Article

Time-dependent Λ in Bianchi Type IX Cosmological Model with Barotropic Perfect Fluid in C-field Theory

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Abstract

Bianchi type IX cosmological model for barotropic fluid distribution in creation field cosmology with varying cosmological term Λ is investigated. To get deterministic solution we assume that $\Lambda=1/R^2$ as considered by Chen and Wu, where R is a scale factor and A = B where A and B are metric potentials. We find that creation field (C) increase with time and $\Lambda \sim 1/t^2$ which matches with the result of HN theory. We have also discussed special cases of model (28) like dust filled universe ($\gamma = 0$), stiff fluid universe ($\gamma = 1$) and radiation dominated universe ($\gamma = 1/3$).

Keywords: Bianchi Type IX, creation field, barotropic perfect fluid, cosmological term.

1. Introduction

The phenomenon of expanding universe, primordial nucleo-synthesis and the observed isotropy of cosmic microwave background radiation (CMBR) are the three very important observations in astronomy. These were successfully explained by big- bang cosmology based on Einstein's field equations. Smoot et al. [1] revealed that the earlier predictions of the Friedman-Robertson-Walker type of models do not always exactly match with our expectations. Some puzzling results regarding the red shifts from the extra galactic objects continue to contradict the theoretical explanations given from the big-bang type of the model.

Also, CMBR discovery did not prove it to be a outcome of big-bang theory. In fact, Narlikar et al. [2] have proved the possibility of non-relic interpretation of CMBR. Also the big bang model is known to have the short comings in the following aspects:

(i) The model has singularity in the past and possibly one in future.

(ii) The conservation of energy is violated in the big-bang model.

(iii) The big-bang models based on reasonable equations of state lead to a very small particle horizon in the early epochs of the universe. This fact gives rise to the 'Horizon problem'.

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(iv) No consistent scenario exists within the frame work of big-bang model that explains the origin, evolution and characteristic of structures in the universe at small scales.

(v) Flatness problem.

So, alternative theories were proposed time to time. Hoyle [3], Bondi and Gold [4] proposed steady state theory in which the universe does not have singular beginning not an end on the cosmic time scale. To overcome this difficulty Hoyle and Narlikar [5] adopted a field theoretic approach by introducing a massless and chargeless scalar field C in the Einstein-Hilbert action to account for the matter creation. In the C-field theory introduced by Hoyle and Narlikar there is nobig-bag type of singularity as in the steady state theory of Bondi and Gold [4].Narlikar [6] has pointed out that, matter creation is accomplished at the expense of negative energy C-field, thus the introduction of a negative energy field may solve horizon and flatness problem faced by big-bang model.

Narlikar and Padmanabhan [7] have obtained the solution of modified Einstein field equation in presence of C-field and they have shown that cosmological model based on this solution is free from singularity, particle horizon and also provides a natural explanation to the flatness problem. The study of Hoyle and Narlikar theory to the space-times with higher dimensions was carried out by Chatterjee and Banerjee [8]. RajBali and Tikekar [9] studied C-field cosmology with variable G in the flat Friedmann-Robertson-Walker model and with non-flat FRW space-time by RajBali and Kumawat [10]. The solutions of Einstein's field equations in the presence of creation field have been obtained for different Bianchi type universes by Singh and Chaubey [11].Bali and Saraf [12, 13] have investigated Bianchi type I dust field universe with decaying vacuum energy in C-field cosmology.

Also, the present day observation of smallness of cosmological constant support to assume that cosmological constant is time dependent. Therefore, the cosmological models linking the variation of cosmological constant having the form of Einstein's field equations unchanged and preserving the energy-momentum tensor of matter content, have been studied by several authors viz. Berman[14], Abdussattar and Vishwakarma[15], Bali and Singh[16], Pradhan et al.[17], Bali and Jain[18], Bali and Tinker[19], Ram and Verma[20]. Tyagi and Singh [21] have investigated time-dependent Λ in C-field theory with LRS Bianchi type III universe and barotropic perfect fluid. LRS Bianchi type V perfect fluid cosmological model in C-field theory with variable Λ is also investigated by Tyagi and Singh [22]. Patil et al. [23] have obtained Bianchi type IX dust filled universe with ideal fluid distribution in creation field.

In this paper, we have investigated Bianchi type IX cosmological model for barotropic fluid distribution in C-field cosmology. For deterministic model, we assumed $\Lambda=1/R^2$, where R is scale factor. We find that creation field (C) increase with time and $\Lambda \sim 1/t^2$.Physical and geometrical parameters of the model are also discussed.

