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Five-dimensional FRW Modified Holographic Ricci Dark Energy Cosmological Models with Hybrid Expansion Law in a Scalar-Tensor Theory of Gravitation

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Abstract

In this paper, we consider a five dimensional FRW space-time filled with dark matter and modified holographic Ricci dark energy in a scalar-tensor theory of gravitation. To solve the field equations of the theory we have used hybrid expansion law which shows a transition from decelerated phase to an accelerated phase in a scalar-tensor cosmology. The EoS parameter, jerk parameter and average density parameter are determined. We have also studied the physical and geometrical properties of the model.

Keywords: FRW model, dark matter, modified Ricci dark energy, scalar - tensor theory.

1. Introduction

In recent years the concepts of dark matter and dark energy have dominated the discussions in modern cosmology .This is because of the fact that the recent scenario of accelerated expansion of the universe (Riess et al. [1]; Perlmutter et al. [2]) is supposed to be driven by new energy with negative pressure known as dark energy. However, dark energy is, still, a cosmological mystery in modern cosmology. Cosmological observations and cosmic microwave background data suggest that the universe is spatially flat and is dominated by dark energy. Wilkinson Microwave Anisotropy Probe (WMAP) measures that dark energy, dark matter and baryonic matter occupies 73%, 23% and 4% respectively, of the energy-mass content of the universe. . In order to explain this accelerated expansion of the universe two different approaches have been advocated: to construct different dark energy candidates and to modify Einstein's theory of gravitation. Some relevant alternatives to Einstein' s theory of gravitation are Brans-Dicke (BD) [3], Saez-Ballester [4] and other scalar-tensor theories of gravitation, $f(R)$ gravity, $f(R,T)$ gravity, $f(T)$ gravity. Cosmological constant with time dependent equation of state $\omega = -1$ is the simplest

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candidate of dark energy. Some other dynamical dark energy models with time dependent equation of state to explain accelerated expansion are quintessence (Steinhardt et al. [5]), Phantom (Caldwell [6]), quintom (Nojiri et al. [7]), tachyon (Padmanabhan [8]), dilaton (Gasperini et al. [9]) with interacting dark energy models like holographic (Setare [10]) and age graphic (Wei and Cai [11]) models.

The problem of dark energy can be studied in a simpler way using holographic dark energy models (Setare [12]; Sheykhi [13]). In recent years, there has been a considerable interest in holographic dark energy models because of the fact that holographic dark energy is an emerging model as a candidate of dark energy constructed by holographic principle (Guberina et al. [14]). Cohen et al. [15], Horova and Minic [16], Thomas [17], Hsu [18], Li [19], Setare [20], Sheyki [13], Setare and Vagenas [21] and Das and Mamon [22] have discussed several aspects of holographic dark energy models both in general relativity and in Brans-Dicke theory of gravitation. It is well known that the holographic principle (Susskind [23]) plays an important role in the black hole and string theory, which is based on the fact in quantum gravity, the entropy of a system scales not with its volume, but with its surface area L^2.

Inspired by the holographic principle, A. Cohen et al. [15] suggested that the vacuum energy density is proportional to the Hubble scale $lH \square H^{-1}$. In this model, both the fine-tuning and coincidence problems can be alleviated, but it cannot explain the cosmic accelerated expansion because the effective equation of state for such vacuum energy is zero. Recently, Li [19] proposed that the future event horizon of the Universe to be used as the characteristic length l. This holographic dark energy model not only presents a reasonable value for dark energy density, but also leads to an acceleration solution for the cosmic expansion. In fact, the choice of the characteristic length l is not the unique for the holographic dark energy model. Gao et al. [24] assumed that the length l is given by the inverse of Ricci scalar curvature, i.e., $1R1^{-\frac{1}{2}}$ which is the so-called holographic Ricci dark energy model. It is argued that this model can solve the coincidence problem entirely. Grand and Oliveros [25] suggested a new holographic Ricci dark energy model with density $\rho_{de} = 3M^2{}_{pl} = \eta H^2 + \varsigma \dot{H}$

Adhav et al. [26], Kumar and Yadav [27], Das and Sultana [28] are some of the authors who have investigated anisotropic Bianchi dark energy and magnetized ghost dark energy models in general relativity. These model play a fundamental role in describing the early stages of evolution of the universe. Also Kiran et al. [29-31], Umadevi and Ramesh [32], Ramesh and Umadevi [33] and Reddy et al. [34, 35] have studied Bianchi type minimally interacting holographic dark energy models in Brans-Dicke and Saez-Ballester theories of gravitation. Das and Sultana [36] have presented a locally rotationally symmetric Bianchi type-II model of the universe filled with dark matter and anisotropic modified holographic Ricci dark energy.

