Article

On the Calculation of Elementary Particle Masses

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Abstract

With an exponential model including the gravitational constant as a time dependent parameter a mass function is derived where elementary particle masses are combined and related. The proton mass m_p is derived as a function of the electron mass m_e and fine structure constant α as m_p=1,672621·10^{-27} kg (1,672622·10^{-27} kg, Δm/m=6·10^{-7}), the measured mass and the relative deviation of both are in brackets. The neutron mass then is calculated as a function of m_p and α as m_n=1,67492745 ·10^{-27} kg (1,67492747 ·10^{-27} kg, Δm/m≈10^{-8}). The tau mass is expressed as a function of m_p and m_e resulting in m_τ=3,16750·10^{-27} kg (3,16750·10^{-27} kg, Δm/m≈10^{-5}). The results for the neutron and tau are within the estimated standard deviation of the experimental values.

Keywords: Hypothetical particle, physics model , composite models, cosmology.

1. Introduction

The precise prediction and calculation of elementary particle masses still is not covered by any theory, e.g. the standard model. Even though the mechanisms that provide the specific particle masses are thought to be understood and confirmed by the discovery of the Higgs boson, a clue to why elementary particles have their specific mass values would be of great importance for our understanding of quantum objects and matter. A correlated unresolved issue is why and how nature offers such a wide scale of masses from neutrinos to galaxies and the observable universe itsself. This paper attempts to provide an approach to elementary particle masses by constructing an exponential model that covers the mass scale of the observable universe and allows to calculate the proton and neutron masses with an accuracy of 6 resp. 8 decimal digits. As a premise the gravitational constant G is assumed to be a function of time G(t) proportional to 1/t, compatible with Dirac´s conclusions from the Large Number Hypothesis LNH [8]. As a result a set of mass dependend integers is observed and introduced. The relation of particle masses and integer values has already been pointed out by the formula of Koide [5] for lepton masses. It is compared with the result for the tau mass calculated with the appoach of this paper.

2. The Exponential Model

Estimations based on measurements of the mass and radius of the observable universe result in a ratio of these values that about matches the conditions of a black hole. Here the mass relates to the so far observable ordinary baryonic matter, thus not including dark matter nor dark energy. Within a space of a given radius the smallest rest mass greater zero that can be observed corresponds to a Compton wavelength that is of the order of that radius. In this model the observable mass m_U of the universe and the such defined smallest mass m_γ inside are fitted by an

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exponential approach. The largest possible wavelength of a particle with mass m_γ thus is of the order of the radius of the universe, which is proportional to m_U. Assuming that the reduced compton wavelength r_C of m_γ is equal to the gravitational radius r_G of the universe, then

$$r_c = \frac{\hbar}{m_\gamma c} = \frac{Gm_u}{c^2} = r_G \tag{2.0a}$$

$$m_{pl} = \sqrt{\frac{\hbar c}{G}}, \quad r_{pl} = \sqrt{\frac{\hbar G}{c^3}} \quad \rightarrow \quad m_\gamma \cdot m_u = m_{pl}^2 \tag{2.0b}$$

where $m_{pl,}$ r_{pl} are the Planck mass and lenght resp. with the gravitational constant G, Planck´s constant ħ and the speed of light c [1]. The observable horizon of the universe is the Schwarzschildt radius r_S=2r_G. The initial mass of the exponential model is the Planck mass. To obtain an exponential scaling the Planck mass is multiplied by successive factors to obtain first the masses for the stable particles proton resp. electron.

The first step results in the proton mass m_p

$$m_p = m_{pl} \frac{m_p}{m_{pl}}$$

followed by the electron mass m_e

$$m_e = m_{pl} \frac{m_p}{m_{pl}} \frac{m_e}{m_p}$$

and finally the smallest mass m_γ

$$m_\gamma = m_{pl} \frac{m_p}{m_{pl}} \frac{m_e}{m_p} \left(\frac{m_x}{m_{pl}}\right)^n = m_e \left(\frac{m_x}{m_{pl}}\right)^n \tag{2.1a}$$

where n is an integer and m_x a mass to be determined. For m_U it follows with Eq. (2.0b)

$$m_u = \frac{m_{pl}^2}{m_\gamma} \quad \rightarrow \quad m_u = \frac{m_{pl}^2}{m_e} \left(\frac{m_{pl}}{m_x}\right)^n \tag{2.1b}$$