2. The Metric and Field Equation

We have considered Bianchi type IX metric of the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + (B^2 \sin^2 y + A^2 \cos^2 y) dz^2 - 2A^2 \cos y dx dz \qquad \text{... (1)}$$

in which, A and B are functions of t alone. Hoyle and Narlikar [24, 5] modifies the Einstein's field equation by introducing C-field with time dependent cosmological term as:

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G \left[T_i^j{}_{(m)} + T_i^j{}_{(c)} \right] + \Lambda g_i^j \qquad \text{... (2)}$$

The energy-momentum tensor $T_i^j{}_{(m)}$ for perfect fluid and creation field $T_i^j{}_{(c)}$ are given by

$$T_i^j{}_{(m)} = (p+\rho) v_i v^j + p g_i^j \qquad \text{... (3)}$$

$$T_i^j{}_{(c)} = -f \left(c_i c^j - \frac{1}{2} g_i^j c_\alpha c^\alpha \right) \qquad \text{... (4)}$$

where f > 0 is coupling constant between the matter and creation field and $c_i = \frac{dc}{dx^i}$.

The co-moving coordinates are chosen such that $v_i = (0, 0, 0, 1)$.

The non-vanishing components of energy-momentum tensor for matter are given by

$$T_1^1{}_{(m)} = T_2^2{}_{(m)} = T_3^3{}_{(m)} = p;$$
$$T_4^4{}_{(m)} = -\rho \qquad \text{... (5)}$$

The non-vanishing components of energy-momentum tensor for creation field are given by

$$T_1^1{}_{(c)} = T_2^2{}_{(c)} = T_3^3{}_{(c)} = -\frac{1}{2} f\dot{c}^2 \; ; \quad T_4^4{}_{(c)} = \frac{1}{2} f\dot{c}^2 \qquad \text{....(6)}$$

Hence, the Einstein's field equation (2) for the metric (1) and EMT (5) and (6) takes the form

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} + \frac{1}{B^2} - \frac{3A^2}{4B^4} = 8\pi G \left(-p + \frac{1}{2} f\dot{c}^2 \right) + \Lambda \qquad \text{... (7)}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{A^2}{4B^4} = 8\pi G \left(-p + \frac{1}{2} f\dot{c}^2 \right) + \Lambda \qquad \text{... (8)}$$

$$\frac{2A_4B_4}{AB}+\frac{B_4^2}{B^2}+\frac{1}{B^2}-\frac{A^2}{4B^4}=8\pi G\left(\rho-\frac{1}{2}f\dot{c}^2\right)+\Lambda \qquad \text{... (9)}$$

The suffix 4 by the symbols A and B denotes differentiation w.r.t. t.

3. Solution of Field Equations

The conservation equation of energy momentum tensor is

$$\left(8\pi GT_j^i+\Lambda g_j^i\right)_{;i}=0 \qquad \text{... (10)}$$

which leads to

$$8\pi G\left[\dot{\rho}-f\dot{c}\ddot{c}+\{(\rho+p)-f\dot{c}^2\}\left(\frac{A_4}{A}+\frac{2B_4}{B}\right)\right]+\dot{\Lambda}=0 \qquad \text{... (11)}$$

Following Hoyle and Narlikar theory, the source equation of C-field i.e. $c_{;i}^i=n/f$ leads to c = t thus $\dot{c}$= 1.

To get determinate solution of equations (7) - (9), we assume condition between the metric potential i.e.

$$A = B \qquad \text{... (12)}$$

Using (12), equations (7) to (9) becomes

$$\frac{2B_{44}}{B}+\frac{B_4^2}{B^2}+\frac{1}{4B^2}=8\pi G\left(-p+\frac{1}{2}f\dot{c}^2\right)+\Lambda \qquad \text{... (13)}$$

$$\frac{2B_{44}}{B}+\frac{B_4^2}{B^2}+\frac{1}{4B^2}=8\pi G\left(-p+\frac{1}{2}f\dot{c}^2\right)+\Lambda \qquad \text{... (14)}$$

$$\frac{3B_4^2}{B^2}+\frac{3}{4B^2}=8\pi G\left(\rho-\frac{1}{2}f\dot{c}^2\right)+\Lambda \qquad \text{... (15)}$$

The barotropic fluid condition leads to

$$p=\gamma\rho \quad ; \quad \text{where } 0\le\gamma\le 1 \qquad \text{... (16)}$$

Using (16) in equation (14) we have

$$\frac{2B_{44}}{B}+\frac{B_4^2}{B^2}+\frac{1}{4B^2}=8\pi G\left(-\gamma\rho+\frac{1}{2}f\dot{c}^2\right)+\Lambda \quad \text{... (17)}$$

