Article

A Scalar Field Cosmological Model in a Modified Theory of Gravitation

T. Kanakavalli[1], G. Ananda Rao[2] & D. R. K. Reddy[3*]

[1]Department of Mathematics, GITAM University, Visakhapatnam, India
[2]Department of Mathematics, ANITS, Visakhapatnam, India
[3]Department of Applied Mathematics, Andhra University, Visakhapatnam -530 003, India

Abstract

This paper is devoted to the discussion of Bianchi type-III cosmological model in the frame work of *f* (R,T) gravity in the presence of two non-interacting matter sources, scalar field (quintessence or phantom) with scalar potential and matter contribution due to *f* (R,T) action where R is the Ricci scalar and T is the trace of the energy tensor. We have solved the field equations using the fact that scalar expansion is proportional to shear scalar of the space-time and presented the scalar field model in this theory. Finally, we have also discussed some physical and kinematical properties of the model.

Keywords: Scalar field cosmology, *f* (R, T) gravity, Bianchi Type III, modified gravity.

1. Introduction

We live in this universe. Hence it has been necessary to know about the origin, evolution and ultimate fate of the universe. A lot of research is going on, in recent years, to unravel the secrets of the universe. In this process, one of the most remarkable discovery of modern cosmology is that the present day universe is in a state of accelerated expansion (Riess et al. 1998; Perlmutter et al. 1999). The cause and the source of this acceleration is still unknown. It is said that an exotic type of unknown repulsive force termed as 'dark energy' is responsible for the accelerated expansion of the universe. The cosmological constant Λ with equation of parameter $\omega = -1$ is considered to be the simplest candidate for dark energy. However, this leads to a theoretical problem called the "five tuning" problem (Weinberg 1989, Copeland et al. 2006).

Many dark energy models have been put forward to discuss the properties of dark energy. Among these models such as quintessence (normal) with equation of state parameter (EoS) $\omega > -1$ (Barreiro et al. 2000), phantom with EoS $\omega < -1$ (Cald well 2003). tachyon (Bagla et al. 2003; Padmanabhan and Choudhury 2002), k-essence (Armendariz it al 2001), Chaplygin gas (Bento et

*Correspondence: Dr. D. R. K. Reddy, Mathematics Department, MVGR college of Engg.Vizianagaram. India.
E-mail: reddy_einstein@yahoo.com

al. 2002) and so on. However, detection of such exotic fields is not possible. This has led to the search of other possible ways to investigate this crucial problem.

Modification of the geometrical part of the Einstein - Hilbert action is one possible way of exploring the problem of dark energy. During the last decade, there has been several modifications of general relativity to provide natural gravitational alternative to dark energy. Some of the proposed modified theories are f (R), f (T), f (G) and f (R, T) theories of gravity . Here we focus our attention on f (R, T) gravity proposed by Harko et al. (2011). In this theory the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace of the stress energy tensor T. Reconstruction of cosmological models based on f (R,T) gravity has become an important area of investigation Jamil et al. (2012), Houndjo (2012). Houndjo and Piatella (2012), Singh and Singh (2014) are some of the authors who have reconstructed several cosmological models in f (R,T) gravity using different physical sources.

Scalar fields play a vital role in cosmology. In particular, in the study of inflationary cosmology (Guth 1981). The discovery of the accelerated expansion of the universe leads to the search of some kind of matter fields which would generate sufficient negative pressure to drive the late time cosmic acceleration (Caldwell et al. 1998; Sahni 2004; Singh and Singh 2014). Studies with a minimally coupled scalar fields representing quintessence have been carried out by Ellis and Madsen (1991) and Barrow and Saich (1993) while non-minimally coupled scalar fields have been investigated by Barrow and Mimoso (1994) and Mimoso and Wands (1995). Also, reconstruction of cosmological models in the presence of perfect fluid and scalar fields have been studied by Singh and Singh (2011) , Sharif and Jawad(2013), Sharif and Zubair (2012) and Singh and Singh (2014) . Very recently, Singh et al. (2016) have analyzed the behavior of scalar field and cosmological constant in f (R, T) theory of gravity and obtained LRS Bianchi type-I cosmological model filled with two non-interacting matter sources namely scalar field (normal or phantom) with scalar potential and matter distribution.

