Article

Inhomogeneous Bianchi Type I Cosmological Model for Stiff Perfect Fluid Distribution

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Abstract

We have investigated inhomogeneous Bianchi type I cosmological model for stiff perfect fluid distribution. To get the deterministic solution of Einstein's field equations, we assume that the isotropic pressure is equal to energy density i.e. $p = \varepsilon$ .Various physical and geometrical properties of the model are also discussed.

Keywords: Inhomogeneous, Cosmology, Stiff fluid, Bianchi Type I.

1. Introduction

In the recent years there has been a lot of interest in the study of large scale structure of the universe because of the fact that the origin of the structure in the universe is one of the greatest cosmological mysteries even today. Spatially homogeneous and anisotropic cosmological models play a significant role in description of the large scale behavior. The choice of anisotropic model in the Einstein system of field equation permits us to obtain cosmological model more general than FRW models. The Einstein field equations are a coupled system of highly non-linear differential equation and we seek physical solution to the field equation for applications in cosmology and astrophysics.

Stiff fluid cosmological models create more interest in the study of early universe because for these models velocity of sound is equal to the velocity of light so no material in this universe could be stiffer such stiffness is comprehensible at the very high densities just after the big bang.The model with stiff fluid was first studied by Zeldovich [4]. In recent years a large number of models have been proposed in studying the various cosmological properties of stiff fluid. In certain models with self-interacting dark matter components, the self-interaction between the dark matter particles is characterized by the exchange of vector mesons via minimal coupling. In such models the self-interaction energy is shown to behave like a stiff fluid [9].

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Stiff fluid is considered in certaincosmological models based on Horava - lifeshitz gravity. In Horava - lifeshitz gravity theories a “detailedbalancing" condition was imposed as a convenient simplification and the usefulness of this detailed balancing condition was discussed in references [10-12]. The stiff is found to be arised in such models where this detailed balancing condition is relaxed [13-16]. Cosmological models with Stiff fluid, based on Horava-Lifeshits gravity, have been studied in references [17-18]. The existence of stiff fluid has been found as exact non-singular solutions in certain inhomogeneous cosmological models [19-22]. The decrease in the density of stiff fluid in the universe is found to be faster than that of radiation and matter, hence its effect on expansion would be larger in the initial stage of the universe.

Primordial Nucleo- sysnthesis is an event took place in the early phase of the universe, a limit on the density of the stiff fluid can be obtained from big bang nucleo sysnthesis constraint as in reference [23]. Barrow [5] has discussed the relevance of stiff equation of state $\rho=p$ to the matter content of the universe in the early state of evolution of universe. Mohanty et al. [6] have investigated cylindrically symmetric Zel’dovich fluid distribution in General Relativity. Götz [7] obtained a plane symmetric solution of Einstein’s field equation for stiff perfect fluid distribution. Bali and Tyagi [8] have investigated stiff magneto fluid cosmological model in General Relativity. Bali and Tyagi [1,2] also obtained cylindrically symmetric inhomogeneous cosmological model and stiff fluid universe with electromagnetic field in general relativity. Sharma et al. [3] have obtained inhomogeneous Bianchi type VI_0 string cosmological model for stiff fluid distribution.

In this paper we have investigated inhomogeneous Bianchi type I cosmological model for stiff perfect fluid distribution . To get the deterministic solution, we assume that the isotropic pressure is equal to energy density i.e. $p = \varepsilon$ and $B= C^n$ where $C = f(x)\, g(t)$.Various physical and geometrical properties of the model are also discussed.

2. Metric & Solution of Field Equation

We consider the metric in the form

$$ds^2 = dx^2 - dt^2 + B^2 dy^2 + C^2 dz^2 \qquad (1)$$

where metric potentials B and C are both functions of x and t.

The energy-momentum tensor is taken as,

$$T_i^j = (\varepsilon + p)\, v_i v^j + p g_i^j \qquad (2)$$

Where v_i satisfy condition,

$$v_i v^i = -1 \tag{3}$$

Here ε, being the density; p, the pressure; and the v^i flow vector satisfying,

$$g_i^j v^i v^j = -1 \tag{4}$$

In present scenario, the co-moving coordinates are taken as,

$$v^i = (0,0,0,1)$$

The Einstein's field equations,

$$-8\pi T_i^j = R_i^j - \frac{1}{2} R g_i^j$$

for the line-element (1) are given by,

$$8\pi p = -\frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{B_4 C_4}{BC} + \frac{B_1 C_1}{BC} \tag{5}$$

$$8\pi p = \frac{C_{11}}{C} - \frac{C_{44}}{C} \tag{6}$$

$$8\pi p = \frac{B_{11}}{B} - \frac{B_{44}}{B} \tag{7}$$

$$8\pi\varepsilon = \frac{B_4 C_4}{BC} - \frac{B_{11}}{B} - \frac{C_{11}}{C} - \frac{B_1 C_1}{BC} \tag{8}$$

$$\frac{B_{14}}{B} + \frac{C_{14}}{C} = 0 \tag{9}$$

Where the sub-indices 1 and 4 in A, B, C and elsewhere denote ordinary differentiation with respect to x and t respectively. From equations (5) and (8), to find the deterministic solution of line element (1) we assume $p = \varepsilon$ and let $B = C^n$ where $C = f(x)\, g(t)$

$$-\frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{B_4 C_4}{BC} + \frac{B_1 C_1}{BC} = \frac{B_4 C_4}{BC} - \frac{B_{11}}{C} - \frac{C_{11}}{C} - \frac{B_1 C_1}{BC} \tag{10}$$