Now equations (15) and (17) together with $\dot{c}=1$ leads to

$$\frac{2B_{44}}{B}+(3\gamma+1)\frac{B_4^2}{B^2}+(3\gamma+1)\frac{1}{4B^2}=4\pi fG(1-\gamma)+\Lambda(1+\gamma) \quad \text{... (18)}$$

To get solution of equation (18) we also assume that

$$\Lambda=\frac{1}{R^2}=\frac{1}{B^2} \quad \text{... (19)}$$

Using equation (19) in equation (18) we have

$$\frac{2B_{44}}{B}+(3\gamma+1)\frac{B_4^2}{B^2}=4\pi fG(1-\gamma)+\frac{1}{4B^2}(\gamma+3) \quad \text{... (20)}$$

Now put B_4 = f (B) which leads to B_{44} = ff'. ... (21)

Now equation (20) with the help of (21) becomes

$$\frac{df^2}{dB}+(3\gamma+1)\frac{f^2}{B}=4\pi fG(1-\gamma)B+(\gamma+3)\frac{1}{4B} \quad \text{... (22)}$$

Equation (22) leads to

$$f^2.B^{(3\gamma+1)}=4\pi Gf\,\frac{(1-\gamma)}{3(\gamma+1)}B^{3\gamma+3}+\frac{1}{4}(\gamma+3)\frac{B^{3\gamma+1}}{(3\gamma+1)} \quad \text{... (23)}$$

which gives

$$f^2=\alpha B^2+\beta \quad \text{... (24)}$$

where $\alpha=\dfrac{4\pi fG(1-\gamma)}{3(1+\gamma)}$, $\beta=\dfrac{1}{4}\dfrac{(\gamma+3)}{(3\gamma+1)}$ ... (25)

Equation (24) leads to

$$\frac{dB}{\sqrt{\alpha B^2+\beta}}=dt \quad \text{... (26)}$$

Hence, equation (26) gives

$$B=\sqrt{\frac{\beta}{\alpha}}\sinh\sqrt{\alpha}t \qquad \text{... (27)}$$

Also the metric (1) reduces to

$$ds^2=-dt^2+\left(\frac{\beta}{\alpha}\sinh^2\sqrt{\alpha t}\right)\left[dx^2+dy^2+\left(\sin^2 y+\cos^2 y\right)dz^2-2\cos y dxdz\right] \qquad \text{... (28)}$$

From equation (19) and (15) we have

$$\Lambda=\frac{\alpha}{\beta}\text{cosech}^2\sqrt{\alpha}t \qquad \text{... (29)}$$

and $$8\pi G\rho=\text{cosech}^2\sqrt{\alpha}t\left(3\alpha-\frac{\alpha}{4\beta}\right)+3\alpha+4\pi Gf \qquad \text{... (30)}$$

Now using p = γρ and equation (12) in equation (11) we have

$$8\pi G\left[\dot{\rho}-f\dot{c}\ddot{c}+\{(1+\gamma)\rho-f\dot{c}^2\}\left(\frac{3B_4}{B}\right)\right]+\dot{\Lambda}=0 \qquad \text{... (31)}$$

Equation (31) leads to

$$4\pi fG\frac{d\dot{c}^2}{dt}+8\pi fG3\sqrt{\alpha}\coth\sqrt{\alpha}t\dot{c}^2$$

$$=\left(3\alpha-\frac{\alpha}{4\beta}\right)2\sqrt{\alpha}\,\text{cos}ech^2\sqrt{\alpha t}\coth\sqrt{\alpha}t+(1+\gamma)9\alpha\sqrt{\alpha}\coth\sqrt{\alpha t}$$

$$+4\pi fG(1+\gamma)3\sqrt{\alpha}\coth\sqrt{\alpha t}+\frac{2\alpha\sqrt{\alpha}}{\beta}\text{cos}ech^2\sqrt{\alpha t}\coth\sqrt{\alpha t}$$

$$-\left(1+\gamma\right)3\sqrt{\alpha}\left(3\alpha-\frac{\alpha}{4\beta}\right)\text{cosech}^2\sqrt{\alpha t}\coth\sqrt{\alpha t} \qquad \text{... (32)}$$

To obtain the solution of equation (32) we assume that $\alpha=1$ and $4\pi fG=3$, so equation (32) leads to

$$\frac{d\dot{c}^2}{dt^2}+6(\coth)\dot{c}^2=6(\coth t) \qquad \text{... (33)}$$

Equation (33) gives

$$\dot{c}^2 = 1 \qquad \text{... (34)}$$

So, we have $\dot{c} = 1$ ... (35)

which agrees with the value used in source equation. Thus, creation field is proportional to time t.