For m_x=m_p and n=2 the result for m_U and the approximate age t_0=r_S/c of the observable universe is

$$m_u = 8{,}805 \cdot 10^{52} kg \;\; (\approx 10^{53} kg), \qquad t_0 = 13{,}822 \cdot 10^9\, a \;\; (13{,}799 \pm 0{,}021 \cdot 10^9\, a)$$

This is in agreement with measurements of the mass and radius resp. the age [7] of the observable universe, where the measured values are in brackets. Now m_U and m_γ are defined as

$$m_\gamma = \frac{m_e m_p^2}{m_{pl}^2\big(G(t)\big)} \tag{2.2a}$$

$$m_u = \frac{m_{pl}^4\big(G(t)\big)}{m_e m_p^2} \tag{2.2b}$$

The model mass m_U is assumed to always fulfil the mass radius relation of Eq. (2.0a). Provided the radius of the universe increases with time, m_U is less for a smaller radius in the past and there is no singularity at an initial radius r_0 that exceeds the mass radius relation of Eq. (2.0a). But with Eq. (2.2b) this implies that the Planck mass cannot be a time independent constant, when assuming that the particle masses m_e and m_p are constant.

Here it is assumed that the gravitational constant G is a parameter G(t) that was larger in the past. Thus the Planck mass $m_{pl}(G(t))$ and $m_U(m_{pl})$ were smaller. The time dependencies of G(t) and $m_U(t)$ are compatible with the conclusions from the LNH, see Appendix C. For $m_U(t)$ beeing smaller in the past, there is eventually a condition reached when it is equal to the smallest observable mass $m_\gamma(t)$, which is referred to as the state of equilibrium. This state is derived and analysed.

3. State of Equilibrium

The state of equilibrium is defined by $m_E=m_\gamma=m_U$. Since the smallest observable mass m_γ cannot exceed the mass of the universe m_U, it is the initial state of the model. In the following m_{pl}, r_{pl} and G_0 are the present values of Planck mass, Planck length and gravitational constant, whereas $m_{pl}(G_x)$ and $r_{pl}(G_x)$ are the values for a specific G_x. With Eq. (2.0b) it follows $m_{pl}(G_E)=m_E$ and then for this state the gravitational radius is equal to the Planck length. Setting Eqs. (2.2a) and (2.2b) equal results in:

$$m_u = m_\gamma \;\rightarrow\; m_{pl}^6(G_E) = \left(m_e m_p^2\right)^2,$$

then

$$m_E = m_{pl}(G_E) = \left(m_e m_p^2\right)^{\frac{1}{3}} \tag{3.0}$$

This result is independent of the definition of the Planck mass in Eq. (2.0b), which may vary depending on it´s derivation. Inserting m_E for m_{pl} into Eqs. (2.2a) and (2.2b) yields

$$m_E = m_{pl}(G_E) = m_u = m_\gamma = \left(m_e m_p^2\right)^{\frac{1}{3}} \tag{3.1}$$

Then with Eq. (2.0b)

$$G_E = \frac{\hbar c}{m_{pl}^2(G_E)} = \frac{\hbar c}{\left(m_e m_p^2\right)^{\frac{2}{3}}} = G_0\frac{m_{pl}^2}{\left(m_e m_p^2\right)^{\frac{2}{3}}} \tag{3.2}$$

and

$$r_E = \sqrt{\frac{\hbar G_E}{c^3}} = \frac{\hbar}{c\left(m_e m_p^2\right)^{\frac{1}{3}}} = r_{pl}\frac{m_{pl}}{\left(m_e m_p^2\right)^{\frac{1}{3}}}$$

The equilibrium values are:

$$G_E = 1{,}694 \cdot 10^{30} \ \frac{m^3}{kg\ s^2}\ ,\ r_E = 2{,}575 \cdot 10^{-15}\ m\ ,\ m_E = 1{,}365929 \cdot 10^{-28}\ kg \qquad (3.3)$$

The equilibrium mass m_E is smaller than the muon, between the electron and proton masses. To roughly estimate the ratio of the strength of gravitational to electromagnetic interaction at equilibrium, the interaction of two masses $m_E/2$ is compared with the interaction of two elementary charges e:

$$\frac{e^2}{4\pi\varepsilon_0 G_E \frac{m_E^2}{4}} = \frac{4e^2}{4\pi\varepsilon_0 \hbar c} = 4\alpha \approx 0{,}03$$

At equilibrium the gravitational and electromagnetic interaction converge and the spatial size of the model mass m_U is with $r_S=2r_E$ of the order of a proton.