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4 C_4}{BC} = \frac{2B_1 C_1}{BC} + \frac{B_{11}}{B} + \frac{C_{11}}{C} = k \tag{11}$$

Where k is constant. Equation (11) leads to

$$gg_{44} + ng_4^2 = \alpha g^2 \tag{12}$$

Where $\alpha = \frac{2k}{n+1}$

Integrating equation (12), we obtain

$$g = \alpha_0 \sinh^{\frac{1}{n+1}}(\sqrt{k}t + t_0) \tag{13}$$

where t_0 and α_0 are constants of integration.

Again from equation (11), we have

$$ff_{11} + nf_1^2 = \alpha f^2 \tag{14}$$

Where $\alpha = \frac{2k}{n+1}$

$$f = \beta_0 \sinh^{\frac{1}{n+1}}\left(\sqrt{k}x + x_0\right) \tag{15}$$

where x_0 and β_0 are constants of integration.

Hence, we obtain

$$C = \beta_0 \alpha_0 \sinh^{\frac{1}{n+1}}\left(\sqrt{k}t + t_0\right) \sinh^{\frac{1}{n+1}}\left(\sqrt{k}x + x_0\right) \tag{16}$$

$$B = (\beta_0 \alpha_0)^n \sinh^{\frac{n}{n+1}}\left(\sqrt{k}t + t_0\right) \sinh^{\frac{n}{n+1}}\left(\sqrt{k}x + x_0\right) \tag{17}$$

Therefore after suitable transformation of coordinates, the metric (1) reduces to

$$ds^2 = (dX^2 - dT^2) + (\beta_0 \alpha_0)^{2n} \sinh^{\frac{2n}{n+1}}\left(\sqrt{k}T\right)$$

$$\sinh^{\frac{2n}{n+1}}\left(\sqrt{k}X\right) dY^2 + (\alpha_0 \beta_0)^2 \sinh^{\frac{2}{n+1}}\left(\sqrt{k}T\right) \sinh^{\frac{2}{n+1}}\left(\sqrt{k}X\right) dZ^2 \tag{18}$$

3. Some Physical & Geometrical Features

The physical and geometrical properties of the model are given as follows:

The rotation ω is identically zero i.e.

$\omega = 0$

Pressure p and Energy density ε of the model are given by

$$8\pi p = \frac{kn}{(n+1)^2}\coth^2\sqrt{k}T + \frac{kn}{(n+1)^2}\coth^2\sqrt{k}X - k \tag{19}$$

$$8\pi\varepsilon = \frac{kn}{(n+1)^2}\coth^2\sqrt{k}T + \frac{kn}{(n+1)^2}\coth^2\sqrt{k}X - k \tag{20}$$

The expansion scalar θ of the model is given by

$$\theta = \sqrt{k}\coth\sqrt{k}T \tag{21}$$

The shear scalar σ of the model is given by

$$\sigma^2 = k\coth^2\sqrt{k}T\left[\frac{1}{3} - \frac{n}{(n+1)^2}\right] \tag{22}$$

The deceleration parameter q of the model is given by

$$q = -3\tanh^2\sqrt{k}T + 2 \tag{23}$$

The proper volume V of the model is given by

$V^3 = B\,C$

$$V^3 = \beta_0\alpha_0\sinh^{\frac{1}{n+1}}\left(\sqrt{k}T\right)\sinh^{\frac{1}{n+1}}\left(\sqrt{k}X\right)(\beta_0\alpha_0)^n\sinh^{\frac{n}{n+1}}\left(\sqrt{k}T\right)\sinh^{\frac{n}{n+1}}\left(\sqrt{k}T\right) \tag{24}$$

From equation (21) and (22) , we obtain

$$\frac{\sigma}{\theta} = \left[\frac{1}{3} - \frac{n}{(n+1)^2}\right]^{\frac{1}{2}} = const.$$

4. Conclusion

The model (18) starts with big bang at T = 0 and goes on expanding till T = ∞ when θ becomes zero. It is clear that as T increases, the ratio of the shear scalar σ and expansion θ tends to finite value i.e. $\frac{\sigma}{\theta} \rightarrow \text{constant}.$ Hence the model does not approach isotropy for large value of T. Since the deceleration parameter $q < 0$, hence the model (18) represents an accelerating universe. In general the model represents expanding, shearing and non rotating universe.

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