Recently, Raju et al. [37] have discussed five dimensional spherically symmetric minimally interacting holographic dark energy model in Saez-Ballester scalar-tensor theory of gravitation. The above investigations have inspired us to consider modified five dimensional FRW universe in the presence of dark matter and modified Ricci dark energy.

This work is organized as follows: The metric and field equations of Saez-Ballester scalar-tensor theory of gravitation in the presence of dark matter and modified holographic Ricci dark energy are obtained in Sec.2. In sect.3 the field equations are solved to obtain cosmological model. The physical properties of the model are presented in Sect.4. The last section contains some conclusions.

2. Metric and field equations

We consider the five dimensional FRW metric in the form

$$ds^2 = dt^2 - a^2(t)[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)d\psi^2] \tag{1}$$

where $a(t)$ is the scale factor of the universe and k= +1,-1,0 for closed, open and flat models respectively. The non-vanishing components of the Einstein tensor for the metric (1) are

$$G_0^0 = -(6\frac{\dot{a}^2}{a^2} + \frac{6k}{a^2}) \tag{2}$$

$$G_1^1 = G_2^2 = G_3^3 = G_4^4 = 3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2}$$

where an overhead dot indicates ordinary differentiation with respect to t.

We consider the universe filled with matter and modified holographic Ricci dark energy fluid. In this case the field equations of Saez and Ballester [4] for combined scalar-tensor and tensor fields are

$$R_{ij} - \frac{1}{2}g_{ij}R - w\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -(T_{ij} + \overline{T_{ij}}) \tag{3}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n\phi_{;i}^{i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{4}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar, w and n are arbitrary dimensionless constants and $8\pi G = c = 1$ in the relativistic units.

The energy momentum tensor for matter and the holographic dark energy are defined as

$$T_{ij} = \rho_m u_i u_j$$

and $\bar{T}_{ij} = (\rho_\lambda + p_\lambda) u_i u_j - g_{ij} p_\lambda$ (5)

where ρ_m, ρ_λ are the energy densities of matter and the modified holographic Ricci dark energy and p_λ is the pressure of the holographic Ricci dark energy. Here

$$u^i u_i = 1, \quad u^i u_j = 0 \quad , i, j=0,1,2,3,4 \tag{6}$$

Also, the energy conservation equation is

$$(T^{ij} + \bar{T}^{ij})_{;j} = 0 \tag{7}$$

In a commoving coordinate system, the field equations (3), (4) and (7) for the metric (1) with the help of Eqns.(2) ,(5) and (6) can be, explicitly, written as

$$6\frac{\left(\dot{a}\right)^2}{a^2} + \frac{6k}{a^2} - \frac{w}{2}\phi^n \dot{\phi}^2 = \rho_m + \rho_\Lambda \tag{8}$$

$$3\frac{\ddot{a}}{a} + 3\frac{\left(\dot{a}\right)^2}{a^2} + \frac{3k}{a^2} + \frac{w}{2}\phi^n \dot{\phi}^2 = -p_\Lambda \tag{9}$$

$$\frac{\ddot{\phi}}{\phi} + 4\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{n}{2}\frac{\left(\dot{\phi}\right)^2}{\phi^2} = 0 \tag{10}$$

$$\dot{\rho}_m + 4\frac{\dot{a}}{a}\rho_m + \dot{\rho}_\Lambda + 4\frac{\dot{a}}{a}(1+\omega_\Lambda)\rho_\Lambda = 0 \tag{11}$$

where $\omega_\Lambda = \dfrac{p_\Lambda}{\rho_\Lambda}$ (12)

is the EoS parameter of the model.

3. Solutions and the model

The field equations (8) – (11) constitute a set of three independent non-linear differential equations [Eq. (11) being the consequence of the field equations (8)-(10)] in five unknowns $a(t), \rho_m, \rho_\Lambda, p_\Lambda$, and ϕ. Hence to obtain a determinate solution we use the following two conditions:

(i) the hybrid expansion law for the average scale factor $a(t)$, proposed by Akarsu et al. [38], which is a combination of power law and exponential law (Das and Sultana [28])

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)} \tag{13}$$

here α and β are non –negative constants and a_0 and t_0 represent the present value of scale factor and age of the universe respectively. Eq. (13) gives the exponential law when $\alpha = 0$ and the power law when $\beta = 0$.