4. Composition of the Equilibrium Mass

When m_U is of the order of m_E and thus exhibits the mass and size properties of an elementary particle, the value for G(t) is too large to neglect binding energy effects. It is assumed that within the space of m_U there is a constituent mass m_b as well as it´s self resp. binding energy $m_S c^2$. Futher it is assumed that there is an energy resp. mass contribution m_Q from an elementary charge e distributed over the sphere of m_U. Thus a general approach for the energy resp. mass content of m_U is made. With Eq. (2.2b) it follows:

$$m_u = \frac{m_{pl}^4(G_u)}{m_e m_p^2} = m_b + m_s + m_Q + m_\alpha \qquad (4.0a)$$

where m_α resembles a second order correction term. The binding energy $m_S c^2$ is

$$m_s c^2 = \frac{3}{5} G_u \frac{m_b^2}{r_c(m_b)}\ ,\quad r_c(m_b) = \frac{\hbar}{m_b c}$$

then with Eqs. (2.0a) and (2.0b)

$$m_s = \frac{3}{5}\frac{m_b^3}{m_{pl}^2(G_u)} \qquad (4.0b)$$

and

$$m_Q c^2 = \frac{e^2}{8\pi\varepsilon_0 r_c(m_b)} = \alpha G_u \frac{m_{pl}^2}{2r_c(m_b)} \quad \rightarrow \quad m_Q = \frac{\alpha}{2} m_b\ , \qquad (4.0c)$$

and

$$m_\alpha = m_b \xi \alpha^2\ , \qquad \alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \qquad (4.0d)$$

where ξ is a constant to be determined later. Then m_U is

$$m_u = \frac{m_{pl}^4(G_u)}{m_e m_p^2} = m_b c_\alpha + \frac{3}{5}\frac{m_b^3}{m_{pl}^2(G_u)} \tag{4.1}$$

with

$$c_\alpha = \left(1 + \frac{\alpha}{2} + \xi\alpha^2\right)$$

Rearranging Eq. (4.1) yields

$$m_{pl}^6(G_u) - m_{pl}^2(G_u) m_e m_p^2 m_b c_\alpha = \frac{3}{5} m_e m_p^2 m_b^3 \tag{4.2}$$

The solution of this equation [4] is

$$m_{pl}(G_u) = 3^{-\frac{1}{4}} 10^{-\frac{1}{6}} \left(Y + \frac{10^{\frac{2}{3}} m_e m_p^2 m_b c_\alpha}{Y}\right)^{\frac{1}{2}} \tag{4.3a}$$

where

$$Y = \left(m_e m_p^2 m_b \left(3^{\frac{5}{2}} m_b^2 + \left(243 m_b^4 - 100 m_e m_p^2 m_b c_\alpha^3\right)^{\frac{1}{2}}\right)\right)^{\frac{1}{3}} \tag{4.3b}$$

A lower limit for m_b is defined by the square root within Y, hence

$$243 m_b^4 - 100 m_e m_p^2 m_b c_\alpha^3 = 0$$

resulting in

$$m_b = \left(\frac{100}{243} m_e m_p^2\right)^{\frac{1}{3}} c_\alpha \tag{4.4}$$

Inserting m_b into Eq. (4.3b) yields

$$Y = \left(3^{\frac{5}{2}} \frac{100}{243} \left(m_e m_p^2\right)^2\right)^{\frac{1}{3}} c_\alpha$$

Inserting Y into Eq. (4.3a) is a straightforward calculation and yields

$$m_{pl}(G_u) = \left(2\left(\frac{10}{81}\right)^{\frac{1}{3}} c_\alpha\right)^{\frac{1}{2}} \left(m_e m_p^2\right)^{\frac{1}{3}} \tag{4.5}$$

For the condition of equilibrium (Eq. (3.1)) the Planck mass in Eq. (4.5) is set equal to the equilibrium mass m_E in Eq. (3.0):

$$m_{pl}(G_u) = m_{pl}(G_E) = \left(m_e m_p^2\right)^{\frac{1}{3}}$$

yielding

$$c_\alpha = \frac{1}{2}\left(\frac{10}{81}\right)^{\frac{1}{3}} = 1{,}004149 \qquad (4.6)$$