The above investigations, in scalar field cosmology, have motivated us to study Bianchi type-III cosmological model in the frame work of f (R, T) gravity in the presence of non-interacting matter and scalar fields. This work is planned as follows: Section2 gives a brief review of scalar field cosmology in f (R, T) gravity. In Section 3, we derive the f (R, T) gravity field equations in the presence of non-interacting matter and scalar fields. Section 4 is devoted to the solution of the field equations and the model. Physical discussion of the model obtained is presented in Sect-5. The last section contains summary and conclusions.

2. Review of f (R, T) gravity with scalar field

Gravitational action for f (R,T) gravity in the presence of minimally coupled scalar field of ϕ with self interacting potential $\Psi(\phi)$ is given by

$$S = \frac{1}{2}\int [\, f\,(R,T) + 2\,£_{\phi}]\sqrt{-g}\,d^4x \tag{1}$$

where f (R, T) is an arbitrary function of the Ricci scalar curvature and the trace T of the energy momentum tensor and $£_{\phi}$ represents matter Lagrangian of the scalar field. Here we use the natural system of units 8 π G = c = 1. The matter energy tensor is given by

$$T_{ij} = \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\,£_{\phi})}{\delta g^{ij}} \tag{2}$$

and its trace is $T = g^{ij}T_{ij}$. The matter Lagrangian depends only on the metric tensor g_{ij} and not on its derivatives. Now equation (2) becomes

$$T_{ij} = g_{ij}\,£_{\phi}\text{-}2\frac{\partial £_{\phi}}{\partial g^{ij}} \tag{3}$$

Now variation of the action S with respect to the metric tensor g_{ij}, yields f (R, T) gravity field equations in the form

$$f_R(R,T)R_{ij} - \frac{1}{2}\,f\,(R,T)\,g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)\,f_R(R,T)$$

$$= T_{ij} - f_T(R,T)(T_{ij} + \Theta_{ij}) \tag{4}$$

where f_R and f_T indicate the derivatives of f (R, T) with respect to R and T respectively. Also ∇_i is covariant derivative, $\Box \equiv \nabla_i\,\nabla_j$ is the d'Alembert's operator and Θ_{ij} is defined as

$$\Theta_{ij} = g^{ij}\,\frac{\delta T_{ij}}{\delta g^{ij}} \tag{5}$$

Now using equation (3) in equation (5) we obtain

$$\Theta_{ij} = -2T_{ij} + g_{ij}\,£_{\phi} - 2g^{lk}\,\frac{\partial^2 £\phi}{\partial g^{ij}\partial g^{lk}} \tag{6}$$

Here it may be noted that the f (R, T) gravity field equations depend on the physical nature of the matter source (i.e. on Θ_{ij}). Hence one can generate various forms of f (R, T) defending on the nature of the matter source. Most of the works in f (R, T) gravity are carried out by assuming a number of suitable forms of f (R, T) since it is very difficult to reconstruct a general form of f (R, T), Harko et al. (2011) has considered the following forms for f (R, T).

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)\, f_3(T) \end{cases} \tag{7}$$

Here we consider the simplest particular form

$$f(R,T) = R + 2f(T) \tag{8}$$

In this paper, we consider R as a function of cosmic time *t* and try to present a general form of *f* (R, T) from known scale factors. Now using equation (8) in equation (4) we obtain the field equations of *f* (R, T) gravity with a scalar field as

$$R_{ij} - \frac{1}{2} R g_{ij} = T_{ij} - 2(T_{ij} + \theta_{ij})\, f^{'}(T) + f\left(T\right) g_{ij} \tag{9}$$

where a prime denotes derivative with respect to the argument.

3. Field equations and the model

We consider the spatially homogeneous and anisotropic Bianchi type-III metric in the form.

