad , n = 0 \tag{26}$$

where c_1 *and* c_2 are constants of integration . Equation (25) implies that the condition for expanding universe is $n(= q + 1) > 0$

Equation (10) can be written as

$$\rho = \frac{56}{9}H^2 - \sigma^2 + \frac{3}{{a_2}^2} - \frac{3}{4}\beta^2 \tag{27}$$

and from Eqs.(8) and (9) , we can write as

$$p = \left(3q - \frac{25}{9}\right)H^2 - \sigma^2 - \frac{3}{2{a_2}^2} - \frac{3}{4}\beta^2 \tag{28}$$

In Eq. (8), adding four times Eq. (10) & three times Eq. (9) we get

$$\frac{\ddot{a_1}}{a_1} + 3\frac{\ddot{a_2}}{a_2} + \frac{6}{{a_2}^2} + 6\left[\left(\frac{\dot{a_2}}{a_2}\right)^2 + \frac{\dot{a_1}\dot{a_2}}{a_1 a_2}\right] = \frac{4}{3}(\rho - p) \tag{29}$$

By using Eq.(3), we obtain

$$\frac{\ddot{V}}{V} = \frac{\ddot{a_1}}{a_1} + 3\frac{\ddot{a_2}}{a_2} + 6\left[\left(\frac{\dot{a_2}}{a_2}\right)^2 + \frac{\dot{a_1}\dot{a_2}}{a_1 a_2}\right] \tag{30}$$

From Eq.(29) and (30) , we get

$$\frac{\ddot{V}}{V} + \frac{6}{{a_2}^2} = \frac{4}{3}(\rho - p) \tag{31}$$

Here we discuss two different physically viable cosmologies n $\neq 0$ and n = 0 respectively, which have physical interests to describe the decelerating and accelerating phases of universe .

Case 1 When n $\neq 0$ and $a_1 = v^m$, where m be any constant number , then

$$a_1(t) = (nlt + c_1)^{\frac{4m}{n}} \tag{32}$$

$$a_2(t) = (nlt + c_1)^{\frac{4(1-m)}{3n}} \tag{33}$$

The directional Hubble's parameter $H_0 \; and \; H_1$ have values given by

$$H_0 = \frac{4ml}{(nlt + c_1)} \tag{34}$$

$$H_1 = \frac{4(1-m)l}{3(nlt + c_1)} \tag{35}$$

From Eq.(4) , the average generalized, Hubble's parameter H has the value given by

$$H = \frac{l}{(nlt + c_1)} \tag{36}$$

With the Eqs. (14), (15) and (16) , the dynamical scalars are given by

$$\theta = \frac{4l}{(nlt + c_1)} \tag{37}$$

$$\sigma^2 = \frac{8l^2(12m^2 - 6m + 1)}{9(nlt + c_1)^2} \tag{38}$$

$$A = \frac{1}{3}(16m^2 - 8m + 1) \tag{39}$$

The Scalar Curvature R for Spherically Symmetric model is defined as

$$R = \frac{2\ddot{a}_1}{a_1} + \frac{6\dot{a_1}\dot{a_2}}{a_1 a_2} + 6\frac{\ddot{a_2}}{a_2} + 6\left(\frac{\dot{a_2}}{a_2}\right)^2 + \frac{6}{{a_2}^2} \tag{40}$$

By using Eqs. (32), (33) and (40), we get

$$R = \frac{8l^2}{3}(8m^2 - 4m - 3n + 8)(nlt + c_1)^{-2} + 6(nlt + c_1)^{\frac{-8(1-m)}{3n}} \tag{41}$$

Here we discuss two possibilities,

A) If we assume that the energy density ρ and pressure p of the matter satisfy the equation of state $= \gamma\rho$, $0 \leq \gamma \leq 1$, then

$$\rho = \frac{3(4-n)l^2}{(1-\gamma)(nlt+c_1)^2} + \frac{9}{2(1-\gamma)}(nlt+c_1)^{\frac{-8(1-m)}{3n}} \tag{42}$$

$$p = \frac{3\gamma(4-n)l^2}{(1-\gamma)(nlt+c_1)^2} + \frac{9\gamma}{2(1-\gamma)}(nlt+c_1)^{\frac{-8(1-m)}{3n}} \tag{43}$$

From Eqs. (12), (27) and (28) , ws get

$$\beta^2 = \frac{(224\gamma-108n+208)l^2}{27(\gamma-1)(nlt+c_1)^2} - \frac{32l^2(12m^2-6m+1)}{27(nlt+c_1)^2} - \frac{2(1+2\gamma)}{(1-\gamma)(nlt+c_1)^{\frac{8(1-m)}{3n}}} \tag{44}$$

Here we observe that, there have no-solution for stiff matter ($\gamma = 1$) . It is evident from the above result that the spatial volume (V) is zero at $= t_0 = \frac{-c_1}{nl}$. The Scalar curvature R , the energy density and pressure are infinite at this epoch . The rate of expansion and the mean anisotropy parameter are infinite at $\rightarrow t_0$. Thus the universe starts evolving with zero volume at $t = t_0$ and expands with cosmic time 't' .

For Large cosmic time t , the spatial volume , expansion parameter , Shear scalar and the mean anisotropic parameter tend to zero .The energy density , the pressure and the guage function β also tend to zero for large cosmic time t .

Here $\lim_{t\rightarrow\infty}\frac{\sigma^2}{\theta} = 0$, so the model approaches isotrophy for large cosmic time t . The condition for homogeneity and isotropization , formulated by Collins and Hawking [34] , are satisfied in present model .