Inserting this into Eq. (4.4) results in

$$m_b = \left(\frac{5}{12}\right)^{\frac{1}{3}} \left(m_e m_p^2\right)^{\frac{1}{3}} = 1{,}020214 \cdot 10^{-28}\, kg \qquad (4.7a)$$

which is about half the muon mass. Eq. (4.4) is rewritten by defining

$$m_c = m_b(c_\alpha = 1) = \left(\frac{100}{243} m_e m_p^2\right)^{\frac{1}{3}} = 1{,}015998 \cdot 10^{-28}\, kg \qquad (4.7b)$$

resulting in

$$m_b = m_c \left(1 + \frac{\alpha}{2} + \xi\alpha^2\right) \qquad (4.7c)$$

The general approach to particle masses is the equilibrium mass in Eq. (3.0). To solve Eq. (4.7c) for the proton mass m_p the equilibrium mass will be generalized.

5. The Generalized Equilibrium Mass

The equilibrium mass in Eq. (3.0) is utilized for finding dependencies between elementary particle masses. Expressing it in units of the electron mass m_e results in

$$\frac{\left(m_e m_p^2\right)^{\frac{1}{3}}}{m_e} = \left(\frac{m_p}{m_e}\right)^{\frac{2}{3}} = 149{,}947$$

Replacing the proton by the neutron mass results in

$$\frac{\left(m_e m_n^2\right)^{\frac{1}{3}}}{m_e} = \left(\frac{m_n}{m_e}\right)^{\frac{2}{3}} = 150{,}085$$

Thus both values group around the integer value 150. The relation of particle masses and integer values have already been pointed out by other authors such as Koide [5] with his pure empirical though astonishing precise and profound formula for lepton masses (see Appendix B). To evaluate whether the integer value for the proton resp. neutron mass is a random result, the electron resp. proton mass in Eq. (3.0) are replaced by other elementary particle masses. Here the lepton and meson masses smaller than the proton, the proton p, neutron n and the tau τ are applied, since this will be sufficient for the derivation of the proton mass. The elementary particles smaller than the proton are the electron e, muon μ, pions π0+, kaons k+0, the eta η, rho ρ, omega ω and K^*+0, see Appendix A and [1,2]. The equilibrium mass is generalized as

$$m'(i,j) = \left(m_i m_j^2\right)^{\frac{1}{3}} \tag{5.0a}$$

Every particle combination i, j is assigned a mass m´. The results for setting j=n and j=p resp. then relate the proton, neutron, pion, kaon, ω and η masses, where the measured values are in brackets:

$$m_{K+} = (m_{\pi 0} m_n^2)^{\frac{1}{3}} \ = 8{,}772 \cdot 10^{-28}\,kg \quad (8{,}801 \cdot 10^{-28}\,kg) \tag{5.0b}$$
$$m_{K0} = (m_{\pi +} m_n^2)^{\frac{1}{3}} \ = 8{,}871 \cdot 10^{-28}\,kg \quad (8{,}871 \cdot 10^{-28}\,kg)$$
$$m_{\omega} = (m_{\eta} m_p^2)^{\frac{1}{3}} \ = 1{,}398 \cdot 10^{-28}\,kg \quad (1{,}395 \cdot 10^{-28}\,kg)$$

It is notable that the kaons and the adjacent pions are related pairwise and that ω is the equilibrium mass of the adjacent η with j=p. This approach is now investigated for i=e. Every elementary particle j is assigned a mass m´(e,j) and a mass number N(j), which is the ratio of m´(e,j) and the electron mass.

$$N(j) = \frac{\left(m_e m_j^2\right)^{\frac{1}{3}}}{m_e} = \left(\frac{m_j}{m_e}\right)^{\frac{2}{3}} \tag{5.1}$$

The results are shown in Fig. 1. where the μ, pion π+, the symmetric grouping of the kaons, K^*+ and p, n build up an integer scheme. Thus the particles values N(j) are supposed to group around a scheme of integers and are assigned these integers:

N(μ) = 35 (34,967)
N(π0+) = 42 (41,168 and 42,097)
N(K+0) = 98 (97,727 and 98,246)
N(η) = 105 (104,753)
N(ρ,ω) = 133 (132,374 and 132,871)
N(K^*+0) = 145 (144,939 and 145,389)
N(p,n) = 150 (149,947 and 150,085