(ii) Grand and Oliveros [25] suggested a new holographic Ricci dark energy model with density

$$\rho_\Lambda = 3M^2_{pl}(\eta H^2 + \varsigma \dot{H}) \tag{14}$$

Chen and Jing [39] modified this as

$$\rho_\Lambda = 3M^2_{pl}(\eta H^2 + \varsigma \dot{H} + \xi \ddot{H} H^{-1}) \tag{15}$$

We consider, as the second condition, the modified holographic Ricci dark energy density given by Eq. (15)) for $M^2_{pl} = 1$. Here $\eta, \varsigma, \xi,$ are constants.

Using Eq. (13) and (1) the modified holographic Ricci dark energy FRW model can be written as

$$ds^2 = dt^2 - a_0{}^2 \left(\frac{t}{t_0}\right)^{2\alpha} e^{2\beta\left(\frac{t}{t_0}-1\right)} \left[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)d\psi^2\right] \tag{16}$$

with the following physical properties.

4. Physical properties of the model

From Eqs. (16), the spatial volume V of the universe is given by

$$V = a^3(t) = a_0{}^3 \left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)} \tag{17}$$

Using Eq. (13), the Hubble parameter H is given by

$$H = \frac{\dot{a}}{a} = \frac{\alpha}{t} + \frac{\beta}{t_0} \tag{18}$$

The deceleration parameter q is given by

$$q = -1 + \alpha t_0^{\ 2} (\alpha t_0 + \beta t)^{-2} \tag{19}$$

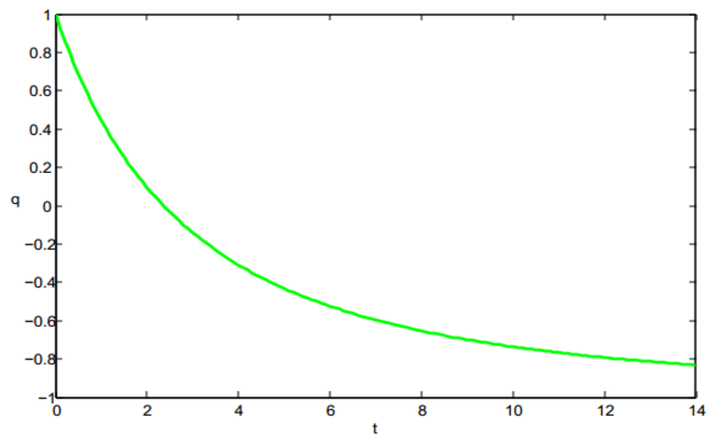

Fig.1: Plot of deceleration parameter versus time for $\alpha = 0.5,\ \beta = 1.2$ and $t_0 = 13.7.$

Figure 1 depicts variation of deceleration parameter (*q*) versus time (*t*) which shows that the model is evolving from early decelerated phase ($q>0$) to present accelerating phase ($q<0$). Recent observations of SNe Ia, expose that the present universe is accelerating and value of deceleration parameter lies in range of $-1 \le q \le 0$. It follows that in our derived model, one can choose values of deceleration parameter consistent with observations.

From Eqs.(18) and (15) we obtain the modified holographic Ricci dark energy density as

$$\rho_\Lambda = \frac{3}{t^2}[\eta t_0^{\ -2}(\alpha t_0 + \beta t)^2 - \alpha \varsigma + 2\alpha \xi t_0 (\alpha t_0 + \beta t)^{-1}] \tag{20}$$

From Eqs.(10) and (13) the Saez-Ballester scalar field φ in the universe is given by

$$\dot{\varphi}\varphi^{\frac{n}{2}} = \varphi_0 a_0^{\ -4} \left(\frac{t}{t_0}\right)^{-4\alpha} e^{-4\beta\left(\frac{t}{t_0}-1\right)} \tag{21}$$

so that we have

$$\phi^{\frac{n+2}{2}} = \left(\frac{n+2}{2}\right)\varphi_0 a_0^{\ -4} \int \left(\frac{t}{t_0}\right)^{-4\alpha} e^{-4\beta\left(\frac{t}{t_0}-1\right)} dt + \psi_0 \tag{22}$$

where φ_0 *and* ψ_0 are constants of integration . Here we set φ_0 =1 and ψ_0 =0.