B) If we assume $= Ka^{-\alpha}$ $(\alpha > 0)$, then we get a Spherically Symmetric model of universe filled with perfect fluid are not satisfying the barotrophic equation of state $p = \gamma\rho$. For this model energy density, pressure and guage function are given by

$$\rho = \frac{56l^2}{9(nlt+c_1)^2} - \frac{8l^2(12m^2-6m+1)}{9(nlt+c_1)^2} + \frac{3}{(nlt+c_1)^{\frac{8(1-m)}{3n}}} - \frac{3K^2}{4}(nlt+c_1)^{\frac{-2\alpha}{n}} \tag{45}$$

$$p = \frac{(27n-52)l^2}{9(nlt+c_1)^2} - \frac{8l^2(12m^2-6m+1)}{9(nlt+c_1)^2} - \frac{3}{2(nlt+c_1)^{\frac{8(1-m)}{3n}}} - \frac{3K^2}{4}(nlt+c_1)^{\frac{-2\alpha}{n}} \tag{46}$$

$$\beta = K(nlt+c_1)^{\frac{-\alpha}{n}} \tag{47}$$

For Large Cosmic time t, the density , pressure and the guage function becomes zero.

Case 2 When n = 0 and $a_1 = v^m$, where m be any constant number , then

$$a_1(t) = {c_2}^{4m} e^{(4ml)t} \tag{48}$$

$$a_2(t) = {c_2}^{\frac{4(1-m)}{3}} e^{\frac{4l(1-m)t}{3}} \tag{49}$$

The directional Hubble's parameter H_0 *and* H_1 have values given by

$$H_0 = 4ml \tag{50}$$

$$H_1 = \frac{4(1-m)l}{3} \tag{51}$$

By using Eq.(4) , the average generalized , Hubble's parameter H has the value given by

$$H = l \tag{52}$$

With the Eqs.(14), (15) , (16) , the dynamical scalars are given by

$$\theta = 4l \tag{53}$$

$$\sigma^2 = \frac{8l^2}{9}(12m^2 - 6m + 1) \tag{54}$$

$$A = \frac{1}{3}(16m^2 - 8m + 1) \tag{55}$$

From Eqs. (41), (48) and (49), the scalar curvature R for Spherically Symmetric model has given by

$$R = \frac{32l^2}{3}(2m^2 - m + 2) + 6c_2^{\frac{-8(1-m)}{3}} e^{\frac{-8l(1-m)t}{3}} \tag{56}$$

Here we discuss two possibilities,

A) If we assume that the energy density ρ and pressure p of the matter satisfy the equation of state $= \gamma\rho$, $0 \leq \gamma \leq 1$, then

$$\rho = \frac{12l^2}{(1-\gamma)} + \frac{9e^{\frac{-8l(1-m)t}{3}}}{2(1-\gamma)c_2^{\frac{8(1-m)}{3}}} \tag{57}$$

$$p = \frac{12\gamma l^2}{(1-\gamma)} + \frac{9\gamma e^{\frac{-8l(1-m)t}{3}}}{2(1-\gamma)c_2^{\frac{8(1-m)}{3}}} \tag{58}$$

By using Eqs. (12) , (27) and (28) , We get

$$\beta^2 = \frac{16(13+14\gamma)l^2}{27(\gamma-1)} - \frac{32l^2}{27}(12m^2 - 6m + 1) - \frac{2(1+2\gamma)}{(1-\gamma)} \frac{e^{\frac{-8l(1-m)t}{3}}}{c_2^{\frac{8(1-m)}{3}}} \tag{59}$$

Here we observe that, there have no-solution for stiff matter $(\gamma = 1)$.

For large cosmic time t , the spatial volume , expansion parameter , Shear Scalar and mean anisotropic parameter tend to zero . The energy density, the pressure and the guage function β tend to zero for large cosmic time t .

Here $\lim_{t\to\infty} \frac{\sigma^2}{\theta} = 0$, so the model approaches isotrophy for large cosmic time t . The condition for homogeneity and isotropization , formulated by Collins and Hawkings [34] are satisfied in present model .

B) If we assume $\beta = Ka^{-\alpha}$ $(\alpha > 0)$, then we get a Spherically Symmetric model of universe filled with perfect fluid are not satisfying the barotrophic equation of state $p = \gamma\rho$. For this model energy density , pressure and guage function are given by

$$\rho = \frac{56l^2}{9} - \frac{8l^2}{9}(12m^2 - 6m + 1) - \frac{3}{4}k^2 {c_2}^{-2\alpha} e^{-2\alpha lt} + \frac{3e^{\frac{-8l(1-m)t}{3}}}{c_2^{\frac{8(1-m)}{3}}} \tag{60}$$

$$p = \left(\frac{27n-52}{9}\right) l^2 - \frac{8l^2}{9}(12m^2 - 6m + 1) - \frac{3}{4}k^2 {c_2}^{-2\alpha} e^{-2\alpha lt} - \frac{3e^{\frac{-8l(1-m)t}{3}}}{2c_2^{\frac{8(1-m)}{3}}} \tag{61}$$

$$\beta = k {c_2}^{-\alpha} e^{-\alpha lt} \tag{62}$$

For Large Cosmic time t, the density, pressure and the gauge function becomes zero.

3. Conclusion

We have discussed two types of solutions of the average scale factor for a Spherically Symmetric model by using a variation law of Hubble's parameter, which yields constant value of the deceleration parameter. We have discussed in detail the two forms of solutions of average scale factor, one is power law of and the other one is exponential form. In power law solutions, the Spatial volume, expansion parameter, Shear scalar and mean anisotropic parameter tend to zero for large cosmic time t . In exponential form solutions, the energy density, the pressure and the gauge function β also tend to zero for large cosmic time t.

Received October 5, 2016; Accepted October 22, 2016

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