Fig.1. N(j) for elementary particles from electron to neutron

Every mass m´(e,j) in the range N(j)=35...133 is located at an integer value N(j)=7n.

$$N(j) = 7n\,, \quad n = 5{,}6{,}14{,}15{,}19 \tag{5.2}$$

The proton is assigned N(p)=150 i.e. a mass m´(e,p) which is equal to the equilibrium mass m_E in Eq. (3.0). In addition there is an exponential scaling for particle masses by an factor f_N to be applied in the calculation of the neutron mass. For the range of particles from pion to tau:

$$f_N = (m'(e,\tau)/m'(e,\pi^+))^{\frac{1}{4}} = 1{,}528065 \tag{5.3a}$$

Then within an error of $\approx 10^{-3}$:

N(π0+) f_N = 64,18 = N(g), which is within the gap between π´s and K´s (5.3b)
N(g) f_N = N(K+0)
N(K+0) f_N = N(p,n)
N(p,n) f_N = N(τ) = 229,52

The reason for this exponential behavior can be approached by assigning each particle a generalized equilibrium mass m´(i,e) with the properties

$$M(i) = \frac{(m_i m_e^2)^{\frac{1}{3}}}{m_e} = \left(\frac{m_i}{m_e}\right)^{\frac{1}{3}} = N(i)^{\frac{1}{2}} \tag{5.4}$$

M(π0+) = 6,48074
M(g) = 8,01116
M(K+0) = 9,89949
M(p,n) = 12,2474
M(τ) = 15,1498

Then the following observation can be made:

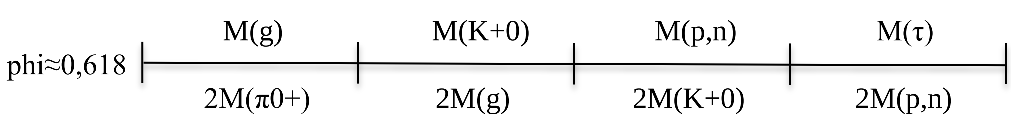

Fig.2. Ratios of M(i)´s for elementary particles from pions to tau

Successive ratios of an M-value and twice the preceeding M-value approach with values between 0,61786 and 0,61859 the golden ratio phi=0,618034 as shown in Fig. 2, which is the perfect ratio of resonances. These perfect proportions already have been found in experiments observing other quantum mechanical systems e.g. quantum phase transitions in atomic chains [3]. Then the ratio of M(i)´s is $f_M \approx 2phi$ and with Eq. (5.4)

$$f_N = f_M^2 \approx (2phi)^2 = 1{,}527864 \tag{5.5}$$

which is consistent with Eq. (5.3a). With Eq. (5.1) particles m´ resp. N ratios are expressed as integer ratios which can be related to each other:

$$\frac{N(i_1)}{N(j_1)} = u\frac{N(i_2)}{N(j_2)} \tag{5.6}$$

where u is a rational resp. integer number. An obvious example for the μ, the π´s and p,n is

$$\frac{150}{35} = u\frac{42}{98}, \qquad u = 10$$

With these approaches and results the proton mass can be calculated as a function of the electron mass.

6. Calculation of the Proton Mass

In Chapter 4 the mass m_b in Eq. (4.7c) has been derived, which is a component of the equilibrium mass and now defined as a mass m´(e,j). Then according to Eqs. (5.1) and (5.2) m_b is assigned a mass number N(b)=7n.

Thus Eq. (4.7c) becomes

$$N(b)m_e = m_c\left(1 + \frac{\alpha}{2} + \xi\alpha^2\right) \tag{6.0}$$

The closest value m´ of a particle to m_b in Eq. (4.7a) is the η with n=15:

$$m_\eta = 9{,}767 \cdot 10^{-28} kg$$

and

$$m'(e,\eta) = 9{,}543 \cdot 10^{-29} kg\,, \quad N(\eta) = 105$$

The next possible mass number is N(b)=7n, n=16, thus

$$N(b) = 112 \tag{6.1}$$

The second order contribution $m_c\xi\alpha^2$ is solved in a seperate approach independently of Eq. (5.0). Considering Eq. (5.1), then Eq. (5.6) is rewritten as

$$\frac{n_1}{n_2} = N(j_2)\frac{N(i_1)}{N(j_1)} = u\frac{m'(e,i_2)}{m_e} \tag{6.2a}$$