4. Physical and Geometrical Properties

For the model (28), the mass density (ρ) is given by

$$8\pi G\rho = \left(3\alpha - \frac{\alpha}{4\beta}\right)\operatorname{cosech}^2\sqrt{\alpha}t + 3\alpha + 4\pi Gf \qquad \text{... (36)}$$

The scale factor R is

$$R = B = \sqrt{\frac{\beta}{\alpha}}\sinh\sqrt{\alpha}t \qquad \text{... (37)}$$

The cosmological constant (Λ) is

$$\Lambda = \frac{\alpha}{\beta}\operatorname{cosech}^2\sqrt{\alpha}t \qquad \text{... (38)}$$

and the deceleration parameter (q) is

$$q = -\tanh^2\sqrt{\alpha}t \qquad \text{... (39)}$$

5. Special Cases

Case I: Dust Filled Universe (γ = 0)

From equation (26), we have

$$\frac{dB}{\sqrt{B^2 + \frac{3}{4}}} = dt \qquad \text{... (40)}$$

The metric (28) for the dust filled universe is given by

$$ds^2 = -dt^2 + \left(\frac{3}{4}\sinh^2(t+t_0)\right)\left[dx^2 + dy^2 + (\sin^2 y + \cos^2 y)dz^2 - 2\cos y dxdz\right]$$

... (41)

So, the mass density (ρ), scale factor (R), cosmological constant (Λ) and the decelerating parameter (q) for the model (41) are given by

$$8\pi G\rho = \frac{8}{3}\cos ech^2(t+t_0) + 6 \qquad ... (42)$$

$$R = \sqrt{\frac{3}{4}}\sinh(t+t_0) \qquad ... (43)$$

$$\Lambda = \frac{4}{3}\mathrm{cosech}^2(t+t_0) \qquad ... (44)$$

and $q = -\tanh^2(t+t_0)$ ... (45)

Case II: Stiff Fluid University (γ = 1)

From equation (26), we have

$$\frac{dB}{1/2} = dt \qquad ... (46)$$

The metric (28) for stiff fluid universe is given by

$$ds^2 = -dt^2 + \left(\frac{t+t_0}{2}\right)^2\left[dx^2 + dy^2 + (\sin^2 y + \cos^2 y)dz^2 - 2\cos y dxdz\right]$$

... (47)

Also the mass density (ρ), scale factor (R), cosmological constant (Λ) and declaration parameter (q) for the model (47) are given by

$$8\pi G\rho = 3 \qquad ... (48)$$

$$R = \frac{t+t_0}{2} \qquad ... (49)$$

$$\Lambda = \frac{4}{(t+t_0)^2} \quad ...(50)$$

and $q = 0$ ... (51)

Case III: Radiation Dominated Universe (γ = 1/3)

From equation (26), we have

$$\frac{dB}{\sqrt{\frac{1}{2}B^2 + \frac{5}{12}}} = dt \quad ...(52)$$

The metric (28) for radiation dominated universe becomes

$$ds^2 = -dt^2 + \left[\frac{5}{6}\sinh^2\left(\frac{t+t_0}{\sqrt{2}}\right)\right]\left[dx^2 + dy^2 + (\sin^2 y + \cos^2 y)dz^2 - 2\cos y dx dz\right] \quad ...(53)$$

Also, the mass density (ρ), scale factor (R), cosmological constant (Λ) and deceleration parameter (q) for the model (52) are given by

$$8\pi G\rho = \frac{6}{5}\operatorname{cosech}^2\frac{(t+t_0)}{\sqrt{2}} + \frac{9}{2} \quad ...(54)$$

$$R = \frac{\sqrt{5}}{\sqrt{6}}\sinh\left(\frac{t+t_0}{\sqrt{2}}\right) \quad ...(55)$$

$$\Lambda = \frac{6}{5}\operatorname{cosech}^2\left(\frac{t+t_0}{\sqrt{2}}\right) \quad ...(56)$$

and $q = -\tanh^2\frac{(t+t_0)}{\sqrt{2}}$ ---(57)

6. Conclusion

The creation field C increases with time and $\dot{c}$=1 which agrees with the value taken in source equation. The scale factor R for the model (28) increases with time and the cosmological term Λ decreases as time increases. The decelerating parameter from equation (39) i.e. $q < 0$ which represent that universe is accelerating.

Further for all special cases i.e. dust filled universe, stiff fluid universe and radiation dominated universe, scale factor R increases and cosmological term Λ decreases with time. In case of stiff fluid, model has uniform motion (51) and the universe is accelerating for dust filled and radiation dominated cases.

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