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{2x} dy^2 - C^2 e^{-2x} dz^2 \tag{10}$$

where A, B, C are functions of cosmic time t only. The energy momentum tensor of a scalar field with self-interacting scalar field potential $\Psi(\phi)$ is given by

$$T_{ij} = \in \phi_{,i}\, \varphi_{,j} - g_{ij} \left(\frac{\in}{2} g^{kl}\, \phi_{,k}\, \varphi_{,l} - \psi(\phi) \right) \tag{11}$$

where $\in = \pm 1$ correspond to normal (quintessence) and phantom scalar fields respectively.

Also, we have

$$T_1^1 = T_2^2 = T_3^3 = -\frac{1}{2} \in \varphi^2 + \psi(\phi) \tag{12}$$

So that the trace $T = g^{ij}\, T_{ij}$ of the energy momentum tensor is

$$T = - \in \varphi^2 + 4\psi(\phi) \tag{13}$$

where an over head dot here and here after denotes derivative with respect to it.

The Lagrangian of the scalar field is

$$£_\phi = -\left(\frac{1}{2}\in\varphi^2 - \psi(\phi)\right) \tag{14}$$

Using equation (14) in Equation (16) we obtain

$$\Theta_{ij} = -2T_{ij} - g_{ij}\left(\frac{1}{2}\in\varphi^2 - \psi(\phi)\right) \tag{15}$$

Now using co-moving coordinates the field equations (9), for the metric (10) with the help of equations (11) - (13) and with the choice (Harkoet al. 2011)

$$f(T) = \lambda T, \tag{16}$$

take the form

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \in\dot{\phi}^2\left(\lambda + \frac{1}{2}\right) - (1+4\lambda)\psi(\phi) \tag{16a}$$

$$\frac{\dot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = \in\dot{\phi}^2\left(\lambda + \frac{1}{2}\right) - (1+4\lambda)\psi(\phi) \tag{17}$$

$$\frac{\dot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} \in\dot{\phi}^2\left(\lambda + \frac{1}{2}\right) - (1+4\lambda)\psi(\phi) \tag{18}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = \in\dot{\phi}^2\left(\lambda + \frac{1}{2}\right) - (1+4\lambda)\psi(\phi) \tag{19}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \tag{20}$$

The following are the dynamical parameters which will be useful to solve the above field equations.

The average scale factor a (t) and the spatial volume are given by

$$V = a^3 = ABC \tag{21}$$

The average Hubble parameter H is

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{22}$$

The scalar expansion θ is

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{23}$$

The shear scalar σ^2 is

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\,\sigma_{ij} = \frac{1}{3}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA}\right] \tag{24}$$

The average anisotropy parameter Δ is

$$\Delta = \frac{1}{2}\sum_{I=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{25}$$

$$\text{where } H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}, H_3 = \frac{\dot{C}}{C} \tag{26}$$

4. Solutions of field equations and the model

In this section, we solve the above field equations (16) - (20) and present a scalar field cosmological model in *f* (R, T) gravity.

Equations (20) immediately yields the solution

$$A = KB \tag{27}$$

where K is a constant of integration which can be set equal to unity without any loss of generality so that we have

$$A = B \tag{28}$$

Using equation (28) the field equations (16) - (19) reduce to

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=\left(1+\frac{\lambda}{2}\right)\in\dot{\phi}^2-(4\lambda+1)\,\psi(\phi) \tag{29}$$

$$2\frac{\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}-\frac{1}{A^2}=\left(1+\frac{\lambda}{2}\right)\in\dot{\phi}^2-(4\lambda+1)\,\psi(\phi) \tag{30}$$

$$\frac{\dot{A}^2}{A^2}+2\frac{\dot{A}}{A}\frac{\dot{C}}{C}-\frac{1}{A^2}=-\left(1+\frac{\lambda}{2}\right)\in\dot{\phi}^2-(4\lambda+1)\,\psi(\phi) \tag{31}$$