where n_1 and n_2 are integers. Since $m_c\xi\alpha^2$ also is assumed to be a mass m´(e,j), the approach is

$$\frac{n_1}{n_2} = u\frac{m_c\xi\alpha^2}{m_e} \tag{6.2b}$$

and

$$\frac{n_1}{n_2} m_e = u m_c \xi \alpha^2 = m_c\, \xi'(u\alpha)^2\,, \qquad \xi' = \frac{\xi}{u} \tag{6.2c}$$

where the right side of the equation is a second order correction term with a constant ξ' to be determined. The number u is assumed to be an integer similar to common fine structure corrections which are of the order $(u\alpha)^2$. With the results of Eq. (5.1), m_e is considered a constituent mass of the generalized equilibrium masses m´(e,j). The left side of Eq. (6.2c) is assumed to be proportional to a self energy contribution of the electron, which is proportional to 3/5 and to m_e. This is evident when replacing m_b by m_e in Eq. (4.0b) and since the Planck mass in the self energy term is proportional to m_e.

Then from Eq. (6.2c) it follows

$$u\xi\alpha^2 = \xi' u^2 \alpha^2 = \frac{3}{5}\frac{m_e}{m_c} \quad\rightarrow\quad \xi' u^2 = \frac{3}{5\alpha^2}\frac{m_e}{m_c} \tag{6.3}$$

Solving Eq. (6.3) by inserting m_c from Eq. (4.7b) results in $\xi' u^2$=101,02. In a first approach the combination with the best fitting integer

$$u = n_3 = 10\,, \quad \xi' = 1{,}0102 \tag{6.4}$$

is utilized to solve Eq. (6.3). The second order term in Eq. (6.0) now is

$$\xi\alpha^2 = \frac{3}{5n_3}\frac{m_e}{m_c} \tag{6.5}$$

which yields

$$N(m_b) m_e = m_c \left(1 + \frac{\alpha}{2} + \frac{3}{5n_3}\frac{m_e}{m_c}\right) \tag{6.6}$$

Inserting N(b) and n_3 yields

$$112 m_e = m_c \left(1 + \frac{\alpha}{2} + \frac{3}{50}\frac{m_e}{m_c}\right)$$

Besides the integers in Eq. (4.4) resulting from the solution of Eq. (4.2) the proton mass depends on two integers N(b)=112 and n_3=10 as a result of the quantisation approach in chapter 5. Solving Eq. (6.6) for m_p with Eq. (4.7b) yields

$$\left(112 - \frac{3}{50}\right) m_e = \left(\frac{100}{243} m_e m_p^2\right)^{\frac{1}{3}} \left(1 + \frac{\alpha}{2}\right)$$

or

$$m_p = \left(\frac{243}{100}\right)^{\frac{1}{2}} \left(2\left(112 - \frac{3}{50}\right)\right)^{\frac{3}{2}} (2+\alpha)^{-\frac{3}{2}} m_e \tag{6.7}$$

and with defining

$$c_m = \left(\frac{243}{100}\right)^{\frac{1}{2}} \left(2\left(112 - \frac{3}{50}\right)\right)^{\frac{3}{2}} = 5{,}2218703 \cdot 10^3$$

then

$$m_p = c_m(2+\alpha)^{-\frac{3}{2}}m_e = 1{,}672621 \cdot 10^{-27}kg \ \ (1{,}672622 \cdot 10^{-27}kg) \tag{6.8}$$

where the measured mass is in brackets, see Appendix A.

7. Calculation of the Neutron Mass

The factor f_N in chapter 5 provides an exponential scaling of particle masses but no precise results as for the proton mass in chapter 6. But the assumption is that this is a function of the mass difference involved, i.e. that for small scales resp. mass differences the precision increases. To verify the splitting of the neutron and proton mass is now approached with the principles previously deployed. With Eqs. (4.0c) and (6.0) it is assumed that the splitting is proportional to α and m_p. Then with Eq. (6.2a) a related approach is

$$\frac{n_1}{n_2}(m_n - m_p) \propto \alpha m_p = f_N^{\frac{3}{2}}\alpha m_p \tag{7.0}$$

where according to Eqs. (5.2) and (5.6) n_1 and n_2 are assumed to be from the set of mass numbers and integers in the previous chapters, thus the number of possible combinations is limited. The factor f_N from Eq. (5.3a) has been adjusted with the correct exponent, since it relates to ratios of m´ and thus of N(j), but masses m are applied here. Then with Eq. (5.1) it follows

$$\frac{m_i}{m_j} = \left(\frac{N(i)}{N(j)}\right)^{\frac{3}{2}} = f_N^{\frac{3}{2}} = 1{,}8889163$$