Using Eqs.(9), (13) and (21) we find the pressure p_Λ of modified holographic Ricci dark energy as

$$p_\Lambda = \frac{3\alpha}{t^2} - 6(\frac{\alpha}{t} + \frac{\beta}{t_0})^2 - \frac{w}{2} a_0^{-8} (\frac{t}{t_0})^{-8\alpha} \exp[-8\beta(\frac{t}{t_0} - 1)] - 3ka_0^{-2}(\frac{t}{t_0})^{-2\alpha} \exp[-2\beta(\frac{t}{t_0} - 1)] \quad (23)$$

From the Eqns.(8), (13),(20) and (21), the energy density of matter ρ_m can be determined as

$$\rho_m = 3(\frac{\alpha}{t} + \frac{\beta}{t_0})^2(2-\eta) + 6ka_0^{-2}(\frac{t}{t_0})^{-2\alpha} \exp[-2\beta(\frac{t}{t_0} - 1)] - \frac{w}{2} a_0^{-8}(\frac{t}{t_0})^{-8\alpha} \exp[-8\beta(\frac{t}{t_0} - 1)] + \frac{3\alpha\varsigma}{t^2} - \frac{6\alpha\xi}{t^3}(\frac{\alpha}{t} + \frac{\beta}{t_0})^{-1} \quad (24)$$

From Eqns. (12), (20) and (22), the EoS parameter takes the form

$$\omega_\Lambda = \frac{\frac{3\alpha}{t^2} - 6(\frac{\alpha}{t} + \frac{\beta}{t_0})^2 - \frac{w}{2} a_0^{-8} (\frac{t}{t_0})^{-8\alpha} \exp[-8\beta(\frac{t}{t_0} - 1)] - 3ka_0^{-2}(\frac{t}{t_0})^{-2\alpha} \exp[-2\beta(\frac{t}{t_0} - 1)]}{\frac{3}{t^2}[\eta t_0^{-2}(\alpha t_0 + \beta t)^2 - \alpha\varsigma + 2\alpha\xi t_0(\alpha t_0 + \beta t)^{-1}]} \quad (25)$$

The behavior of EoS parameter of dark energy in term of cosmic time *t* is shown in Fig. 2. It is observed that for open, flat and closed universes, the EoS parameter of dark energy is a decreasing function of time. For open universe it is always varying in quintessence region and finally approaches to *-1*, whereas for flat and closed models the EoS parameter crosses the phantom divide line (PDL) (i.e., $\omega_\Lambda = -1$) and attains a constant value in phantom region. The current cosmological data from SNIa (Riess et al. [1]; Astier et al. [40]), CMB (Komatsu et al. [41]; MacTavish et al. [42]) and large scale structure (SDSS) (Eisenstein et al. [43]) data rule out that $\omega_\Lambda << -1$, they mildly favor dynamically evolving dark energy crossing the PDL (Rodrigues [44]; Nesseris and Perivolaropoulos [45]). Thus our DE model is in good agreement with recent observations.

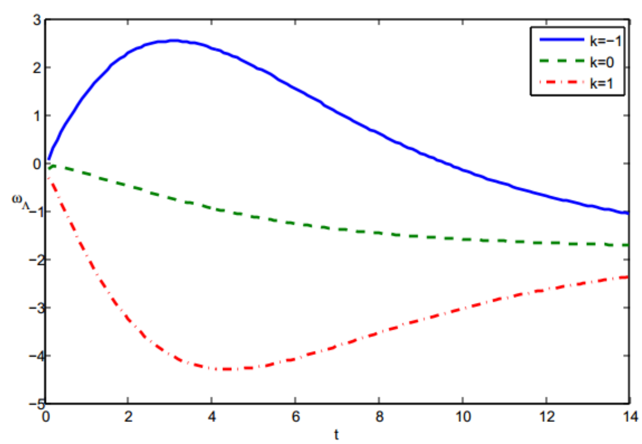

Fig.2: Plot of EoS parameter versus time for $\alpha = 0.5,\ \beta = 1.2, a_0 = 0.1,\ \xi = 0.4, \zeta = 0.5$ and $t_0 = 13.7$..