The field equations (29) - (31) are a system of three independent equations in four unknowns A, C, ϕ and $\Psi(\phi)$. Hence we need an extra condition to determine an exact solution. Since the scalar expansion θ is proportional to the shear scalar σ^2 we take

$$A=C^n \tag{32}$$

where $n\neq 1$ is a positive constant

Now using equation (32) and solving the field equations (29) -(31) we obtain expressions for metric potentials as

$$A=B=\left[\frac{nt}{\sqrt{n^2-1}}+nk_1\right] \tag{33}$$

$$C=\left[\frac{nt}{\sqrt{n^2-1}}+nk_1\right]^{1/n}$$

where K_1 is an integrating constant. Also the scalar potential and the scalar $\Psi(\phi)$ and the scalar field ϕ are obtained as

$$\psi(\phi)=\frac{2(n+1)}{n^2(1+4\lambda)(t+\sqrt{n^2-1}k_1)^2} \tag{34}$$

$$\phi=\sqrt{\frac{-(2n^2+4n+1)}{\in(1+2\lambda)\,2n^2}}\;\frac{1}{(t+\sqrt{n^2-1}\,k_1)} \tag{35}$$

Here, we may observe that for $\in$ = -1 phantom scalar field only is possible. Now using equation (33) in equation (10) we can write the cosmological model in *f* (R, T) gravity in the form

$$ds^2 = dt^2 - \left[\frac{nt}{\sqrt{n^2-1}} + nk_1 \right]^2 \left(dx^2 + e^{2x} dy^2 \right)$$

$$- \left(\frac{nt}{\sqrt{n^2-1}} + nk_1 \right)^{\frac{2}{n}} dz^2 \qquad (36)$$

where the scalar field and the self interacting scalar field potential in the model is given by the equations (34) and (35).

5. Physical discussion of the model

The scalar field cosmology with the self interacting scalar field potential in *f*(R,T) gravity is now described by equation (33), (34) and (35) with the following physical and kinematical parameters of the model.

The spatial volume is

$$V = \left(\frac{nt}{\sqrt{n^2-1}} + nk_1 \right)^{\frac{2n+1}{n}} \qquad (36)$$

The scale factor of the universe is

$$a(t) = \left(\frac{nt}{\sqrt{n^2-1}} + nk_1 \right)^{\frac{2n+1}{3n}} \qquad (37)$$

The mean Hubble parameter is

$$H = \frac{\dot{a}}{a} = \frac{2n+1}{3n(t+\sqrt{n^2-1}\,k_1)} \qquad (38)$$

The expansion scalar is

$$\theta = 3H = \frac{(2n+1)}{n(t+\sqrt{n^2-1}\,k_1)} \qquad (39)$$

The shear scalar is

$$\sigma^2 = \frac{(n-1)^2}{3n^2\,(t+\sqrt{n^2-1}\,k_1)^2} \tag{40}$$

The average anisotropy parameter is

$$\Delta = \frac{2\,(n-1)^2}{(2n+1)^2} \tag{41}$$

The deceleration parameter is

$$q = \frac{n-1}{2n+1} \tag{42}$$

It may be observed that the space-time (36) is defined only when n $\neq$ 1. The spatial volume of the model (36) increases with time and attains infinite value when t$\to\infty$. It may also be observed also be observed that when t = 0, the Hubble parameter, scalar expansion, shear scalar become constant and when t$\to\infty$, they all vanish. It may be observed that when n = 1, the space - time is not defined and hence the model never becomes isotropic and shear free. Also for 0 < n < 1, q is negative Therefore for this value of n the universe accelerates. For n > 1, it decelerates. In this case we have only phantom scalar field since $\in$ = -1 is the only possibility. The average scale factor in this universe is constant when t = 0 and attains infinite value when t$\to\infty$.

6. Summary and concluding remarks

In this investigation, we have considered the Bianchi type - III space-time in f (R,T) gravity in the presence of matter fields and non-interacting matter fields and a scalar field with self-interacting scalar potential. The field equations of this theory have been solved analytically using the fact that the scalar expansion of the space-time is proportional to the shear scalar. We have presented the scalar field cosmological model using the above solution. We have also discussed the physical and kinematical behavior of the model. It is observed that the model will never attain isotropy throughout the evolution of the universe. It is interesting to note that the universe accelerates. We also observe that we obtain a phantom scalar field model only.

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