Considering the previous results, then a supposable combination from the set of integers resp. mass numbers is n_1=n_3 from the solution for the proton mass in Eq. (6.4) and n_2=N(e)=1, resulting in

$$n_3(m_n - m_p) = f_N^{\frac{3}{2}}\alpha m_p \tag{7.1a}$$

yielding a mass formula that with f_N relates the pion, proton, neutron and tau masses

$$n_3 = \frac{f_N^{\frac{3}{2}}\alpha m_p}{(m_n - m_p)} = 9{,}99993 \approx 10 \tag{7.1b}$$

Then solving for the neutron mass and inserting f_N and n_3=10 yields

$$m_n = \left(\frac{f_N^{\frac{3}{2}}\alpha}{n_3} + 1\right)m_p = 1{,}67492745 \cdot 10^{-27}kg \quad (1{,}67492747 \cdot 10^{-27}kg) \tag{7.2}$$

The factor f_N from Eq. (5.3a) is the best fit for Eq. (5.3b). Replacing it by the theoretical value from the observation presented in Fig. 2 and Eq. (5.5) results in

$$m_n = \left(\frac{(2phi)^3\alpha}{n_3} + 1\right) m_p = 1{,}6749270 \cdot 10^{-27} kg \tag{7.3}$$

with an accuracy of seven decimal digits. Solving Eq. (7.2) for m_p yields

$$m_p = \left(\frac{f_N^{\frac{3}{2}}\alpha}{n_3} + 1\right)^{-1} m_n = 1{,}67262191 \cdot 10^{-27} kg \quad (1{,}67262190 \cdot 10^{-27} kg\,) \tag{7.4}$$

The results of Eqs. (7.2) and (7.4) are within the standard deviation of the measured masses, see Appendix A.

8. Calculation of the Tau Mass

The results of Eq. (5.1) shown in Fig. 1. can be written as:

$$m'(e,j) = \left(m_e m_j^2\right)^{\frac{1}{3}} = N(j) m_e \tag{8.0}$$

Here N(J) is the ratio of an equilibrium mass and the electron mass. The ratio of an equilibrium mass and the proton mass then is approached with

$$m'(e,j) = \left(m_e m_j^2\right)^{\frac{1}{3}} = \frac{m_p}{I(j)} \tag{8.1}$$

since $m'(e,j)$ is smaller than m_p in the considered mass range, where I(j) are integers. For $j=\tau$ an accurate result for the tau mass m_τ is obtained:

$$I(\tau) = \frac{m_p}{(m_e m_\tau^2)^{\frac{1}{3}}} = 8{,}00006 \approx 8 \tag{8.2a}$$

or

$$(m_e m_\tau^2)^{\frac{1}{3}} = \frac{1}{8} m_p \tag{8.2b}$$

yielding

$$m_\tau = \left(\frac{\left(\frac{m_p}{8}\right)^3}{m_e}\right)^{\frac{1}{2}} = 3{,}16750 \cdot 10^{-27} kg \quad (3{,}16747 \cdot 10^{-27} kg) \tag{8.2c}$$

Inserting m_p from Eq. (6.8) yields

$$m_\tau = \left(\frac{c_m}{8}\right)^{\frac{3}{2}} (2+\alpha)^{-\frac{9}{4}} m_e = 3{,}16750 \cdot 10^{-27} kg \tag{8.3}$$

which is within the standard deviation of the measured mass in brackets, see Appendix A. For a comparison the formula of Koide in Appendix B is used to calculate the tau mass [4] as a function of the muon and electron mass with the result $m_\tau = 3{,}16773 \cdot 10^{-27}$ kg. The accuracy of both results in principle allows to equate the tau mass in Eq. (8.2c) and in the Koide formula to relate the proton, electron and muon masses.