The average matter energy density of Ω_m and the average modified holographic Ricci dark energy density parameter Ω_Λ are defined as

$$\Omega_m = \frac{\rho_m}{3H^2} \text{ and } \Omega_\Lambda = \frac{\rho_\Lambda}{3H^2} \tag{26}$$

and the total energy density parameter is

$$\Omega = \frac{1}{3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^2}\{3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^2(2-\eta)+6ka_0^{-2}\left(\frac{t}{t_0}\right)^{-2\alpha}\exp[-2\beta\left(\frac{t}{t_0}-1\right)]$$
$$-\frac{w}{2}a_0^{-8}(\frac{t}{t_0})^{-8\alpha}\exp[-8\beta\left(\frac{t}{t_0}-1\right)]+\frac{3\alpha\varsigma}{t^2}-\frac{6\alpha\xi}{t^3}\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{-1}$$
$$+\frac{3}{t^2}[\eta t_0^{-2}(\alpha t_0+\beta t)^2-\alpha\varsigma+2\alpha\xi t_0(\alpha t_0+\beta t)^{-1}]\} \tag{27}$$

Variation of density parameter with cosmic time has been shown in Fig. 3. It is observed that in open, flat and closed universes the total energy density parameter is varying in different phases at the initial epoch, later they approach to constant value. Since our model predicts a flat universe for large times and present-day universe is very close to flat. Hence derived model is also compatible with observational results.

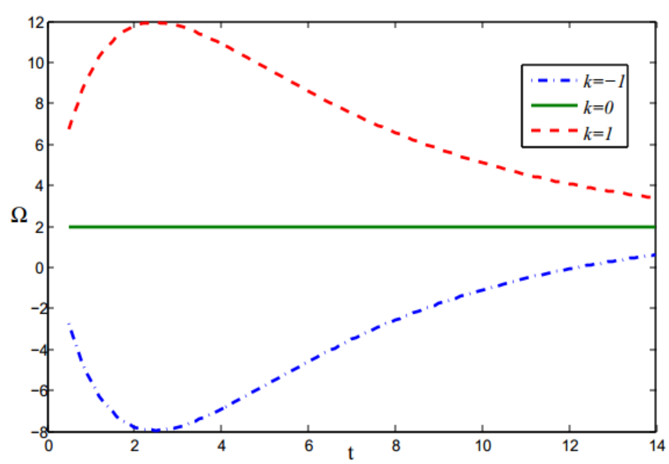

Fig.3: Plot of total average density parameter versus time for $\alpha = 0.5,\ \beta = 1.2, a_0 = 0.1,\ \xi = 0.4, \zeta = 0.5$ and $t_0 = 13.7$.

The cosmic jerk parameter is defined as

$$j(t) = \frac{\dddot{a}}{H^3 a} = q + 2q^2 - \frac{\dot{q}}{H} \tag{28}$$

In cosmology, it is believed that the transition from the decelerating to accelerating phase of the universe is due to a cosmic jerk parameter. This transition of the universe occurs for different models with positive value of the jerk parameter and the negative value of the deceleration parameter (Chiba and Nakamura [46]; Visser [47]). For example, ΛCDM models have a constant jerk.

Using Eq.(18),(19) in (27),we get the expression for the jerk parameter as

$$j(t) = 1 - 3\alpha t_0{}^2 (\alpha t_0 + \beta t)^{-2} + 2(\alpha t + \beta t)^{-4} (\alpha^2 t^4{}_0 + \alpha\beta t t_0{}^3) \tag{29}$$

It may be seen that the jerk parameter has a positive value. Also we have a negative value of deceleration parameter. Hence there will be a smooth transition of the universe from decelerate phase to an accelerated phase at late times which is in good agreement with the modern cosmological observations.

5. Discussion and Conclusions

In this paper, we have discussed five dimensional FRW model filled with dark matter and modified holographic Ricci dark energy in Saez-Ballester [4] scalar-tensor theory of gravitation. To obtain a determinate solution of the field equations we have used hybrid expansion law for the average scale factor. We have also discussed physical properties of the models. It is observed that the spatial volume of the models increase with cosmic time showing the spatial expansion of the universe. The matter energy density, modified Ricci dark energy density, scalar field and the dark energy pressure approach to zero at late times and they all diverse at initial epoch. The Hubble's parameter becomes constant at late times showing a uniform expansion of the universe. It can be seen that there is a smooth transition of the universe from decelerated phase to accelerating phase (see fig. 1). We can observe that (fig. 3) the average density parameter in FRW open, flat and closed models attains constant value at late times i.e., the models are very close to flat space time. The behavior of EoS parameter of dark energy shows that the open model lies in quintessence region while the flat and closed models cross the PDL and lie in phantom region. This exhibits quintom behavior which will help to avoid the big-rip. Thus our models are in good agreement with observations of modern cosmology.

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