Discussion

The proton, neutron and tau as well as meson masses in the range probed can be calculated by defining a generalized equilibrium mass as a combination of two particle masses. For particular equilibrium masses which are a function of the electron mass, quantum structures i.e. mass numbers can be observed which allow to relate elementary particle masses. The proton and neutron masses then are a function of the fine structure constant, the electron mass and integers resp. an exponential factor, which does not seem to be accidentally due to the accurate results of six resp. eight decimal digits. The exponential dependencies of mass numbers can be traced back to the golden ratio, but their origin remains unresolved. This result is similar to the empirical formula of Koide (Appendix B), which relates the e, μ, and τ masses with two integers only with the accuracy of four to five decimal digits. Zenczykowski [6] is pointing out that this does not seem to be an ‘accident’ and thus strongly suggests an algebraic origin of mass. For the exponential model of mass scales which is needed for the calculations, the gravitational constant becomes a parameter G(t) that was larger in the past and results in a strength converging with electromagnetic interaction at the proton sized elquilibrium state. The results suggest that the origin of the observed elementary particle masses is linked to fundamental constants, an inherent algebraic structure and that the constancy of the gravitational constant has to be questioned.

Appendix A

Elementary particle masses in kg from [1,2]

$m_e = 9{,}10938356 \cdot 10^{-31}$
$m_\mu = 1{,}8835316 \cdot 10^{-28}$
$m_{\pi 0} = 2{,}40618 \cdot 10^{-28}$
$m_{\pi +} = 2{,}48807 \cdot 10^{-28}$
$m_{K+} = 8{,}8006 \cdot 10^{-28}$
$m_{K0} = 8{,}87078 \cdot 10^{-28}$
$m_{K+} = 8{,}8006 \cdot 10^{-28}$
$m_\eta = 9{,}76653 \cdot 10^{-28}$
$m_{rho} = 1{,}38203 \cdot 10^{-27}$
$m_\omega = 1{,}39520 \cdot 10^{-27}$
$m_p = 1{,}672621898(21) \cdot 10^{-27}$
$m_n = 1{,}674927471(214) \cdot 10^{-27}$
$m_\tau = 3{,}16747(29) \cdot 10^{-27}$

The values in brackets are the estimated standard deviations of the last digits of the measured masses.

Appendix B

The formula of Koide [5]:

$$Q = \frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2} = 0{,}666659(10) \approx \frac{2}{3}$$

The value of Q should be a random number, but it is exactly halfway between 1/3 when all lepton masses would be equal, and 1 when two masses would be neglectible, which suggests an unresolved physical meaning of the formula.

Appendix C

The time dependencies of G and m_u derived by Dirac from the LNH [8], resulting in G proportional 1/t and m_u proportional t^2, are solutions of G(t) and $m_u(G)$ in Eqs. (2.0b) and (2.2b). In the following $m_{pl}(G_0)$ and G_0 are the present values of the Planck mass and gravitational constant, whereas $m_{pl}(G_x)$ is the value for a specific G_x.

When m_U was smaller by a factor x, then with Eq. (2.2b)

$$x m_u = x\frac{m_{pl}^4(G_0)}{m_e m_p^2} = \frac{\left(\frac{\hbar c}{G_0}\sqrt{x}\right)^2}{m_e m_p^2},\ \ x = \frac{m_x}{m_u}$$

Then with Eqs. (2.0a) and (2.0b) it follows

$$m_{pl}(G_x) = \left(\frac{\hbar c}{G_0}\sqrt{x}\right)^{\frac{1}{2}} \ \rightarrow\ G_x = \frac{G_0}{\sqrt{x}} \ \rightarrow\ \frac{G_x}{G_0} = \sqrt{\frac{m_u}{m_x}} = \sqrt{\frac{r_G}{G_0}\frac{G_x}{r_x}}$$

and

$$\left(\frac{G_x}{G_0}\right)^2 = \frac{r_G}{G_0}\frac{G_x}{r_x} \ \rightarrow\ \frac{G_x}{G_0} = \frac{r_G}{r_x} \tag{A1}$$

The solutions for G(t) thus depend on r(t). With Eq. (A1) and the approximation for the ages of the observable universe $t_0=2r_G/c$ and $t_x=2r_x/c$ resp. we get

$$G_x = G_0\frac{t_0}{t_x} \ \rightarrow\ G \propto \frac{1}{t} \tag{A2}$$

Inserting G from Eq. (A2) into Eqs. (2.0b) and (2.2b), then the mass of the universe is:

$$m_u \propto \frac{\left(\frac{\hbar c}{t^{-1}}\right)^2}{m_e m_p^2} \propto t^2$$

These are the time dependencies of the gravitational constant and the mass of the universe concluded from LNH.

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